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MANAGING THE RISK OF FAILURE OF THE WATER SUPPLY NETWORK USING THE MASS SERVICE SYSTEM

ZARZĄDZANIE RYZYKIEM AWARII SIECI WODOCIĄGOWEJ Z WYKORZYSTANIEM SYSTEMU MASOWEJ OBSŁUGI

The aim of this paper is to analyse the functioning of the repair brigades in the process of failure removal in the water distribution subsystem using the mass service system (MSS). An example is presented using queuing model which takes into account notifications with various scheduling algorithms of failures occurring to the system. The functioning analysis of mass service system can be used in the optimization of the repair teams' actions and in the management of water supply companies.

Keywords: *water distribution subsystem, queuing theory, management, modelling.*

Celem pracy jest analiza funkcjonowania brygad naprawczych w procesie usuwania awarii w podsystemie dystrybucji wody przy użyciu systemu masowej obsługi (SMO). Przykład został przedstawiony przy użyciu modelu kolejek, który uwzględnia zgłoszenia napływające do systemu z różnymi algorytmami planowania awarii. Funkcjonująca analiza systemu masowego świadczenia usług może być wykorzystana w optymalizacji działań zespołów naprawczych oraz w zarządzaniu przedsiębiorstwem wodociągowym.

Słowa kluczowe: *podsystem dystrybucji wody, teoria kolejek, zarządzanie, modelowanie.*

1. Introduction

The theory of mass service called interchangeably the queuing theory [15, 16] is widely used in the search for mathematical models (analytical) allowing the most precise description of the services related to all branches of industry. The queuing theory is used in the analysis and description of the phenomena in which there is a problem of mass customer service [7, 14, 17, 32]. The main problem with the practical use of the theory of mass service system (MSS) is to determine the optimal decisions at random arriving queries (notifications, events). A pioneer in this field was the Danish mathematician A.K. Erlang. Erlang published studies on the load of call centres in 1909 [9]. In 1917, he presented the formulas for the probability of call blocking, called the Erlang model [8]. According to [8, 9, 27] the MSS can be classified in two categories, as follows:

– systems with exponential service time and many servers,
– systems constant service time and a single server.

Other equally important works are associated with the name David G. Kendall. In the years 1951 and 1953 Kendall published the works on the queuing systems in which he systematized the mass service systems using the so-called Kendall's notation. Thanks to those works Kendall is considered as the founder of the science of mass service [15, 16]. Since then many valuable works, presenting the theoretical basis of various queuing models, were published [6, 10-12, 14, 17-19, 21, 22, 32, 40]. Nowadays, the queuing models and the theory of queues are constantly applied and implemented in various technical systems [1, 2, 5, 7, 13, 25, 26, 39]. It is also possible to link MSS with the modelling of adverse events occurring in systems included in critical infrastructures [3, 4, 20, 23, 24, 28-31, 33-38, 41, 42].

The main task of the theory of queues is to optimise the waiting time before satisfying the arriving queries. Solving the tasks for the MSS requires knowledge of two basic characteristics: queries (notifications) arrival rate and the waiting time before supplying the service.

In the water distribution subsystems (WDS) the MSS is a trade-off platform for random competition between "needs of service" and "servers". In the WDS, the needs of service can be represented by the notification about failures occurrence, while as the servers can be represented by the repair brigades which are capable of handling these notifications. The ultimate target is to supply water with the required pressure, adequate quality and quantity, to all recipients. The exploitation of the water supply system mainly includes:

- supervision, i.e. activities aim at getting information about the state of the system and its current changes through inspections,
- genesis, i.e. analysing the causes that led to the occurrence of a particular state,
- diagnosis, i.e. inference about the state of the system's components on the basis of the results of performed examination,
- forecasting, i.e. predicting the conditions of the system or its components in the future,
- water pressure measurements,
- sampling for microbiological, physical and chemical examinations,
- flushing water mains,
- repairs in the water supply network,
- patrolling the exploitation area.

Exploitation of water supply network requires not only maintaining its operation and its proper management, but also restoring its technical capability and utility.

The aim of the study is to analyse the functioning of the repair brigades in the process of failure removal in the water distribution system using the mass service models. The analysis of the MSS functioning can be used, amongst others, in the management of water supply companies. Moreover, the article presents the topic of using the mass service system in a water supply system. It is noteworthy that there are few publications in the field. However, a wide application can be seen especially for telecommunications systems. The novelty

of presented analysis is the fact, that notifications with priorities for failures arriving to the WDS have been given. The presented analysis can significantly contribute to increasing the efficiency of the repair brigades and thus to increasing the reliability of water supply to all water recipients.

2. Research methodology – The Queuing Systems

2.1. Classification of queuing systems and basics models

We can talk about MSS when, on one hand, we have the notifications of events which arrive at the system with specific intensity and which require to be handled, and, on the other hand, there are servers capable of handling these notifications. Sometimes, however, not all notifications will be handled so there is a possibility to cancel the notification.

Queuing systems can be classified [27]:
- the algorithm (the method) for the inflow of notifications (input stream characteristics). Input stream can be characterized by the average number of notifications per time unit, or by the mean time between the arrivals of two successive notifications. The algorithm can be deterministic (notifications arrive in regular intervals) or stochastic (the mean time between notifications is the expected value of a random variable - the time between successive notifications),
- the way of handling notifications by servers. The server can be busy when the notification enters the system, then we assume the mean time needed for handling a single notification or there is a lack of notifications at server. The service algorithm can be deterministic (time to handle the notification is constant) or stochastic (we can distinguish different time distribution for handling the notification),
- the task scheduling algorithm (rules for selecting notifications from the queue to be handled by server) which can be summarised as:
  - FIFO (First in, first out) - first notification is handled first,
  - LIFO (Last in, first out) - last notification is handled first,
  - SIRO (Service in random order) - "random" selection, regardless of the arrival order,
  - priority scheduling – notifications with higher priority are selected to be handled as the first, regardless how many notifications with lower priority are in the system:
  - the absolute priority,
  - the relative priority,
- the number of places in the queue,
- the number of service channels.

The MSS operations can be modelled as stochastic processes [8, 10, 12, 15, 16, 41, 42]. To mark different MSS types the Kendall's notation is commonly used and the system can be described by some parameters: $A / B / \mathrm{r} : (L, N)$ where:

$A = T_p$ – the distribution of the random variable T_1, i.e. the time between successive notifications,

$B = T_n$ – the distribution of the random variable T_2, i.e. the distribution of service time,

r – the number of servers,

L – the number of places in the queue,

N – the size of serviced population.

If L and N in the notation are omitted, it means that there are infinitely large (∞, ∞).

It was assumed that the probability distributions of the time intervals between notifications of needs of service and in service are exponential distributions. Furthermore, the functions T_p and T_n are independent. The number of places in the queue and the number of serviced people are infinite.

The parameters of the process are [41]:

λ – parameter of exponential distribution of the random variable T_p, which is the intensity of the inflow of notifications to the MSS:

$$\lambda = \frac{1}{T_p} \qquad (1)$$

μ – parameter of exponential distribution of the random variable T_n, repair rate :

$$\mu = \frac{1}{T_n} \qquad (2)$$

k – the number of notifications arriving to the MSS (number of failures),

r – the number of servers (number of repair brigades), $r \geq 1$,

ρ – utilization rate of the MSS [15]

$$\rho = \frac{\lambda}{\mu} \qquad (3)$$

To avoid the so-called jamming the condition of the inequality must be fulfilled:

$$r \geq \rho = \frac{\lambda}{\mu} \qquad (4)$$

The probability of state $P_k (t)$ in which the WDS at time interval t has k notifications, the intensity of notifications $\lambda_k(t)$ and the intensity of service $\mu_k(t)$ are calculated per unit of time and are dependent on the number of notifications at time interval t (or interval of time). With the stationary nature of the process and other assumptions we obtain: $P_k(t) = P_k$, $\lambda_k(t) = \lambda_k$, $\mu_k(t) = \mu_k$. The system of equations describing the process for a stationary process is as follows [41, 42]:

$$\begin{cases} \lambda_z P_0 = \mu_z P_1 \\ (\lambda_z + k\mu_z)P_k = \lambda_z P_{k-1} + (k-1)\mu_z P_{k+1} & k > r \\ (\lambda_z + r\mu_z)P_k = \mu_z P_{k-1} + r\mu_z P_{k+1} & k \geq r \end{cases} \qquad (5)$$

The distribution of the probabilities of the number of damaged elements given by A.K. Erlang is:

$$P_k = \begin{cases} \dfrac{M!}{k! \cdot (M-k)!} \cdot \rho^k \cdot P_0 ; \quad k = 0,1,...,r \\[4mm] \dfrac{M!}{r! \cdot r^{k-r} \cdot (M-k)!} \rho^k \cdot P_0 ; \quad k = r, r+1,...M \end{cases} \qquad (6)$$

where :

$$P_0 = \frac{1}{[1 + \sum\limits_{k=1}^{M} (\rho_1 \cdot \rho_2 \cdot ... \rho_k)]} \qquad (7)$$

wherein:

$$\rho_k = \frac{\lambda_{k-1}}{\mu_k}; \quad k = 1, 2, \ldots, M \tag{8}$$

The probability that during the stationary period the system is in the k-th state takes the form:

$$P_k = \begin{cases} \dfrac{\rho^k}{k!} \cdot P_{01} & k < r \\[2ex] \dfrac{\rho^k}{r! \cdot r^{k-r}} \cdot P_{01} & k \geq r \end{cases} \tag{9}$$

The intensity of transitions in the k-th state is as follows:

$$\mu_k = \begin{cases} k \cdot \mu_z; & k = 0, 1, 2, \ldots, r \\ r \cdot \mu_z; & k = r, r+1, \ldots, M \end{cases} \tag{10}$$

$$\lambda_k = (M - k) \cdot \lambda_z; \quad k = 0, 1, \ldots, M \tag{11}$$

where:

μ_k – the intensity of service (service rate) in the k-th state,

λ_k – the intensity of notification inflow (arrival rate) in the k-th state.

The average number of notifications in the MSS is:

$$E(N_w) = \sum_{k=1}^{r+L_w} k P_k \tag{12}$$

The average number of notifications in the queue is:

$$E(U_w) = E(U_w) = \sum_{k=1}^{L_w} k P_{r+k} \tag{13}$$

The average number of free repair brigades is:

$$E(O_r) = \sum_{k=0}^{r-1} (r - k) \cdot P_k \tag{14}$$

There is the equality:

$$E(N_w) - E(U_w) + E(0_r) = r \tag{15}$$

The average waiting time for service is:

$$E(T_w) = E(T_w) = \frac{E(U)}{\lambda_z} \tag{16}$$

The average time when notification is in the MSS is:

$$E(T_z) = E(T_w) + \frac{1}{\mu_z} \tag{17}$$

The average waiting time for service when all the repair brigades are busy is [41]:

$$E(T_v) = \frac{E(T_w)}{P_{k \geq r}} \tag{18}$$

The number of repair brigades for the WDS should be:

$$r = r_{min} + r_{dod} \tag{19}$$

where:

r_{min} – the number of repair brigades necessary to avoid a queue or blocking the queue, which is defined by the condition (4),

r_{dod} – the number of additional repair brigades, which may be adopted from the condition for the required system reliability K_w [41].

2.2. Models of service with priorities

Reliability and safety of the WDS operation can be also considered assuming the priority MSS model. Among the priorities one can distinguish the absolute priority and the relative priority.

The notification arriving to the system gets the absolute priority if handling the notification causes that handling any other notification would be interrupted. However, the notification has the relative priority if it does not interrupt the handling of the other notifications. Giving priority to the notifications should be performed individually for each WDS taking into account its specificity.

The intensity of the notifications (arrival rate) of the first kind is λ_{z1} and the notifications of the second kind is λ_{z2}. It is assumed that the service time for both types of notifications is the same and that the intensity of service (service rate) is then equal to μ_z. Therefore it can be written [39]:

$$\lambda_z = \lambda_{z1} + \lambda_{z2} \tag{20}$$

$$\rho_i = \frac{\lambda_{zi}}{\mu_z}; \quad i = 1, 2 \tag{21}$$

It can consider the cumulative probability $P\{N_1(t) = k_1, N_2(t) = k_2\}$, $N_1(t)$ and $N_2(t)$ - the number of notifications of the first and second kind arriving to the MSS at time interval t. It can be assumed that streams of the first and second kind failures are stationary streams (Poisson). In such case the probability that in a short period of time there will be more than one failure has small value of higher order than Δt [14].

When the first notification type, has the absolute priority, the second type notification has to wait and it is not possible to serve the second kind notification as the first one. Handling the notifications with the absolute priority takes place independently of the notifications with the relative priority. To describe handling the first kind notifications in the WDS the following model can be used: $A / B / r : (L, N)$. The expected value of notifications that are in the system [39]:

$$E(N_{w1}) = \frac{\rho_1}{1 - \rho_1} \tag{22}$$

The expected value of notifications waiting for service:

$$E(U_{w1}) = \frac{\rho_1^2}{1 - \rho_1} \tag{23}$$

The probability that the server is free:

$$P_1 = 1 - \rho_1 \qquad (24)$$

The expected value of the waiting time for service:

$$E(T_{w1}) = \frac{\lambda_1}{\mu \cdot (\mu - \lambda_1)} \qquad (25)$$

The expected value and the waiting time for service if server is busy:

$$E(T_{V1}) = \frac{1}{(\mu_z - \lambda_{z1})} \qquad (26)$$

The number of the second kind notifications can be calculated according to the formula [22]:

$$E(N_{w2}) = \frac{\rho_2}{1 - \rho_1 - \rho_2}[1 + \frac{\rho_1}{1 - \rho_1}] \qquad (27)$$

The order of handling the notifications of one kind can be according to the following rules:
- first arrived - first served,
- last arrived - first served,
- random selection of service.

Handling the notifications of the first and second kind can be performed independently if:

$$\rho_1 + \rho_2 \leq r \qquad (28)$$

The Erlang formula for both streams of notifications takes the form [10]:

$$p_{k1,k2} = \frac{(\rho_1 + \rho_2)^{k_1+k_2}}{(k_1 + k_2)! \sum\limits_{i=1}^{M}(\rho_1 + \rho_2) \cdot \dfrac{i}{i!}} \qquad (29)$$

$$\rho_i = \frac{\lambda_{zi}}{\mu_z} \qquad (30)$$

where:
$k_1 + k_2 \leq M$, M – number of population in service, i - the number of different streams of notifications
$i = 1,2,$
$\lambda_{z \leq i} = \lambda_{z1} + ... + \lambda_{zi}$

Downtime brigades indicator can be estimate using the formula [39]:

$$z = \frac{E(O)}{r} \qquad (31)$$

3. Application case

3.1. Research object

The daily production capacity at the end of 2011 amounted to 84000 m³/d. The average daily production of treated water in Water Treatment Plans amounts to 37700 m³/d. The system of collective water supply at the end of 2011 covered 184152 inhabitants. The main network is made of cast iron and steel pipes. The distribution network is constructed from cast iron, steel, PE and PVC pipes. The skeleton of water supply system consists of four mains transporting treated water from the second stage pumping station. 80% of the network is made in a closed system. In addition to the operation of the water supply system and water supply connections in the city operate also: emergency deep-seated intake with a capacity of 240 m³/d, 32 water pumping stations, 12 clean water compensating tanks with a total capacity of 34100 m³/d, 187 public wells.

Calculation example for the water distribution subsystem was based on the list of failures in the water supply system and water supply connections. The data taken for the analysis are summarized in Table 1.

Figure 1 presents percentage distribution of the number of failures in the water supply network and water connections.

In the city, water and sewage emergency services operate six repair brigades. At the beginning of a shift, a task is assigned to each brigade based on the notification from the water and sewer emergency services. The order of handling failures is set every time (handling

Table 1. Failures of water network and water connections over the years 2005-2012

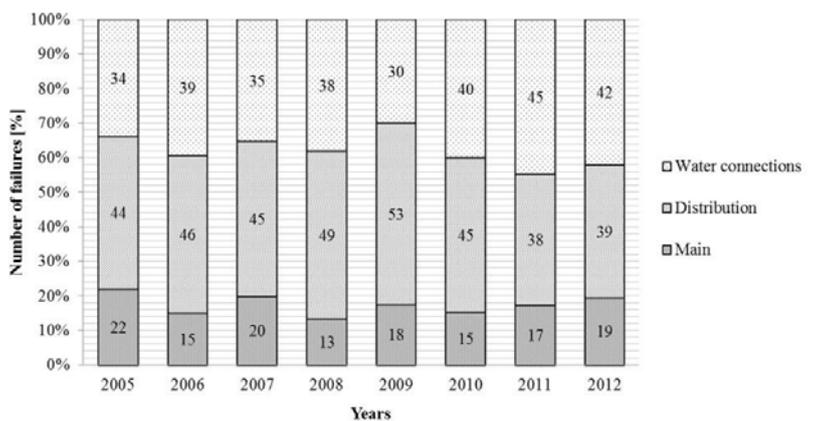| Years | Type of pipes | | | | | |
| | Main | | Distribution | | Water connections | |
	Length (km)	Number of failures	Length (km)	Number of failures	Length (km)	Number of failures
2005	49,5	54	350,5	108	283,8	83
2006	49,5	45	384,4	136	287,7	117
2007	49,5	51	443,5	114	315,8	90
2008	49,5	29	447,7	106	322,8	83
2009	49,8	38	468	114	323,2	65
2010	49,8	39	490,5	114	323,8	102
2011	49,8	52	504,1	113	323,8	134
2012	49,8	55	520,5	109	323,8	119

Fig. 1. The number of failures in the water supply network and water connections, in %

the failures is determined based on their priority service given by the master of the brigade). The brigades with the materials necessary to handle the failure are carried to the designated place. The application example were analysed for the brigades: 4 (service of water network) + 2 (service of water connections).

3.2. Results of modelling

a) **Service of water network. System works without the priority of notifications (all notifications have the same priority).**

The MSS model type $A/B/r:(\infty,\infty)$ was adopted. Assumptions for modelling based on exploitation data:
- the mean time between failures $Tp_{śr}$ (d), $Tp_{śr} = 2{,}23$ d
- unit intensity of notifications (arrival rate) $\lambda_z = 0{,}448$ 1/d
- the mean time of repair $Tn_{śr}$ (d), $Tn_{śr} = 0{,}132$ d
- repair rate $\mu_z = 7{,}57$ 1/d
- mass service system utilization rate $\rho = 0{,}059$
- the number of repair brigades, r = 4
- minimum number of repair brigades $r_{min} = 1$

The results of the calculations are presented in Table 2.

b) **Service of water network. System works without the priority of notification (all notifications have the same priority).**

Assumptions for the modelling were the same as in section 1a with the change in the number of repair brigades according to the condition $r \geq \rho$. To avoid a queue or queue blocking. The number of necessary repair brigades is $r_{min} = 1$, where r_{min} is the smallest natural number satisfying the above inequality. The results of the calculations are presented in Table 3.

c) **Service of water network. System works with the absolute priority of notification.**

For the calculations it was assumed, that the notifications with the absolute priority will arrive to the system.

c1) Assumptions for modelling of the first kind of notifications (based on exploitation data):
- the number of notifications with priority k_1 is assumed to be 95% of all the notifications arriving to the MSS.
- $k_1 = 156$,
- the mean time between failures $T_{pśr}$ (d), $T_{pśr} = 2{,}34$ d
- unit intensity of notifications inflow (arrival rate) $\lambda = 0{,}427$ 1/d
- the mean time of repair $T_{nśr}$ (d), $T_{nśr} = 0{,}132$ d
- repair rate $\mu_z = 7{,}57$ 1/d
- mass service system utilization rate $\rho_1 = 0{,}056$

Table 2. Summary of results

The number of notifications k	Number of free repair teams r_k	The probability of state P_k	E(N)	E(U)	E(O)	z
		SMO1a, r = 4, ρ = 0,059				
0	4	0,8416				
1	3	0,1494	0,1676	0,0002	3,8326	0,958
2	2	0,0088				
3	1	0,0002				

Table 3. Summary of results

The number of notifications k	Number of free repair teams r_k	The probability of state P_k	E(N)	E(U)	E(O)	z
		SMO1b, r = 1, ρ = 0,059				
0	1	0,8335				
1	0	0,1480	0,1861	0,0010	0,8149	0,814
2	0	0,0175				
3	0	0,0010				

Table 4. Summary of results for the first type of notifications

The number of notifications k	Number of free repair teams r_k	The probability of state P_k	E(N)	E(U)	E(O)	z
		SMO1c, r = 4, ρ = 0,056				
0	4	0,8482				
1	3	0,1435	0,1603	0,0002	3,8399	0,960
2	2	0,0081				
3	1	0,0002				

Table 5. Summary of results for the second type of notifications

The number of notifications k	Number of free repair teams r_k	The probability of state P_k	E(N)	E(U)	E(O)	z
		SMO1c, r = 4, ρ = 0,003				
0	4	0,9913				
1	3	0,0086	0,0087	0,0000	3,9913	0,998
2	2	0,0001				
3	1	0,0000				

Table 6. *Summary of the results for handling water supply connections.*

The number of notifications k	Number of free repair teams r_k	The probability of state P_k	E(N)	E(U)	E(O)	z
		SMO2, r = 2, ρ = 0,032				
0	2	0,9108				
1	1	0,0864	0,0920	0,0000	1,9080	0,954
2	0	0,0028				
3	0	0,0000				

Table 7. *Summary of the results for the whole work of repair brigades*

The number of notifications k	Number of free repair teams r_k	The probability of state P_k	E(N)	E(U)	E(O)	z
		SMO3, r = 6, ρ = 0,116				
0	6	0,7191				
1	5	0,2507				
2	4	0,0291				
3	3	0,0011	0,3123	0,0011	5,6877	0,948
4	2	0,0000				
5	1	0,0000				
6	0	0,0000				

c2) Assumptions for modelling of the second kind of notifications (based on exploitation data):
- the number of notifications with priority k_2 is assumed to be 5% of all the notifications arriving to the MSS,
- $k_2 = 8$,
- the mean time between failures Tp$_{śr}$ (d), Tp$_{śr}$ = 45,63 d
- unit intensity of notifications inflow (arrival rate) $λ_z$ = 0,022 1/d,
- the mean time of repair Tn$_{śr}$ (d), Tn$_{śr}$ = 0,132 d,
- repair rate $μ_z$ = 7,57 1/d,
- mass service system utilization rate $ρ_2$ = 0,003.

Since ρ1 + ρ2 <r handling arriving notifications can be carried out independently.

In Table 4 and 5 the calculation results for the sequence of notifications of the first type and second type are presented.

d) Service of water connections

Assumptions for modelling based on exploitation data:
- the mean time between failures Tp$_{śr}$ (d), T$_{pśr}$ = 3,07 d
- unit intensity of notifications inflow (arrival rate) $λ_z$ = 0,326 1/d
- the mean time of repair Tn$_{śr}$ (d), Tn$_{śr}$ = 0,097 d
- repair rate $μ_z$ = 10,31 1/d

- mass service system utilization rate ρ = 0,032
- the number of repair brigades r = 2

Table 6 summarizes the results of calculations for the work of brigades serving the water supply system connections.

e) Service of water network and water connections. Teamwork of all the brigades without priority.

Assumptions for modelling based on exploitation data:
- the mean time between failures Tp$_{śr}$ (d), Tp$_{śr}$ = 1,29 d
- unit intensity of notifications inflow (arrival rate) $λ_z$ = 0,7751/d
- the mean time of repair Tn$_{śr}$ (d), Tn$_{śr}$ = 0,150 d
- repair rate $μ_z$ = 6,67 1/d
- mass service system utilization rate ρ = 0,116
- the number of repair brigades r = 6

Table 7 summarizes the results of calculations for the work of brigades serving the water supply system and water connections.

On Figure 2 calculated average numbers of notifications in the MSS was presented.

After performing the calculations it is possible to check the reliability condition, in order to check whether the given MSS has the required level of reliability [41]:
- the availability index of one repair brigade: K_g = 0,9923077,
- the required level of MSS reliability: K_w = 0,9965225. The availability index of MSS takes the form:

$$K_g(SMO) = \sum_{i=r\min}^{r} \binom{r}{i} \cdot K_g^{i} \cdot \left(1 - K_g\right)^{r-1} \tag{31}$$

The reliability condition:

$$K_g(SMO) \geq K_w \tag{32}$$
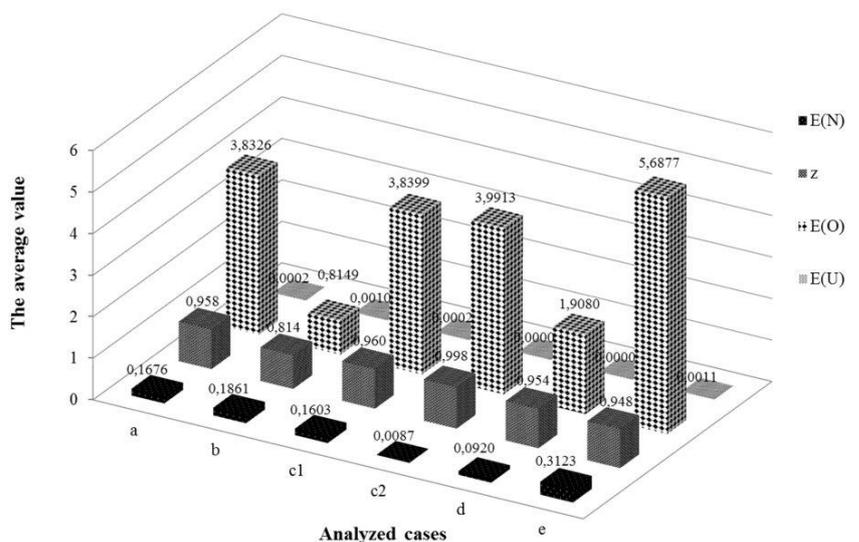$$K_g(SMO3) \geq K_w \rightarrow 0,9999999 \geq 0,9965225.$$

Fig.2. *The comparison of calculated average numbers of notifications for MSS for water supply system*

The required level of MSS reliability has been maintained because the condition (32) was met when the repair brigades worked together.

4. Conclusions

- The conducted calculations show that the most effective model of work of the water and sewage emergency service is the work of brigades in case e), when there is a teamwork of all the brigades without priority of failures. It results from obtaining the lowest outage index of repair brigades.
- The proposed methodology allows checking whether with the given number of repair brigades in the water supply company the jamming is not created while notifications are handled (idem).
- The fulfilment of the reliability condition with repair brigades working as a team causes that MSS has the required degree of reliability.
- The method can be used to search for solutions with the smallest possible number of repair brigades with maintaining the required level of reliability (minimum cost - maximum reliability).
- However, for the proper functioning of the emergency service the distribution of notifications arriving to the system should be predicted. The classification would predict the division of notifications on those that require immediate service and those whose handling is not so urgent. During such classification should be considered the economic aspects as well as the aspects related to safety of water supply to consumers.
- Analysing different variants of repair brigades work allows to make decisions concerning the improvement of work of water and sewer emergency service, which in consequence causes that the satisfaction of water consumers who use the service increases.
- The proposed method allows to analyse the repair brigades work in every water supply company, regardless of its size.
- The MSS analysis can be used in the management process in water supply companies in order to increase the efficiency in the process of management and making decisions related to operation of the water supply system.
- The article presents the topic of using the mass service system in a water supply system. It is noteworthy that there are few publications in the field. However, a wide application can be seen especially for telecommunications systems. The novelty of presented analysis is the fact, that notifications with priorities for failures arriving to the WDS have been given. The presented analysis can significantly contribute to increasing the efficiency of the repair brigades and thus to increasing the reliability of water supply to all water recipients.
- Service with priorities can be applied for different water recipients depending on water supply regime. For example, absolute priority can be attributed to failures on water pipes supplying in water health care institutions, nursing homes or nurseries. Water for firefighting purposes should have independent source of water. The presented model can be used in crisis management plans for urban agglomerations.

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