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ASSESSMENT OF THE TRACK CONDITION USING THE GRAY RELATIONAL ANALYSIS METHOD

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The article concerns the developed methodology for assessing the technical condition of a tramway track. Thanks to the data collected from multiple tram journeys equipped with an on-board vibration recording system, it was possible to create profiles of crossings through track sections in different technical condition. In order to identify the track condition, an algorithm based on the gray-scale modeling was proposed, and a similarity comparison between the obtained track profiles. A new measure of similarity has been proposed that has not been used so far in gray-scale modeling. The obtained results confirm the applicability of the proposed methodology.

Keywords: *track, maintenance, monitoring, tramway, GRA.*

Praca dotyczy opracowanej metodyki do oceny stanu technicznego toru tramwajowego. Dzięki zgromadzonym danym z wielokrotnych przejazdów tramwaju wyposażonego w pokładowy system rejestracji drgań, udało się stworzyć profile przejazdów przez odcinki torów w różnym stanie technicznym. W celu identyfikacji stanu toru zaproponowano algorytm oparty na metodzie modelowania szarych systemów oraz badanie podobieństwa pomiędzy uzyskanymi profilami przejazdów. Zaproponowano także nową miarę podobieństwa nie stosowaną do tej pory w zagadnieniach modelowania szarych systemów. Uzyskane wyniki potwierdzają aplikacyjność zaproponowanej metodyki.

Słowa kluczowe: *tor, utrzymanie, monitorowanie, tramwaj, GRA.*

1. Introduction

The execution of current research on recording the acceleration caused by tram vibrations in operating conditions, using on-board diagnostics systems and wireless data transmission, enables the track condition assessment based on the vehicle dynamic response analysis [5 ÷ 7]. This issue is very important from the infrastructure maintenance point of view, as it allows for an ongoing assessment of its technical condition in normal operating conditions. Such systems are particularly suitable for rail networks where driving conditions are constant, reproducible and without significant interference or changes in driving behavior [e.g. 1; 3; 12]. In urban conditions, this is not a trivial task, as it results from the substantial spread of data received even from the same vehicle type and the same measuring section. This is due to the fact that the vehicle is moving at different speeds within the same track, variable load (number of passengers), driving behavior of the motorists (rapid or gentle acceleration and deceleration), weather conditions, traffic at different hours and days, technical condition of the vehicle, etc. The measurement uncertainty of the monitoring system itself should also be taken into account. All of these factors make it difficult to estimate the track condition for light rail vehicles using the acceleration level measured in the vehicle.

In order to eliminate some of the above mentioned factors and to propose a methodology for evaluating the track condition, it was decided to, at the first stage, select the data from different track sections (in different parts of the city) of one type, i.e. with 60R2 tram rail, excluding areas using a classic railway track (mainly 49E1). In addition, it was decided to include the tram speed recordings for a given track, forming a certain profile characteristic for a particular track condition (the relation between the effective acceleration values and the tram speed). For each passing, the maximum speed was taken into account, assuming that the information about the technical condition of the track will be most visible for such driving speed. Another factor, whose impact was eliminated, was the technical condition of the vehicle itself. The data considered were from a new vehicle, but this does not limit the application of the proposed methodology. In practice, it is always possible to eliminate this factor by installing a vibration measurement system on a new or renovated vehicle.

The presented analysis used data collected from more than two months of operation of a modern low-floor tram in normal passenger traffic. The information on the vibration acceleration value determined from a 1 second time window in the range of 0 to 100 Hz, recorded on the vehicle body located above the first bogie. Thus this is in a way a measure of travel comfort (there are currently no official legal acts in this field dedicated to light rail vehicles such as a tram). The effective value of vibration acceleration was selected after a comparative analysis of various statistical measures [7].

Evaluation of the track sections actual technical condition was determined on the basis of independent information obtained from maintenance services, assisted by independent measurements of track geometry. Finally, the data presented in Table 1 and presented in Figure 1 were taken into account. The proposed grading scale of the track technical condition assessment is deliberately coincidental with that adopted in MPK Poznan (local tramway operator).

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Table 1. Main data included in the analysis

Technical condition	Number of track sections	Total number of dynamic response measurements of the vehicle
Good	8	1086
Satisfactory	2	278
Poor	3	103
Critical	2	240

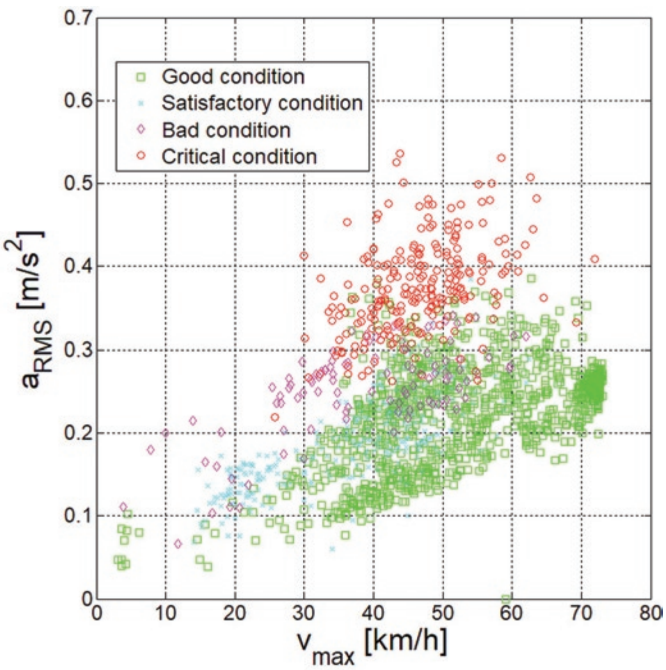

Fig. 1. Speed and acceleration profiles on track sections with different technical condition

As shown in Figure 1, data on various track technical conditions strongly intertwine and are strongly dependent on the maximum vehicle travel speed. In addition, the available speed range for certain data may vary due to the fact that sections of different technical conditions are located in a different "urban environment". These can be i.e. sections where the tram travels without stopping and sometimes it may be necessary to brake and accelerate. Hence, the range of maximum speed values should also be considered. Available data also differ in their number (different number of passes through a particular track section), which needs to be taken into account in the proposed methodology.

While it is easy to determine good and critical track conditions based on average or maximum effective vibration acceleration, the remaining intermediate states are no longer easily distinguishable due to large scattering (see Table 2, columns 1 and 2). Another simple solution would be to create a linear regression model of the passage profile for each of the track conditions and to evaluate the y-intercept or the slope of the line accordingly. Unfortunately, the slope does not carry information about the track condition, and the information contained in the y-intercept does not distinguish between good and satisfactory. The relevant data is given in Table 2.

Table 2. Simple statistical parameters describing the collected data

Track technical condition	RMS vibration acceleration value [m/s^2]	Maximum RMS vibration acceleration value [m/s^2]	Regression line slope describing the travel profile	Regression line y-intercept describing the travel profile
Good	0.224	0.386	0.0024	0.0918
Satisfactory	0.184	0.385	0.0034	0.0632
Poor	0.250	0.341	0.0029	0.1408
Critical	0.375	0.537	0.0030	0.2330

Since the use of the aforementioned simple methods is not effective in unequivocally determining the track condition, it was decided to resort to methods based on the similarity of specific data to the reference values. The reference will be based on the passage profile for the track section in good condition. The idea of the method will be to compare the obtained passage profile with the previously constructed model. In actual operating conditions, it will take only a few days to collect certain data from a particular controlled track section, because of the repeated passes on the same route by the same vehicle. This is a relatively short measurement time.

Due to the fact that the data can be very scattered, it was decided to use gray-scale modeling tools that can be used not only when there is little data available, but also when the data is uncertain. This is where the gray GM models can be used to model a particular profile. It is also necessary to define the similarity measure of the individual driving profiles. This can also be performed using methods for modeling gray systems (GRAs).

2. Track technical condition determination methodology

The main part of the research activities will be based on the gray systems modeling methods, so it is worthwhile to present some of the foundations of this theory. Theory of gray systems was proposed by prof. J-L. Deng, and has many different research areas and uses [4]. One of them is the study of similarity between data sequences (GRAs) [10]. Studying the similarity of data in different collections is of great importance in this methodology as it allows for a comparison of a given drive profile with the reference for a good track condition. The specified measure of similarity in the conditions of maximum travel speed can then be easily parameterized giving a single number indicating the degree of compliance with the model, and thus the technical condition of the track.

For this purpose, the travel profiles similarity measures should be defined. There are a number of measures in the GRA literature that define the relation between the data. An example is a generalized GRA model, which is used to analyze relationships between sequences and measures based on distance and similarity. A detailed overview of the methods can be found in [10]. Certain other measures have been proposed in [13].

An important role in this approach is played by the gray GM systems modeling in relation to the data set. As a result of certain operations, it can be treated as a series which allows it to be modeled with a gray model, such as GM(1,1) [4; 8; 11; 14; 16; 17]. This provides an opportunity for a model representation of primary data that is characterized by high uncertainty and dispersion. One feature of this model is the smoothing of local fluctuations (series) by the use of AGO (Accumulated Generating Operation), which allows for the replacement of the original data with model data, which are largely smoothed out.

Figure 2 shows a flowchart illustrating an algorithm for modeling passage profiles, determining similarity measures, and determining the technical condition of a track.

Assuming that further realizations of passages for a given track section are available. Let $X_i(k)$, $X_j(k)$, $i=1,2, ..n, j = 1,2,...,m$ denote vectors whose elements are the measure of the value of travel comfort for the passage through travel profiles i and j. Original data obtained using GM(1,1) models must be removed in cases where exactly the same speed values correspond to different values of effective vibration acceleration. This is necessary due to the fact that the GM(1,1) models describe a series. Although the speed values are determined to the nearest 0,01 km/h, the situation for which different acceleration readings are obtained for the exact same speed is quite common and should be taken into account.

In the next step of the algorithm, the original data is replaced by the results of linear interpolation. This is due to the fact that the basic GM(1,1) model requires a constant interval between the data, and that the compared vectors Xi and Xj must have the same number of elements. This is a condition for calculating the similarity measure of both profiles. For this purpose, it may also be necessary to cut out some data so that the compared sets cover the same maximum speed range in both comparable passage profiles – the tested one and the reference.

In order to model the resulting series, it is necessary to use the AGO, which according to [8] can be represented for the Xi vector as:

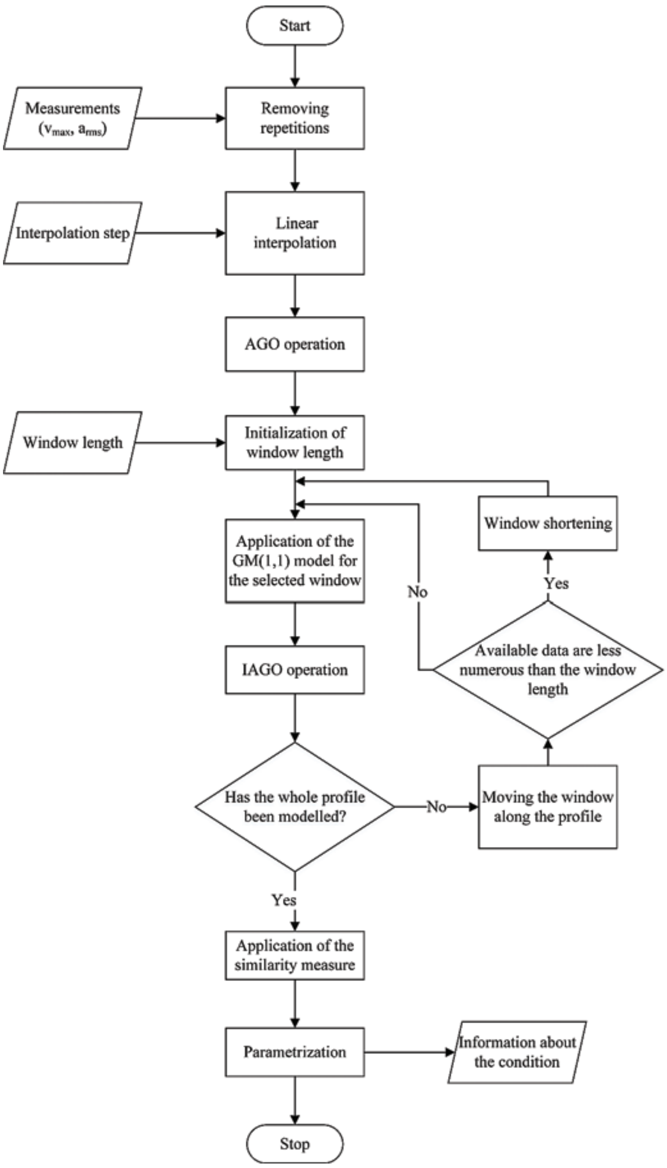

Fig. 2. Algorithm for the determination of the technical condition of the track on the basis of the similarity study of the profile of crossings: recognized and referenced

$$X^{(1)}{}_i(k) = \sum_{r=1}^{k} x_i(r) . \tag{1}$$

The previously mentioned GM(1,1) model is derived from the general description of the gray system in the form of a differential equation (2). In general for the case where the equation of the p order with excitation of m order GM(p,m) as described in [2] the following equation will be obtained:

$$\sum_{l=0}^{p} a_l \frac{d^{p-l} X_i^{(1)}}{dt^{p-l}} = \sum_{i=1}^{m-1} b_i X_{i+1}^{(1)} , \tag{2}$$

where: X_1 is a vector of original observations $x_1(\mathrm{r})$, $X_1^{(1)}$ is a system state variable vector derived from the original observation vector after the AGO operation according to (1) , X_{i+1} is the input vector, a_l , b_h are constant coefficients.

Model GM(1,1) for a given data set X can be expressed as:

$$\frac{dX^{(1)}(t)}{dt} + aX^{(1)}(t) = b \tag{3}$$

The solution of equation (3) with unit step k can be represented [8] as:

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - b/a\right]\exp(-ak) + b/a \tag{4}$$

where $\hat{x}^{(1)}$ is the predicted value of the cumulative series element. Using finite differences and expressing equation (3) as a series of equations for discrete values, according to [8] the following approximation is obtained:

$$x^{(1)}(k+1) - x^{(1)}(k) = -\frac{a}{2}\left[x^{(1)}(k) + x^{(1)}(k+1)\right] + b \tag{5}$$

Model parameters are calculated based on the equation (5) using the least squares method [16]:

$$\begin{bmatrix}\hat{a}\\ \hat{b}\end{bmatrix} = \left(\mathbf{Z}^T\mathbf{Z}\right)^{-1}\mathbf{Z}^T\mathbf{Y} \tag{6}$$

where:

$$z(k) = -\frac{1}{2}\left(x^{(1)}(k+1) + x^{(1)}(k)\right),$$

$$\mathbf{Y} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ .... \\ x^{(0)}(n) \end{bmatrix}, \quad \mathbf{Z} = \begin{bmatrix} z(1) & 1 \\ z(2) & 1 \\ ... & ... \\ z(n-1) & 1 \end{bmatrix}$$

The parameters of the GM(1,1) model can be estimated using all available rolling window methods [15]. Estimating parameters based on all data can cause the model to excessively smooth the values and as a result not capture certain changes in their trend. Using a narrow window causes the model to adapt to the trend and reflect it. The model can be used when there is little data, so using narrow windows is feasible. A window with a length of 80 measurements was arbitrarily chosen as a compromise between good data averaging and the ability to adapt the model to the data at an interpolation step for the track passage profile made up of 360 measurement points. At each step of the model construction, the window was shifted one measurement and the smoothed modeled values were estimated. In cases where the number of data points available for the model parameters evaluation was less than the window length, the window was shortened respectively. Theoretically, the window can only be shortened to four measurements that are necessary for the estimation of the GM(1,1) model parameters. The last four model values are derived from forecasts using the model and the last parameters estimated.

The next step is to calculate the similarity of modeled passage profiles. With constant interpolation k, it is possible to define a matrix of mutual change own similarity [13]:

$$£_k = \begin{bmatrix} \sigma_{11}(k) & \dots & \sigma_{1m}(k) \\ \dots & \dots & \dots \\ & \sigma_{jj}(k) & \sigma_{jm}(k) \\ \dots & \dots & \dots \\ & & \sigma_{mm}(k) \end{bmatrix} \tag{7}$$

where: $\sigma_{ij}(k)$ is a measure of similarity for passage profiles i and j for a given step k, corresponding to a given maximum speed. Here the most interesting are the relative measures: $\sigma_{21}(k)$, $\sigma_{31}(k)$, $\sigma_{41}(k)$, that relate to the travel profiles associated with particular technical conditions of the track (satisfactory, poor and critical) and the reference (labeled as good track conditions).
The proposed definition of similarity measure of profiles may be expressed as:

$$\sigma_{ij} = \frac{\alpha A_{ij} + \beta B_{ij} + \gamma C_{ij}}{\alpha + \beta + \gamma} \tag{8}$$

where:

$$A_{ij} = \begin{cases} \dfrac{\hat{x}_i^{(0)} \hat{x}_j^{(0)}}{\max(\hat{x}_i^{(0)}) \cdot \max(\hat{x}_j^{(0)})} & \text{for } \hat{x}_i^{(0)} \neq \hat{x}_j^{(0)} \\ 1 & \text{for } \hat{x}_i^{(0)} = \hat{x}_j^{(0)} \end{cases},$$

$$B_{ij} = \begin{cases} \dfrac{\max(\Delta_{ij}) - \Delta_{1ij}}{\max(\Delta_{ij}) - \min(\Delta_{ij})} & \text{for } \max(\Delta_{ij}) \neq \min(\Delta_{ij}) \\ 1 & \text{for } \max(\Delta_{ij}) = \min(\Delta_{ij}) \end{cases},$$

$$C_{ij} = \begin{cases} \dfrac{\max(\Delta_{ij}) - \Delta_{ij}}{\max(\Delta_{ij}) - \min(\Delta_{ij})} & \text{for } \max(\Delta_{ij}) \neq \min(\Delta_{ij}) \\ 1 & \text{for } \max(\Delta_{ij}) = \min(\Delta_{ij}) \end{cases},$$

$$\Delta_{1ij} = \left|\hat{x}_{1i}^{(0)} - \hat{x}_{1j}^{(0)}\right|, \; \Delta_{ij} = \left|\hat{x}_i^{(0)} - \hat{x}_j^{(0)}\right|$$

$\hat{x}^{(0)}$ – series values vector after smoothing with the model GM(1,1),

$\hat{x}_1^{(0)}$ – first series value, A – a component characterizing the similarity of "shapes" of the compared passages, B – a component taking into account different profile values for the smallest travel speed, C – a component characterizing the differences in values, α, β, γ – individual components influence coefficients (the weight of individual characteristics taken into account).

The sum of all components in formula (8) does not exceed 1.0 and they represent partial similarities in terms of individual characteristics. By adjusting the influence coefficients, different characteristics can be given a different level of significance. Part C is used in the GRA literature as a measure of similarity, for example [9], while B is an adaptation of this measure for the first value of the series.

The use of the GM(1,1) adaptive model in the proposed methodology is important in that it allows to capture the similarity features associated with the „local" changes in the compared travel profiles values. In the case of linear regression modeling of these profiles, the information would be lost. It should be noted that the proposed measure of similarity is universal and can be used to compare different sets of data concerning aspects other than the discussed problem.

Ultimately, the obtained similarity values can easily be parameterized by calculating the average or maximum value and on this basis, operational decisions or decisions on additional checks on a particular track section can be made.

3. Results

The described method was applied to the collected data presented in Figure 1. Interpolated and smoothed passage profiles with the GM(1.1) model with a measuring window of 80 points are shown in Figure 3. Different change rates of the vibration effective acceleration value as a function of the vehicle driving speed can be seen. The resulting smoothed profiles represented the input for the similarity calculation procedure $\sigma_{21}(k)$, $\sigma_{31}(k)$, $\sigma_{41}(k)$, where index 1 refers to the track profile for the good track condition. The mean values of the similarity calculation for the various weight values are shown in Table 2.

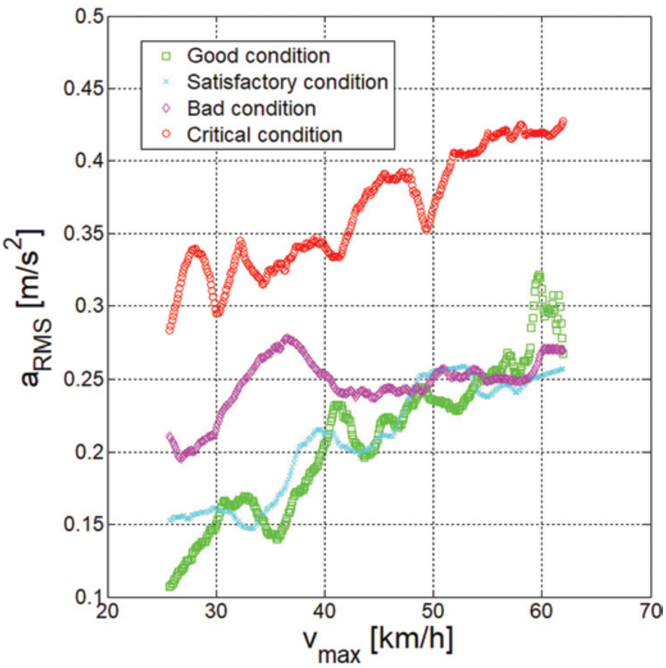

Fig.3. Result of GM model (1,1) with sliding window

Table 3. Sample results of the mean similarity measure for the data in Figure 1 and different weights

Similarity	α=1, β=1, γ=1	α=2, β=1, γ=1	α=1, β=2, γ=1	α=1, β=1, γ=2	α=0, β=0, γ=1
Satisfactory – good condition	0.538	0.539	0.489	0.586	0.731
Poor – good condition	0.483	0.507	0.471	0.526	0.654
Critical – good condition	0.448	0.478	0.413	0.453	0.469

The data in Table 3 indicates that in all cases the track technical condition becomes distinctive (as in Table 1). A smaller number means less similarity of a given profile to the profile corresponding to good track condition. The profiles that were obtained from passages in satisfactory track condition were most similar to the pattern defined for the track sections as good technical condition. A lower similarity can be seen between the data of passages through tracks in poor technical condition, and the smallest, for those in critical condition.

If greater significance is assigned to the distance of these profile values from the reference profile values, the distinction becomes particularly pronounced, hence this feature becomes the most important in the obtained profiles. According to the analyzes, this feature (and thus the commonly used GRA measure) is sufficient to clearly distinguish between the track conditions and, in this case, to better distinction of these states, it seems however, that a more flexible definition may have wider applications also to other data.

Thanks to the methodology used it is possible to clearly distinguish between the technical conditions of the tracks when measuring their exploitation in real operating conditions, which is very important from the practical point of view.

3. Conclusions

The problem of evaluating the technical condition of the track in real operating conditions is not trivial due to a number of factors influencing the measurement results, which are difficult to directly account for in the models. The idea of recording the vibration acceleration by the on-board system mounted on the tram (provided from the vehicle in good technical condition) and the creation of passage profiles on a given tested track section enables the classification of the technical track condition. However, this can be difficult due to the large spread of measurement data values. This classification can be performed through modeling of such a profile and then calculating the similarity of the measured profile and the reference profile. The gray systems theory provides a good foundation for this type of modeling, as in principle, it allows the modeling of uncertain data, and thus also data sets with large scattering. Using the GRA methodology in this case gives unambiguous results and allows to distinguish between the technical conditions of the track by simple parameterization of the mutual similarities between the modeled passage profiles. The proposed methodology allows for a relatively quick track condition diagnosis. Due to the multiple passages of a given vehicle on a given track, gathering the necessary data and creating a profile is not a difficult task. This underlines the practicality of the proposed methodology.

The similarity measure proposed in the article is very flexible and can be applied to a variety of problems. It embraces various aspects of the similarity between series. In the case of the data used, the obtained results are very good, although in this case the simpler GRA method also fulfills the task.

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