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AN ALGORITHM FOR ESTIMATING THE EFFECT OF MAINTENANCE ON AGGREGATED COVARIATES WITH APPLICATION TO RAILWAY SWITCH POINT MACHINES

ALGORYTM DO OCENY WPŁYWU KONSERWACJI NA ZAGREGOWANE ZMIENNE TOWARZYSZĄCEI JEGO ZASTOSOWANIE W ODNIESIENIU DO KOLEJOWYCH NAPĘDÓW ZWROTNICOWYCH

We propose an algorithm for estimating the effectiveness of maintenance on both age and health of a system. One of the main contributions is the concept of virtual health of the device. It is assumed that failures follow a nonhomogeneous Poisson process (NHPP) and covariates follow the proportional hazards model (PHM). In particular, the effect of maintenance on device's age is estimated using the Weibull hazard function, while the effect on device's health and covariates associated with condition-based monitoring (CBM) is estimated using the Cox hazard function. We show that the maintenance effect on the health indicator (HI) and the virtual HI can be expressed in terms of the Kalman filter concepts. The HI is calculated from Mahalanobis distance between the current and the baseline condition monitoring data. The effect of maintenance on both age and health is also estimated. The algorithm is applied to the case of railway point machines. Preventive and corrective types of maintenance are modelled as different maintenance effect parameters. Using condition monitoring data, the HI is calculated as a scaled Mahalanobis distance. We derive reliability and likelihood functions and find the least squares estimates (LSE) of all relevant parameters, maintenance effect estimates on time and HI, as well as the remaining useful life (RUL).

Keywords: *virtual health indicator, virtual age, maintenance effectiveness, preventive and corrective maintenance, Cox-Weibull hazard function, proportional hazards model.*

W artykule zaproponowano algorytm służący do szacowania skuteczności utrzymania ruchu w odniesieniu do wieku i stanu technicznego (kondycji) systemu. Główny wkład proponowanej metody stanowi koncepcja wirtualnego stanu urządzenia. Metoda zakłada, że uszkodzenia można zamodelować za pomocą niejednorodnego procesu Poissona, a zmienne towarzyszące za pomocą modelu proporcjonalnego hazardu. Mówiąc precyzyjniej, wpływ konserwacji na wiek urządzenia szacuje się z wykorzystaniem funkcji hazardu Weibulla, natomiast wpływ na stan urządzenia i zmienne towarzyszące związane z monitorowaniem stanu ocenia się stosując funkcję hazardu Coxa. W artykule pokazujemy, że wpływ konserwacji na wskaźnik stanu i wskaźnik stanu wirtualnego można wyrazić w kategoriach filtra Kalmana. Wskaźnik stanu oblicza się na podstawie odległości Mahalanobisa między bieżącymi a początkowymi danymi z monitorowania stanu. Ocenia się także wpływ utrzymania na wiek i kondycję systemu. Proponowany algorytm zastosowano w odniesieniu do napędów zwrotnicowych. Zapobiegawcze i naprawcze typy konserwacji zamodelowano jako różne parametry utrzymania ruchu. Korzystając z danych z monitorowania stanu, obliczono wskaźnik stanu jako skalowaną odległość Mahalanobisa. Wyprowadzono funkcje niezawodności i wiarygodności oraz obliczono metodą najmniejszych kwadratów szacunkowe wielkości wszystkich istotnych parametrów, a także szacunkowy wpływ konserwacji na wskaźniki czasu i stanu technicznego oraz pozostały okres użytkowania (RUL).

Słowa kluczowe: *wirtualny wskaźnik stanu technicznego, wiek wirtualny, skuteczność konserwacji, konserwacja zapobiegawcza i korygująca, funkcja hazardu Coxa–Weibulla, model proporcjonalnego hazardu.*

Notation

Typesetting Convention: vectors, matrices and arrays are indicated by arrows above the letters.

Latin Symbols

C	Cox model identifier.
CM	Corrective maintenance.
E	Expectation.
$\mathcal{L}$	Likelihood function.
L	Lubrication.
M	Maintenance.
PM	Preventive maintenance.
$\mathcal{R}$	Reliability.

TA	Thickness adjustment.
TS	Tightening of screws.
UCL	Upper confidence limit.
W	Weibull model identifier.
$\bar{X}$	Vector of covariates.
d	Mahalanobis distance.
i	Device index.
j	Manoeuvre index.
l	Number of observations in the baseline.
m	Number of covariates.
n	Total number of manoeuvres.
t	Time.
$t_{V\,j}$	Virtual age after j th manoeuvre.
z	Health indicator.
$z_{V\,j}^{+}$	Virtual health indicator after j th manoeuvre.
*	Optimality.

Greek Symbols

Λ	Cumulative hazard function.
α	Confidence level.
β	Shape parameter in Weibull distribution.
η	Scale parameter in Weibull distribution, characteristic life.
θ	Effect of maintenance on the health (as gauged by the health indicator) of the system.
λ	Power-law intensity (hazard) function.
φ	Effect of maintenance on the age of the system.
ω	Length of planning horizon (life cycle).

1. Introduction and background

Maintenance is critical for the longevity, reliability and availability of a vast majority of industrial, consumer and specialised systems and devices. However, a well-known postulate from reliability theory states that maintaining an entity (i.e. anything from the most basic component to a complex system) is justified and is beneficial only if the system displays a certain degradation in its performance with the passage of time. Such a deteriorating behaviour is called “aging”, for the obvious analogy with the biological world. For this reason, in identifying the most effective maintenance, a common criterion for categorising maintenance actions is by effects these have on some general system metric, or parameter, which is usually age. In this regard, a common approach found in the literature on complex maintenance models of various industrial systems divides maintenance actions into four categories: worse repairs (increase the age when applied), minimal repairs (do not change the age when applied, leaving the system in the as-bad-as-old (ABAO) state), imperfect repairs (reduce the age by some factor between 0 and 1) and perfect repairs (effectively reduce the age to 0, amounting to as-good-as-new (AGAN) state) (Pulcini, 2003; Wu & Zuo, 2010). A preventive or corrective maintenance action affects the system’s health state, and the effect of maintenance ranges from minimal (ABAO) to that equivalent to a complete renewal (AGAN). We are interested in measuring the maintenance effect and investigating how it impacts the system’s health indicator (HI). The maintenance effect can range from 0 for AGAN state to 1 for ABAO state of the system.

Because the majority of real-life maintenance actions do not result in either ABAO or AGAN states, it is fair to state that, generally, maintenance actions amount to imperfect repairs (Pham & Wang, 1996), which may be classified into models featuring age reduction (Kijima & Nakagawa, Replacement policies of a shock model with imperfect maintenance, 1992), hazard rate reduction (Chan & Shaw, 1993), combined age-hazard reduction (Zhou, Xi, & Lee, 2007) and other models (Corman, Kraijema, Godjevac, & Lodewijks, 2017; Syamsundar, Muralidharan, & Naikan, General repair models for maintained systems, 2012). However, the age of a machine or even of a component is not always known. As an example, components or subsystems in protective devices, such as batteries in uninterruptible power supplies, may exhibit hidden failures, which are not manifested immediately, therefore making estimation of the age at failure difficult. Alternative methods for finding the optimal maintenance policy have been developed for different arrangements and systems subject to both evident and hidden failures, such as estimating the optimal number of minimal repairs before replacement (Babishin & Taghipour, Optimal maintenance policy for multicomponent systems with periodic and opportunistic inspections and preventive replacements, 2016; Babishin, Hajipoiur, & Taghipour, Optimisation of Non-Periodic Inspection and Maintenance for Multicomponent Systems, 2018; Babishin & Taghipour, Joint Maintenance and Inspection Optimization of a k-out-of-n System, 2016; Babishin & Taghipour, Joint Optimal Maintenance and Inspection for a k-out-of-n System, 2016).

Historically, imperfect repair has been quantified through improvement factors (Malik, 1979), (*p, q*) rule (Brown & Proschan, 1983), virtual age process (Uematsu & Nishida, 1987; Kijima, Some results for repairable systems with general repair, 1989) and superposed renewal process (Kallen, 2011), among others. Of those listed, the virtual age Models I and II due to Kijima assumed general repairs and utilised conditionally-distributed failure times (Kijima, Some results for repairable systems with general repair, 1989). Kijima’s models were subsequently further developed by Dagpunar (Dagpunar, 1998), where functional dependency of the maintenance effect on both the time since previous maintenance action and the previous virtual age was assumed. Fuqing and Kumar (Fuqing & Kumar, 2012) generalised Kijima’s Models I and II from constant to time-dependent repair effectiveness parameter (Fuqing & Kumar, 2012). Using Kijima’s modelling framework, Doyen and Gaudoin classify the effects of maintenance as having a failure intensity-reducing, or an age-reducing effect, also allowing for a Markovian memory property (Doyen & Gaudoin, Classes of imperfect repair models based on reduction of failure intensity or virtual age, 2004). Furthering the framework of Kijima (Kijima, Some results for repairable systems with general repair, 1989) and Doyen and Gaudoin (Doyen & Gaudoin, Classes of imperfect repair models based on reduction of failure intensity or virtual age, 2004), in the present paper, virtual age and virtual health indicator are used, and the effects of maintenance are considered simultaneously on both intensity and age.

Maintenance optimisation in railway-related applications is considered, for example, by Corman et al. (Corman, Kraijema, Godjevac, & Lodewijks, 2017), where they propose a data-driven approach to optimising preventive maintenance of a light rail braking system in terms of reliability, availability and maintenance cost. Based on the data, they model reliability degradation by a Weibull distribution and use sequential optimisation to find optimal preventive maintenance intervals resulting in 30 % cost reduction (Corman, Kraijema, Godjevac, & Lodewijks, 2017). Corman et al. further suggest using multicomponent optimisation to capture complex economic and structural dependence (Corman, Kraijema, Godjevac, & Lodewijks, 2017).

In the context of many repairable systems, “events” can be considered points at which a system changes its state, or exchanges information with its surroundings. Common events include failures, inspections and various kinds of maintenance. Identifying these properly and unambiguously, however, can be challenging, if the effects of such events are not readily observable.

An aspect of interest to the present investigation is the type of maintenance, classified into preventive maintenance (PM) and corrective maintenance (CM). Doyen and Gaudoin proposed a model for each type of PM and CM, each with just one maintenance policy available (Doyen & Gaudoin, Imperfect maintenance in a generalized competing risks framework, 2006). Nasr *et al.* consider failure-point virtual age for CM and repair-point virtual age for PM (Nasr, Gasmi, & Sayadi, 2013). Said and Taghipour further expanded this by considering three maintenance types for PM events and minimal repair for CM events (Said & Taghipour, 2016). They derive the likelihood function for estimating the parameters of the failure process and the effects of preventive maintenance, as well as provide the conditional reliability and the expected number of failures between two consecutive PM types (Said & Taghipour, 2016). Other methods included using feed-forward artificial neural networks (ANN) on condition monitoring data with asset targets’ being asset survival probabilities estimated by Kaplan-Meier (KM) and degradation failure probability density function (PDF) estimator (Heng, et al., 2009).

Reliability and availability of multicomponent systems were obtained, for example, in (Babishin & Taghipour, Optimal maintenance policy for multicomponent systems with periodic and opportunistic inspections and preventive replacements, 2016; Babishin, Hajipoiur, & Taghipour, Optimisation of Non-Periodic Inspection and Maintenance for Multicomponent Systems, 2018). Chen *et al.* use queueing theory to find reliability and availability expressions for a 2-component cold standby system with repairman who may have vacation under Poisson shocks (Chen, Meng, & Chen, 2014). For more complex systems, however, Monte Carlo simulation is widely used, such as in Wang and Cotofana (Wang & Cotofana, 2010), Conn *et al.* (Conn, Deleris, Hosking, & Thorstensen, 2010) and Lim and Lie (Lim & Lie, 2000). Bayesian methods have also been used to estimate the parameters for reliability and maintainability in Nasr *et al.* (Nasr, Gasmi, & Sayadi, 2013), Yu *et al.* (Yu, Song, & Cassady, 2008) and Fuqing and Kumar (Fuqing & Kumar, 2012). In addition, Nasr *et al.* (Nasr, Gasmi, & Sayadi, 2013) derive log-likelihood functions corresponding to failure-point and repair-point virtual age models (Nasr, Gasmi, & Sayadi, 2013). In this paper, both reliability and log-likelihood expressions are provided.

Presently, a large-scale move towards Internet of Things (IoT) is being implemented in various industries. This makes the data from monitoring equipment and sensors ever more ubiquitous and accessible. With this in mind, a question arises as to how to incorporate such operating condition data into the reliability models. One widely-used method is to treat condition monitoring or operating condition data as covariates within the Cox proportional hazard models’ framework (Syamsundar & Naikan, Imperfect repair proportional intensity models for maintained systems, 2011; Cox, 1992; Bendell, Wightman, & Walker, 1991). An obstacle to the universality of such models is that they assume that covariates are time-independent, thus ignoring any influence of changing operating conditions. Previously, accelerated failure time model (AFTM) has been incorporated with virtual age model by Martorell *et al.* (Martorell, Sanchez, & Serradell, 1999). However, combining imperfect repair models with either proportional hazards model or AFTM and considering the effect of covariates is rare, and the attempts found in the literature adopt some simplifying assumptions, such as piecewise-constant operating conditions (Hu, Jiang, & Liao, 2017). Proportional hazards model has also been applied to covariate data for railway maintenance effectiveness estimation in (Babishin & Taghipour, Maintenance Effectiveness Estimation with Applications to Railway Industry, 2019).

Cha and Finkelstein (Cha & Finkelstein, 2016) considered periodic and age-based imperfect PM and minimal repairs in-between (Cha & Finkelstein, 2016). In the present paper, however, neither PM, nor CM events are limited to minimal or perfect repairs, which makes the model more general and widely applicable.

Predicting degradation of a system, machine or device and choosing the best maintenance actions allow preventing or reducing its damage or failure. This is where prognostics and health management (PHM) becomes important. We make use of condition monitoring data, which are observations of different parameters (e.g. temperature, weather, current, voltage). Galar et al. previously proposed feature extraction through data reduction, where only significant data are retained, and irrelevant information is discarded (Galar, Gustafson, Tormos, & Berges, 2012). These observations are aggregated into a health indicator, which represents the system’s condition. Health indicator was used by Kumar et al. for detecting the degradation of electronic products (personal computers) (Kumar, Vichare, Dolev, & Pecht, 2012). Their health indicator represents a weighted sum of the fractional contributions of each bin in a time window (Kumar, Vichare, Dolev, & Pecht, 2012).

In repairable systems, the passage of time, the number of operating cycles and/or the changes in the system’s operating conditions signify deterioration of the system and its approaching failure. This motivates preventive maintenance, which improves the system’s condition and extends its remaining useful life (RUL). RUL is defined as “the expected number of remaining manoeuvres that can be achieved before reaching the failure state” (Letot, et al., 2015).

The main objectives of the present research are to demonstrate an algorithm for quantifying the effectiveness of corrective and preventive maintenance performed on a machine, and to estimate the machine’s degradation rate and remaining useful life, given the maintenance effectiveness.

In the current paper, condition monitoring data are used for estimating the effect of maintenance on both the age of a railway point machine and its covariates. A railway switch, or point machine, is a device for allowing the trains to pass from one railway track onto another one, which makes these devices both necessary and ubiquitous for simultaneous operation of trains in multiple directions. A manoeuvre is a 7-phase sequence of operations performed by components of a point machine (Letot, et al., 2015).

Because of the function point machines perform, they greatly affect the service of rail transportation. This, in turn, affects the safety of passengers, the economic benefits, efficiency and timeliness of train travel. All of these factors can potentially incur huge costs and penalties, including loss of life from accidents, if the system does not perform as expected. For this reason, excessive funds are spent every year on inspection and maintenance of such systems as point machines in order to minimise their failures and to ensure they perform correctly and reliably. For example, the Swedish Rail Administration estimates the costs of railway track maintenance falling under the category of switches and crossings to account for almost 1/3 of the total maintenance costs (Innotrack, 2009). Thus, improving reliability and maintainability in this sector may not only result in the improved safety and lower accident occurrence, but can also bring significant cost reductions to the railroad industry.

Condition monitoring and health management of railway assets, such as point machines, has received some coverage in the literature (Atamuradov, Medjaher, Dersin, Lamoureux, & Zerhouni, 2017; Ardakani, et al., 2012). For example, Ardakani et al. (Ardakani, et al., 2012) use feature extraction techniques and principal component analysis (PCA) as the methods for prognostics and health management for analysing the degradation of electromechanical point machines for railway turnouts. A turnout is a point machine with the switch rails connected to it.

The present article is structured as follows: Section 2 contains the relevant background; Section 3 presents the model; Section 4 contains reliability and likelihood functions; Section 5 illustrates the models by providing numerical examples; lastly, Section 6 summarises the conclusions.

2. Model

2.1. Health indicator calculation

Since a maintenance action can affect the age of a system as well as the condition monitoring data, we investigate both effects. More specifically, we estimate how much reduction in the system's age is caused by a maintenance type, and how the health indicator (which is constructed solely based on the condition monitoring data) is affected by the maintenance action. Health indicator is a measure quantifying the deterioration of the system.

At each operational actuation of the machine, readings from the sensors and diagnostic modules monitoring such parameters, as temperature, humidity, voltage, current, etc. are recorded. Each of the monitoring parameters is designated an index m (e.g. for temperature, $m=1$, for humidity $m=2$, etc.). The ordinal number of an actuation is designated as j and used as a counting index (e.g. for the 2000th actuation of a point machine, j=2000). These are then aggregated to form covariate X_{m_j}. The health indicator, denoted as z_j, is obtained from Mahalanobis distance (MD) calculation as follows:

$$z_j = \frac{\sqrt{\left(\vec{X}_j - \vec{\mu}\right)^T \left[\operatorname{cov}\left(\vec{X}_j\right)\right]^{-1} \left(\vec{X}_j - \vec{\mu}\right)}}{\left(\chi^2\right)^{-1}(0.999999, m)} =$$

$$= \frac{\sqrt{\left[X_{1_j} - \mu_{1_l} \quad \cdots \quad X_{m_j} - \mu_{m_l}\right]\left[\operatorname{cov}\left(\vec{X}_j\right)\right]^{-1} \begin{bmatrix} X_{1_j} - \mu_{1_l} \\ \vdots \\ X_{m_j} - \mu_{m_l} \end{bmatrix}}}{\left(\chi^2\right)^{-1}(0.999999, m)} \quad (1)$$

where j denotes the number of actuations, $\vec{X}_j$ is the vector of m covariates for j th actuation, $\vec{X}_j = \left[X_{1_j} \quad \cdots \quad X_{m_j}\right]$, where $j = 1,2,\ldots,n$, $(\chi^2)^{-1}(0.999999, m)$ is the value of inverse cumulative distribution of the 0.999999th quantile of a chi-squared distribution with m degrees of freedom, which denotes the threshold for the "healthy" values of the HI, $\vec{\mu}$ is the vector of means over l observations, also called "baseline", such that:

$$\vec{\mu} = \left[\mu_{1_l} \quad \cdots \quad \mu_{m_l}\right]$$

$$= \left[\sum_{l=1}^{m} X_{1_l} / m \quad \cdots \quad \sum_{l=1}^{m} X_{m_l} / m\right], \quad (2)$$

Also, $\left[\operatorname{cov}\left(\vec{X}_j\right)\right]^{-1}$ is the inverse of the covariate matrix $\left[\operatorname{cov}\left(\vec{X}_j\right)\right]$, given as:

$$\left[\operatorname{cov}\left(\vec{X}_j\right)\right] =$$

$$= \begin{bmatrix} \operatorname{var}\left(X_{1_j}\right) & \operatorname{cov}\left(X_{1_j}, X_{2_j}\right) & \cdots & \operatorname{cov}\left(X_{1_j}, X_{m_j}\right) \\ \operatorname{cov}\left(X_{2_j}, X_{1_j}\right) & \operatorname{var}\left(X_{2_j}\right) & \cdots & \operatorname{cov}\left(X_{2_j}, X_{m_j}\right) \\ \vdots & \cdots & \ddots & \vdots \\ \operatorname{cov}\left(X_{m_j}, X_{1_j}\right) & \operatorname{cov}\left(X_{m_j}, X_{2_j}\right) & \cdots & \operatorname{var}\left(X_{m_j}\right) \end{bmatrix}. \quad (3)$$

Thus, when HI < 1, the MD is considered to be chi-squared distributed, and the system is "healthy". When HI ≥ 1, the probability that the covariates are normally distributed and their covariances are chi-squared distributed is very small, which suggests that the system is demonstrating "abnormal" behaviour.

In general, the extent to which a machine has moved away from its "baseline", or usual operation, is quantified by the HI. The expectation here is that a large deviation from baseline signals an ongoing degradation of the system and, as a result, increases failure risk. When the health indicator is below the predetermined threshold ($HI < 1$), the system is operating normally. Consequently, defining the alternative event, have $HI \geq 1$, which corresponds to the "failed" operational state of the system.

2.2. Virtual health and the effect of maintenance on the system's health indicator ("Cox model")

When the ratio of the hazards for different treatments does not change with time, proportional hazards models can be used to describe the reliability of the system.

2.2.1. Virtual health indicator algorithm

We consider failures as having a negative effect on the HI. The effect of failures on the HI is modelled using a Cox proportional hazards model, where the hazard function λ_C is given for each machine as:

$$\lambda_C\left(z, \theta_M\right) = \exp\left\{\theta_M \gamma z\right\}, \quad (4)$$

where z is the HI of the machine, γ is the Cox regression coefficient used for scaling the covariates and θ_M is the maintenance effect on machine's HI.

In order to capture the effects of each maintenance type and isolate them from the cumulative effects of maintenance events which have taken place in the past history, the health indicator values (Mahalanobis distances) have to be scaled by the maintenance effect factor (MEF) θ_M after the maintenance events. The virtual health indicator is denoted as $z_{V_j}^{+}$, with "*V*" standing for "virtual" and "+" indicating that it is recalculated after a maintenance event has taken place in order to account for the effect of the most recent maintenance.

The procedure to calculate the maintenance effect factor is as follows.

Given:

$z_i \geq 0, z_0 = 0, z_{Vj}^- = z_j^-, z_{V1}^+ = z_1^+$, $0 \leq \theta_{PM} \leq 1$, $0 \leq \theta_{CM} \leq 1$.

Obtain:

1. Take the HI before the first maintenance event to be z_1^- and after it to be z_1^+.
2. Calculate the first maintenance effectiveness using the following expression:

$$\theta_{M_1} = \frac{z_{V1}^+}{z_1^-} = \frac{z_1^+}{z_1^-} \tag{5}$$

3. Take the HI after the first maintenance and just prior to the second maintenance to be z_2^-.
4. Taking the HI just after the second maintenance z_2^+ from the data for the manoeuvre immediately following the second maintenance event, calculate preliminary estimate of maintenance effect $\hat{\theta}_{M_2}$ as:

$$\hat{\theta}_{M_2} = \frac{z_2^+}{z_2^-} \tag{6}$$

5. Estimate the value of the virtual HI z_{V2}^+ after the second maintenance event using the following formula:

$$z_{V2}^+ = \left(z_2^- - z_1^+\right)\hat{\theta}_{M_2} + z_1^-\theta_{M_1} = \left(z_2^- - z_1^+\right)\hat{\theta}_{M_2} + z_1^+ \tag{7}$$

6. Calculate the new estimate of maintenance effectiveness θ_{M2} using the virtual health indicator as follows:

$$\theta_{M2} = \frac{z_{V2}^+}{z_2^-}. \tag{8}$$

7. Repeat the steps above to calculate new maintenance effectiveness estimates for events 3,4,…,*i* by induction using the following recursive formula for step 5:

$$\begin{aligned} z_{Vj}^+ &= \left(z_j^- - z_{Vj-1}^+\right)\hat{\theta}_{Mj} + z_{Vj-1}^+ = \\ &= \left(z_j^- - z_{Vj-1}^+\right)\hat{\theta}_{Mj} + \left(z_{j-1}^- - z_{Vj-2}^+\right)\hat{\theta}_{Mj-1} + \cdots + \left(z_2^- - z_{V2}^+\right)\hat{\theta}_{M2} + z_1^-\hat{\theta}_{M1} = \\ &= \left(z_j^- - z_{Vj-1}^+\right)\frac{z_j^+}{z_j^-} + \left(z_{j-1}^- - z_{Vj-2}^+\right)\frac{z_{j-1}^+}{z_{j-1}^-} + \cdots + \left(z_2^- - z_{V2}^+\right)\frac{z_2^+}{z_2^-} + z_1^+. \end{aligned} \tag{9}$$

For step 6 of the current procedure, use the following formula:

$$\theta_{Mj} = \frac{z_{Vj}^+}{z_j^-} \tag{10}$$

In order to better visualize the calculation procedure and the formulae, Figure 1 below represents a general case of a deteriorating machine or device subject to imperfect maintenance. In such case, the first maintenance action (denoted as M_1) will result in the virtual HI closest to the baseline, thus representing the largest health-improving effect, followed by the virtual HI for the second maintenance M_2 and so on. Note that the horizontal axis in the figure represents the distance from the baseline (or 0), and not the time progression. In Figure 1, the segment [z_0 ; z_1^+] represents the virtual health $\theta_{M_1}z_1^-$ of the device after the first maintenance action has been performed (i.e. the distance from the baseline to the manoeuvre right after the first maintenance event). The segment [z_1^+ ; z_2^+] represents deterioration of the virtual health $\theta_{M_2}\left(z_2^- - z_1^+\right)$, occurring between the first and the second maintenance events and calculated right after the second maintenance event. The segment [z_0 ; z_2^+] equal in length to the combined segments [z_0 ; z_1^+] and [z_1^+ ; z_2^+] represents the virtual health $\theta_{M2}\left(z_2^- - z_1^+\right) + z_1^+$ after the second maintenance.

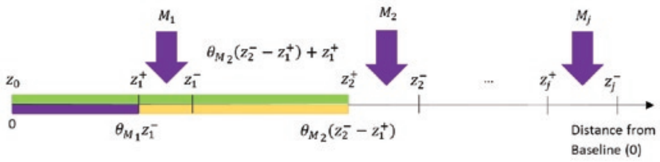

Fig. 1. Visualisation of maintenance events and procedure for estimating their effects

The virtual HI is calculated for each machine using θ_{PM} and θ_{CM} to denote the effect of, respectively, preventive and corrective maintenance on the former as follows:

$$\begin{aligned} &j = 1: \quad z_{V1}^+ = \theta_{M1}z_1^-, \\ &j = 2: \quad z_{V2}^+ = \theta_{M2}z_{V2}^- = \theta_{M2}\left(z_{V1}^+ + (z_2^- - z_1^+)\right) = \theta_{M2}\left(\theta_{M1}z_1^- + (z_2^- - z_1^+)\right), \\ &j = 3: \quad z_{V3}^+ = \theta_{M3}\left(z_{V2}^+ + (z_3^- - z_2^+)\right) = \theta_{M3}\left(\theta_{M2}\left(\theta_{M1}z_1^- + (z_2^- - z_1^+)\right) + (z_3^- - z_2^+)\right), \\ &\quad \vdots \\ &j = n: \quad z_{Vn}^+ = \theta_{Mn}z_{Vn}^- = \theta_{Mn}\left(\theta_{Mn-1}\left(\ldots\left(\theta_{M2}\left(\theta_{M1}z_1^- + (z_2^- - z_1^+)\right) + (z_3^- - z_2^+)\right) + \cdots\right) + (z_n^- - z_{n-1}^+)\right), \\ &\theta_{Mj} = \begin{cases} \theta_{PMj}, \text{if maintenance event } j \text{ is a PM;} \\ \theta_{CMj}, \text{if maintenance event } j \text{ is a CM.} \end{cases} \end{aligned} \tag{11}$$

where z_j^- is the value of HI calculated right before the maintenance action, z_j^+ is the value of HI calculated right after the maintenance action, and superscript M denotes the type of maintenance action.

It can be noted from Eq. 9 that the form of the virtual health indicator estimate is identical to the current state estimate of a Kalman filter [21, 26]:

$$z_{Vj}^+ = \left(z_j^- - z_{Vj-1}^+\right)\hat{\theta}_{Mj} + z_{Vj-1}^+ \quad \text{cf.} \quad Est_t = (Meas - Est_{t-1})K_G + Est_{t-1}, \tag{12}$$

where Est_t is the current estimate of the state, $Meas$ is the initial measurement, Est_{t-1} is the initial estimate of the state, K_G is the Kalman gain, and so have:

$$z_{Vj}^+ = Est_t, \quad z_j^- = Meas, \quad z_{Vj-1}^+ = Est_{t-1}, \quad \hat{\theta}_{Mj} = K_G. \tag{13}$$

Furthermore, maintenance effectiveness estimate $\hat{\theta}_{Mj}$ can be compared to Kalman filter gain using Eq. 6 and identities from Eq. 13, so that:

$$\hat{\theta}_{Mj} = \frac{z_j^+}{z_j^-} = \frac{z_j^+}{Meas} \quad \text{cf.} \quad K_G = \frac{Er_{Est_t}}{Er_{Est_t} + Er_{Meas_t}}, \tag{14}$$

and from $\hat{\theta}_{Mj}$=K_G (Eq. 13) and Eq. 14 it follows that:

$$K_G = \frac{z_j^+}{Meas},$$

$$z_j^+ = K_G \cdot Meas = \frac{Meas \cdot Er_{Est_t}}{Er_{Meas_t} + Er_{Est_t}} \quad (15)$$

where Er_{Est_t} is the error in the estimate of the state and Er_{Meas_t} is the error in the measurement of the state. Thus, the health indicator after a failure or maintenance event can be interpreted using Kalman filter theory as the initial measurement of the state multiplied by Kalman gain. It can also be expressed through the initial measurement of the state multiplied by the error in the current estimate and divided by the total error of the initial measurement and that of the current estimate.

In addition, from Eq. 10 and Eq. 13 have:

$$\theta_{M\,j} = \frac{Est_t}{Meas} \quad (16)$$

Analysing the formulae for the calculation of the maintenance effect factors θ_{PM} and θ_{CM}, it can be seen that:

$$\theta_{M\,j} < 0 \text{ if and only if either :}$$
$$z_{V\,j}^+ < z_j^+ < z_j^-, \text{ or}$$
$$z_j^- < z_{V\,j-1}^+ < z_{V\,j}^+ . \quad (17)$$

Similarly, rewriting Eq. 17 using Kalman filter notation:

$$\theta_{M\,j} < 0 \text{ if and only if either :}$$
$$Est_t < \frac{Meas \cdot Er_{Est_t}}{Er_{Meas_t} + Er_{Est_t}} < Meas, \text{ or} \quad (18)$$
$$Meas < Est_{t-1} < Est_t .$$

Both Eq. 17 and Eq. 18 describe cases in which the system experiences improvement in HI as it ages and which violate the basic characteristics of repairable systems. Thus, $\theta_{M\,j} < 0$ can serve as an indicator that the system experiences "early mortality" and its hazard function is decreasing with the system's age.

2.3. Virtual age and the effect of maintenance on the system's age ("Weibull model")

Whenever a system is subject to degradation with time, the latter is commonly modelled as affecting the system's age. In the context of the present problem, it is assumed that each machine is subject to a nonhomogeneous Poisson process (NHPP) with the time-dependent power law intensity function λ_W of the general form:

$$\lambda_W(t, \varphi_M) = \frac{\beta}{\eta}\left(\frac{\varphi_M t}{\eta}\right)^{\beta-1}, \quad (19)$$

where β is the Weibull shape parameter, η is the Weibull scale parameter, t is the time to failure, φ_M is the maintenance effect on system's age and $M = \begin{cases} PM, \text{if preventive maintenance is performed} \\ CM, \text{if corrective maintenance is performed} \end{cases}$

Assuming that the effect of maintenance on age is cumulative, it is modelled through the concept of virtual age.

2.3.1. Virtual age

Using φ_{PM} and φ_{CM} to denote the effect of, respectively, preventive and corrective maintenance on machine's age, so that $0 \le \varphi_{PM} \le 1$, $0 \le \varphi_{CM} \le 1$, where 0 corresponds to the as-good-as-new (AGAN) state and 1 to the as-bad-as-old (ABAO) state, and designating virtual age for the jth aintenance action as t_{V_j}, obtain:

$$j=1: \begin{cases} t_{V_1}^{PM} = \varphi_{PM}(t_1 - t_0), \text{if current event is a PM;} \\ t_{V_1}^{CM} = \varphi_{CM}(t_1 - t_0), \text{if current event is a CM;} \end{cases}$$

$$j=2: \begin{cases} t_{V_2}^{PM} = \varphi_{PM}\left(t_2 - t_1 + t_{V_1}^{PM}\right), \text{if current event is} \\ \text{a PM and previous event was a PM;} \\ t_{V_2}^{PM} = \varphi_{PM}(t_2 - t_1), \text{if current event is} \\ \text{a PM and previous event was a CM;} \\ t_{V_2}^{CM} = \varphi_{CM}\left(t_2 - t_1 + t_{V_1}^{PM}\right), \text{if current event is} \\ \text{a CM and previous event was a PM;} \\ t_{V_2}^{CM} = \varphi_{CM}(t_2 - t_1), \text{if current event is} \\ \text{a CM and previous event was a CM;} \end{cases} \quad (20)$$

$$j=n: \begin{cases} t_{V_n}^{PM} = \varphi_{PM}\left(t_n - t_{n-1} + t_{V_{n-1}}^{PM}\right), \text{if current event is a} \\ \text{PM and previous event was a PM;} \\ t_n^{PM} = \varphi_{PM}(t_n - t_{n-1}), \text{if current event is a} \\ \text{PM and previous event was a CM;} \\ t_{V_n}^{CM} = \varphi_{CM}\left(t_{n,} - t_{n-1} + t_{V_{n-1}}^{PM}\right), \text{if current event is a} \\ \text{CM and previous event was a PM;} \\ t_n^{CM} = \varphi_{CM}(t_n - t_{n-1}), \text{if current event is a} \\ \text{CM and previous event was a CM;} \end{cases}$$

It can be noted that the value of 0 for the effect of maintenance on the age indicates a complete renewal of the system, and the value of 1 is analogous to the minimal repair.

In the present subsection, a Weibull model for an NHPP failure process has been discussed for identifying the effect of a particular maintenance type on the age of a component or a device. The available condition monitoring data are incorporated into maintenance decision-making trough the Cox proportional hazards model. This is a useful technique for estimating reliability and related metrics.

2.4. Combined (Cox-Weibull) model

Point machines have subassemblies and components that experience age-dependent deterioration (e.g. gearbox) and those that do not (e.g. electronic control and diagnostic module). Thus, the importance of condition-based vs. age-based maintenance estimation techniques depends on the particular component. Moreover, modern monitoring and diagnostic capabilities within the IoT framework provide plenty of condition monitoring data in addition to the age-based data.

In the preceding subsections, two models were discussed: a Cox PHM model, which quantifies the effect of maintenance on the health indicator, and a Weibull model, which identified the effect of a particular maintenance type on age. Thus, in estimating the hazard function for a point machine as a whole, the available data can be taken into consideration by combining the age-based hazard in the form of Weibull hazard function with the condition-based monitoring hazard in the form of Cox proportional hazards model. In the present section, these models are combined to obtain a more powerful model.

In order to improve the sensitivity and applicability of the model, the Cox-Weibull model was enhanced with the maintenance effectiveness estimates multiplicative to the virtual age and virtual health indicator. The model allows to reset the health indicator to the value reflecting the maintenance effectiveness and the system's state by multiplying the health indicator after the specific type of maintenance by the maintenance effect factor for that particular maintenance type. The visualization of the model is given in Figure 2 below.

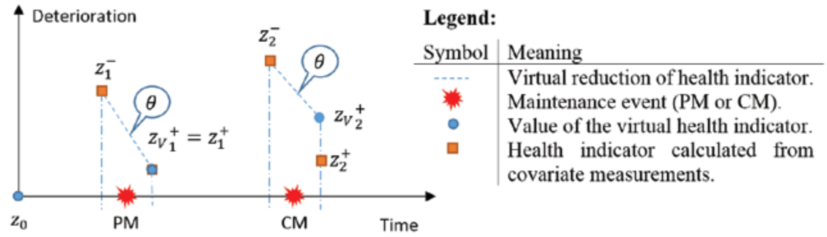

Fig. 2. Visualisation of two sample maintenance events with virtual health indicator

In Figure 2, squares indicate points at which condition monitoring data, or covariates are recorded just before and after a system event (such as failure, or maintenance). Circles represent points at which virtual health indicator is calculated. Following the performance of preventive maintenance (PM) (indicated by an oval callout with θ inside), the device's health is improved and its deterioration is reduced. This reduction is reflected in the changes within the condition monitoring and/or covariate data, which results in a decrease of HI as shown by the square markers. With the use of the device and the passage of time, it keeps deteriorating to failure. At this point, corrective maintenance (CM) is performed, HI is reduced and the device's health is improved. While HI shows a large improvement as represented by square markers, it is not clear how much of a contribution did the most recent maintenance action have compared to the previous maintenance history. Such a reduction in HI is likely due to the cumulative effect of all the previous maintenance actions. However, of interest is the isolated effect of each maintenance type, such as PM and CM, since these most likely happened intermittently in the past operational history.

With this goal, the previously-presented Weibull and Cox models are combined together to improve the sensitivity of the model and to quantify the effects of PM and CM maintenance types on the age and health of the device or system. The hazard function $\lambda(t,z,\varphi_M,\theta_M)$ for the new combined Cox-Weibull model has the following form:

$$\lambda(t,z,\varphi_M,\theta_M)=\frac{\beta}{\eta}\left(\frac{\varphi_M t}{\eta}\right)^{\beta-1}\exp\{\theta_M\gamma z\}=\frac{\beta}{\eta}\left(\frac{t_V^M}{\eta}\right)^{\beta-1}\exp\left\{\gamma z_{Vj}^{+}\right\} \tag{21}$$

where all the terms are as previously described.

The cumulative hazard function is then given as follows:

$$\Lambda\left(t_{V_j}^M\right)=\int_0^t\lambda(t,z,\varphi_M,\theta_M)=\frac{\theta_M\gamma z}{\varphi_M}\left(\frac{\varphi_M t_{V_j}^M}{\eta}\right)^{\beta}. \tag{22}$$

In order to establish the dynamics of the hazard function and to infer whether its form is suitable for a particular case at hand, we take the derivative of $\lambda(t,z,\varphi_M,\theta_M)$ with respect to time as follows:

$$\lambda'(t,z,\varphi_M,\theta_M)=\frac{d}{dt}\left[\frac{\beta}{\eta}\left(\frac{\varphi_M t}{\eta}\right)^{\beta-1}\exp\{\theta_M\gamma z\}\right]=\frac{\beta(\beta-1){\varphi_M}^{\beta-1}t^{\beta-2}}{\eta^{\beta}}\exp\{\theta_M\gamma z\} \tag{23}$$

It should be noted that both maintenance effect indicators φ_M,θ_M satisfy the Markovian property, since they depend only on the preceding state and not the entire evolution of the states up to the present. Thus, they can be treated as time-independent.

Setting the derivative of the hazard function equal to 0, we can find the critical points:

$$\lambda'(t,z,\varphi_M,\theta_M)=0$$
$$\frac{\beta(\beta-1){\varphi_M}^{\beta-1}t^{\beta-2}}{\eta^{\beta}}\exp\{\theta_M\gamma z\}=0. \tag{24}$$

Solving Eq. 24, obtain different cases:

$$\begin{cases}\beta=1:\lambda=\text{const.}\\ \beta=0:\lambda=0\\ \varphi_M=0:\text{purely AGAN maintenance effect}\end{cases} \tag{25}$$

In the case of $\lambda=$ const., failure distribution is an exponential distribution, and there is no benefit from performing any maintenance activities, since failures result not from deterioration, but rather from random events. In the case of $\lambda=0$, the entire hazard function is 0, and the system is not deteriorating. In the case of $\varphi_M=0$, each maintenance is perfect and results in as-good-as-new state, thus being equivalent in effect to replacement.

Using the hazard and cumulative hazard functions as given in Eq. 21 and Eq. 22, reliability and likelihood functions are constructed in order to estimate the optimal parameters of interest.

3. Reliability and likelihood functions

The goal of the present methodology is to estimate simultaneously the parameters β and η of the power law intensity function, as well as the maintenance effectiveness estimates φ_{PM}, φ_{CM}, θ^{PM}, θ^{CM}, and the coefficients of the covariates γ_i. All of these can be aggregated into a vector $\vec{p}$:

$$\vec{p}=(\beta,\eta,\varphi_{PM},\varphi_{CM},\theta_{PM},\theta_{CM},\gamma). \tag{26}$$

First, the reliability function is calculated by taking into account the suspension histories due to preventive maintenance, as well as failures and pseudo failures (i.e. when the health indicator crosses some threshold). Then, the likelihood function of the model is calculated.

3.1. Reliability

Different cases require different reliability function calculations, as shown below. All of the expressions are given for each device *i*.

Case 1: event *j* is a failure, immediately followed by CM:

- Previous event (j-1) is a failure, followed by CM:

$$f\left(t_{V_j}^{CM}-t_{V_{j-1}}^{CM}\middle|t_{V_{j-1}}^{CM},z_{V\,j-1}^{+}\right)= \tag{27}$$
$$=\lambda\left(t_{V_j}^{CM}\right)\exp\left\{-\left(\Lambda\left(t_{V_j}^{CM}\right)-\Lambda\left(t_{V_{j-1}}^{CM}\right)\right)\right\}.$$

• Previous event $(j-1)$ is a PM:

$$f\left(t_{V_j}^{CM}-t_{V_{j-1}}^{PM}\middle|t_{V_{j-1}}^{PM},z_{V\,j-1}^{+}\right)=\lambda\left(t_{V_j}^{CM}\right)\exp\left\{-\left(\Lambda\left(t_{V_j}^{CM}\right)-\Lambda\left(t_{V_{j-1}}^{PM}\right)\right)\right\}$$

$$=\frac{\beta}{\eta}\left(\frac{\varphi_{CM}t_{V_j}^{CM}}{\eta}\right)^{\beta-1}\exp\left\{\theta_{CM}\gamma_i z_i-\left(\frac{\theta^{CM}\gamma_i z_i}{\varphi_{CM}}\left(\frac{\varphi_{CM}t_{V_j}^{CM}}{\eta}\right)^{\beta}-\right.\right.$$

$$\left.\left.-\frac{\theta^{PM}\gamma_i z_i}{\varphi_{PM}}\left(\frac{\varphi_{PM}t_{V_{j-1}}^{PM}}{\eta}\right)^{\beta}\right)\right\}=\frac{\beta}{\eta}\left(\frac{\varphi_{CM}t_{V_j}^{CM}}{\eta}\right)^{\beta-1}\exp\left\{\frac{\gamma_i z_i}{\varphi_{CM}}\left(\theta^{CM}\varphi_{CM}-\right.\right.$$

$$\left.\left.-\theta^{CM}\left(\frac{\varphi_{CM}t_{V_j}^{CM}}{\eta}\right)^{\beta}+\theta^{PM}\left(\frac{\varphi_{PM}t_{V_{j-1}}^{PM}}{\eta}\right)^{\beta}\right)\right\}. \quad (28)$$

Case 2: event j is a PM:

• Previous event $(j-1)$ is a failure, followed by CM:

$$R\left(t_{V_j}^{PM}-t_{V_{j-1}}^{CM}\middle|t_{V_{j-1}}^{CM},z_{V\,j-1}^{+}\right)=\exp\left\{-\left(\Lambda\left(t_{V_j}^{PM}\right)-\Lambda\left(t_{V_{j-1}}^{CM}\right)\right)\right\}$$

$$=\exp\left\{-\gamma_i z_i\left(\frac{\theta_{PM}}{\varphi_{PM}}\left(\frac{\varphi_{PM}t_{V_j}^{PM}}{\eta}\right)^{\beta}-\frac{\theta_{CM}}{\varphi_{CM}}\left(\frac{\varphi_{CM}t_{V_{j-1}}^{CM}}{\eta}\right)^{\beta}\right)\right\}. \quad (29)$$

• Previous event $(j-1)$ is a PM:

$$R\left(t_{V_j}^{PM}-t_{V_{j-1}}^{PM}\middle|t_{V_{j-1}}^{PM},z_{V\,j-1}^{+}\right)=\exp\left\{-\left(\Lambda\left(t_{V_j}^{PM}\right)-\Lambda\left(t_{V_{j-1}}^{PM}\right)\right)\right\}$$

$$=\exp\left\{-\gamma_i z_i\left(\frac{\theta_{PM}}{\varphi_{PM}}\left(\frac{\varphi_{PM}t_{V_j}^{PM}}{\eta}\right)^{\beta}-\frac{\theta_{PM}}{\varphi_{PM}}\left(\frac{\varphi_{PM}t_{V_{j-1}}^{PM}}{\eta}\right)^{\beta}\right)\right\}. \quad (30)$$

3.2. Likelihood function

The likelihood function $\mathcal{L}(\vec{p})$ is calculated from reliability as follows:

$$\mathcal{L}(\vec{p})=\frac{1}{n}\prod_{j=1}^{n}\left(\lambda\left(t_{V_j}^{M}\right)\right)^{\Psi_j}\exp\left\{-\left(\Lambda\left(t_{V_j}^{M}\right)-\Lambda\left(t_{V_{j-1}}^{M}\right)\right)\right\},$$

$$\Psi_j=\begin{cases}1,\text{if event j is a failure / CM.}\\0,\text{if event j is a PM.}\end{cases} \quad (31)$$

The parameter vector $\vec{p}$ is then estimated by the least squares estimation (LSE) method using Levenberg-Marquardt algorithm (Levenberg, 1944; Marquardt, 1963) implemented in MATLAB.

4. Case study

Based on the maintenance logs and procedures, 3 maintenance actions were identified: thickness adjustment, tightening of a screw and lubrication. At each manoeuvre of a point machine, health indicator is calculated according to Equation 1.

Maintenance effectiveness and other parameters are found for 19 point machines from Italy. The naming convention adopted for the present article is as follows: model (“X” or “Y”), material type used for sliding chairs (“I” or “J”), turnout number and machine sequence letter (1st machine activated in a manoeuvre at a particular turnout is designated as “A”, while 2nd machine – as “B”).

Observations spanning June 2015–June 2017 were used, with baseline calculated according to the clients’ rules to obtain HI values. The data were divided into 3 parts: ‘Normal’ and ‘Reverse’, which refer to the operating direction, and ‘Both’ (the latter combining the former two). Separate parameter estimates were calculated for each. The results, separated by direction, are presented further below.

Maintenance effectiveness estimates with their corresponding 95 % lower and upper confidence limits for the normal direction manoeuvres are presented for 19 point machines in Figure 3. Maintenance effectiveness estimates with their corresponding 95 % lower and upper confidence limits for the reverse direction manoeuvres are presented for 19 point machines in Figure 4. Maintenance effectiveness estimates with their corresponding 95 % lower and upper confidence limits for the combined normal and reverse direction manoeuvres are presented for 19 point machines in Figure 5.

As can be seen from Figures 3-5, the largest variation occurs for the CM effect estimate on virtual age, regardless of the direction of manoeuvres. The variation in the maintenance effect estimates is summarized in Table1.

The maximal age-reducing effect in both directions was most effective maintenance action in both directions is PM effect on virtual age, with the spread of only 13 %, closely followed by the PM effect on virtual health (15 %). For both normal and reverse directions, the smallest spread in estimates is encountered for the PM effect on virtual health (13 % and 8 %, respectively). With $\varphi_{CM}\in[0.01;0.03]$, corrective maintenance appears to have a nearly-AGAN effect on the virtual age for point machine XI10A in all directions of operation. With $\theta_{PM}\in[0.90;0.91]$, preventive maintenance appears to have a nearly-ABAO effect on the virtual HI for point machine XI2A.

For both preventive and corrective maintenance, the majority of the point machines experience imperfect maintenance effects between ABAO and AGAN. Corrective maintenance has an effect closer to that of AGAN on the HI for all point machines in all directions. Corrective maintenance has an effect closer to that of AGAN on the virtual age in 8 out of 19 point machines in both directions, 15 out of 19 point machines in normal direction and 15 out of 19 point machines in reverse direction.

4.1. Estimating the remaining useful life (RUL)

The RUL can be calculated as a pdf:

$$f_{RUL}\left(t_c\mid t_{V_{j-1}}^{M},z_{V\,j-1}^{+}\right)=\frac{f\left(t+t_{V_j}^{M}-t_{V_{j-1}}^{M}\middle|t_{V_{j-1}}^{M},z_{V\,j}^{+}\right)}{R\left(t_{V_j}^{M}-t_{V_{j-1}}^{M}\middle|t_{V_{j-1}}^{M},z_{V\,j}^{+}\right)}$$

$$=\lambda\left(t+t_c,z,\varphi_M,\theta_M\mid t_{V_{j-1}}^{M},z_{V\,j-1}^{+}\right)\frac{R\left(t+t_{V_j}^{M}-t_{V_{j-1}}^{M}\middle|t_{V_{j-1}}^{M},z_{V\,j-1}^{+}\right)}{R\left(t_{V_j}^{M}-t_{V_{j-1}}^{M}\middle|t_{V_{j-1}}^{M},z_{V\,j-1}^{+}\right)}, \quad (32)$$

where t is time, T is the lifetime, t_c is the value of RUL random variable $T_c=\left\{t_c:T-t\middle|T\right\rangle t,t_{V_{j-1}}^{M}\}$. The results are shown in Figure 6.

As can be seen from the figure, the predicted RUL is not too far from the actual failure data. The RUL can be predicted without an exact failure threshold based on failure data and condition monitoring (CM) information. The estimated values form a smoother curve than the actual values. This suggests that the estimating procedure is able to smooth the predictions. However, sufficient failure and CM data

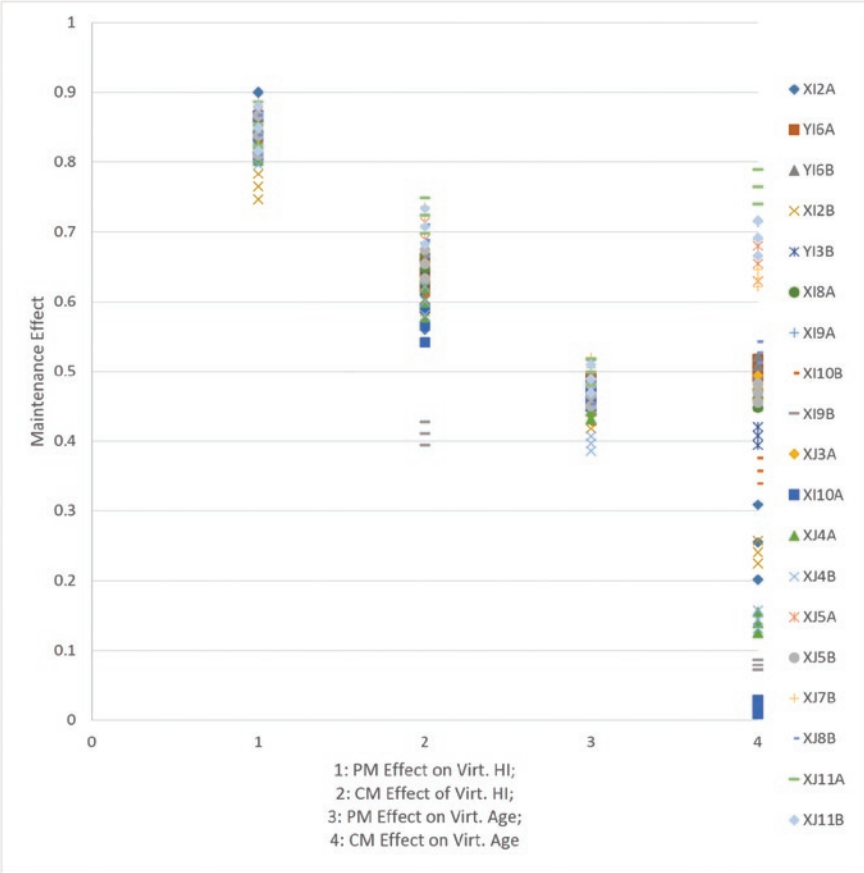

Fig. 3. Maintenance effect on virtual hi and virtual age for point machines in 'normal' direction

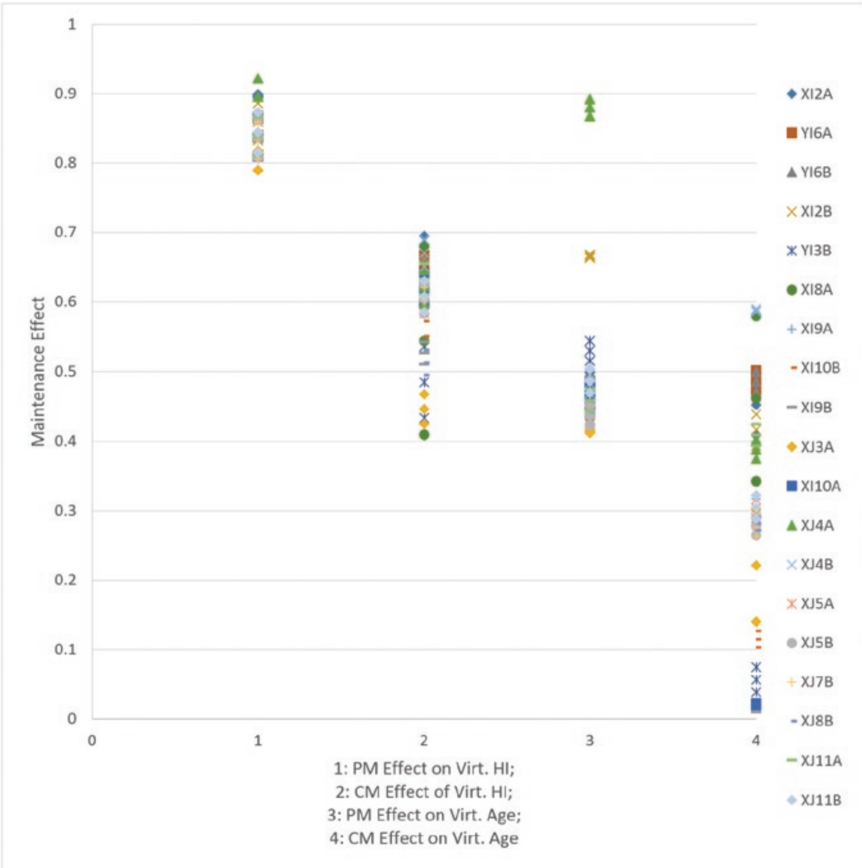

Fig. 4. Maintenance effect on virtual hi and virtual age for point machines in 'reverse' direction

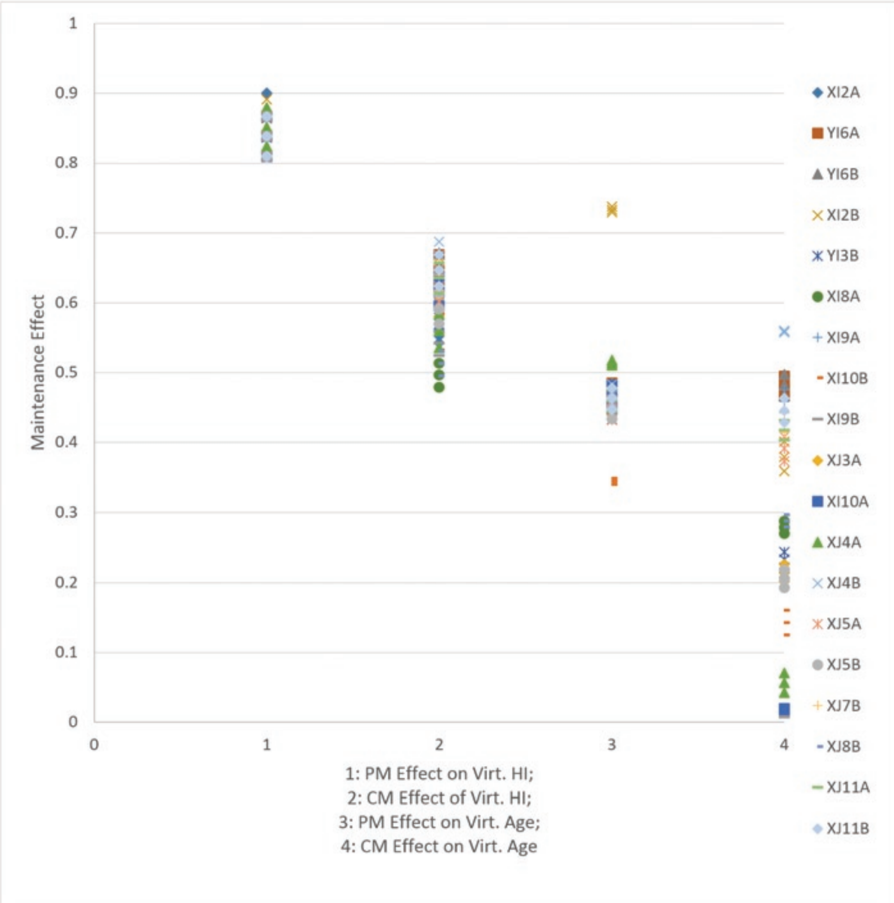

Fig. 5. Maintenance effect on virtual hi and virtual age for point machines in 'both' (i.E. Normal and reverse combined) directions

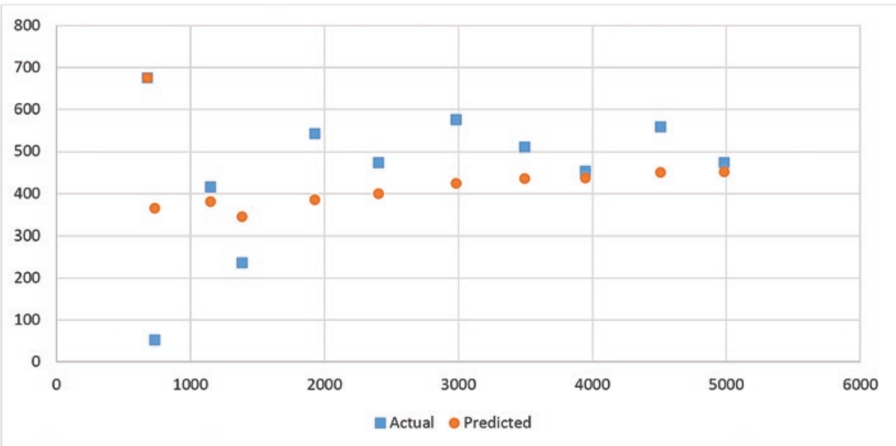

Fig. 6. Actual failures and predicted remaining useful life (rul) estimates

are required, unlike for filtering-based models, where parameters in initial life distribution can be estimated separately.

5. Conclusions

In this paper, a model is proposed for quantifying the effects of different types of maintenance on a device subject to condition monitoring. It is assumed that failures follow a nonhomogeneous Poisson process (NHPP) and covariates follow the Cox proportional hazards model. In particular, the multiplicative effect of maintenance on the age of a device is estimated using the Weibull hazard function, while the multiplicative effect on the health of a device and covariates associated with condition-based monitoring (CBM) is estimated using the Cox hazard function.

The proposed algorithm for estimating the impact and effectiveness of maintenance uses the concept of virtual age and introduces the concept of virtual health. It is shown that virtual health and the effect of maintenance on the health indicator of a device can be described using the concepts of Kalman filter.

An example of practical application of the algorithm is provided to a real case of railway point machines. In this example, preventive or corrective types of maintenance are modelled as different maintenance effect parameters. Using condition monitoring data, the health indicator is calculated as a scaled Mahalanobis distance. The reliability and the likelihood functions are derived and the least squares estimates (LSE) of the covariate coefficient, Weibull shape and scale parameters, as well as the preventive and corrective maintenance effect estimates on time and health indicator are found using the Levenberg-Marquardt algorithm.

The effect of corrective maintenance was closer to that of "as-good-as-new" (AGAN) state across all point machines, with point machine XI10A demonstrating the most dramatic AGAN virtual health improvement. The effect of preventive maintenance on the health indicator was the closest to "as-bad-as-old" (ABAO) across all point machines, with point machine XI2A demonstrating the least improvement in virtual health.

Remaining useful life (RUL) calculations were performed and predicted RUL estimates were obtained. The predicted RUL estimates

Table 1. Estimated Maintenance Effects and Upper and Lower 95 % Confidence Limits [LCL; UCL] on the for 'Both', 'Normal' and 'Reverse' Directions

Direction	PM Effect on Virt. HI, θ_{PM}	CI on PM Effect on Virt. HI, θ_{PM}	CM Effect on Virt. HI, θ_{CM}	CI on CM Effect on Virt. HI, θ_{CM}
Both	0.83	[0.75; 0.90]	0.57	[0.39; 0.75]
Normal	0.86	[0.79; 0.92]	0.55	[0.41; 0.69]
Reverse	0.85	[0.81; 0.89]	0.59	[0.48; 0.69]

Direction	PM Effect on Virt.Age, φ_{PM}	CI on PM Effect on Virt.Age, φ_{PM}	CM Effect on Virt.Age, φ_{CM}	CI on CM Effect on Virt.Age, φ_{CM}
Both	0.46	[0.39; 0.52]	0.4	[0.01; 0.79]
Normal	0.65	[0.42; 0.88]	0.31	[0.02; 0.59]
Reverse	0.54	[0.34; 0.73]	0.29	[0.01; 0.56]

were generally smoother than the actual data, thus displaying filtering qualities.

As a future work, application of fuzzy logic to estimate the health indicator, based on the covariate values appears to be promising. Yet another avenue is to perform clustering analysis using Gaussian mixture model (GMM) and identify the clusters corresponding to normal, failed and/or borderline devices.

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