Article citation info:
ZHANG Y, MA Y, OUYANG L, LIU L. A novel reliability model for multi-component systems subject to multiple dependent competing risks with degradation rate acceleration. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2018; 20 (4): 579–589, http://dx.doi.org/10.17531/ein.2018.4.9.

Yanjing ZHANG
Yizhong MA
Linhan OUYANG
Lijun LIU

A NOVEL RELIABILITY MODEL FOR MULTI-COMPONENT SYSTEMS SUBJECT TO MULTIPLE DEPENDENT COMPETING RISKS WITH DEGRADATION RATE ACCELERATION

NOWATORSKI MODEL NIEZAWODNOŚCI DLA SYSTEMÓW WIELOELEMENTOWYCH NARAŻONYCH NA LICZNE ZALEŻNE RYZYKA KONKURUJĄCE UWZGLĘDNIAJĄCY PRZYSPIESZENIE TEMPA DEGRADACJI

The purpose of this paper is to establish a new reliability model of the system subject to multiple dependent competing risks. For a system subject to multiple dependent competing risks, the total degradation consists of natural degradation amount and sudden degradation increments (SDIs) caused by random shocks arriving at the system. Most researchers on this topic only focus on the SDIs. However, the impact of random shocks on degradation rate is ignored. In this paper, a novel reliability model considering degradation rate acceleration (DRA) caused by random shocks is proposed, in which the degradation model is based on the degradation path. The dependence relationship between multiple degradation processes is dealt with by copula method, and the arrival time of shocks is assumed to follow a non-homogeneous Poisson process (NHPP). Finally, the effectiveness of the proposed reliability model is demonstrated by an example of a series system. Moreover, the effect of model parameters is evaluated through sensitivity analysis.

Keywords: *dependent competing risks; degradation rate acceleration; reliability model; copula method; sensitivity analysis.*

Celem niniejszej pracy było stworzenie nowego modelu niezawodności systemu narażonego na liczne zależne ryzyka konkurujące. W przypadku systemu eksponowanego na wiele zależnych ryzyk konkurujących, na wartość całkowitą degradacji składa się wartość degradacji naturalnej oraz wartość nagłych przyrostów degradacji (sudden degradation increments, SDI) powodowanych przez losowe zaburzenia systemu. Większość badaczy tej tematyki koncentruje się wyłącznie na SDI, ignorując tym samym wpływ zaburzeń losowych na tempo degradacji. W niniejszym artykule zaproponowano nowy model niezawodności uwzględniający przyspieszenie tempa degradacji powodowane zaburzeniami losowymi, w którym model degradacji opiera się na krzywej degradacji. Zależność między mnogimi procesami degradacji rozpatrywano za pomocą metody funkcji kopuły przy założeniu, że czas wystąpienia zaburzenia odpowiada niejednorodnemu procesowi Poissona. Skuteczność proponowanego modelu niezawodności zademonstrowano na przykładzie systemu szeregowego. Ponadto, wykorzystano analizę czułości do oceny wpływu parametrów modelu na niezawodność systemu.

Słowa kluczowe: *zależne ryzyka konkurujące; przyspieszenie tempa degradacji; model niezawodności; metoda funkcji kopuły; analiza czułości.*

1. Introduction

The failure of a system is usually caused by internal degradation or external random shocks. The failure caused by internal degradation is called soft failure, such as erosion, fatigue, wear, etc. And the hard failure is caused by external random shocks, such as device breakdown, short circuit, etc. The degradation processes for components in a system and a shock process arriving at the system compete with each other. The occurrence of any failure mode may lead to the failure of systems. A system deteriorates with its use and age, which is a continuous accumulation of degradation. However, the hard failure may or may not happen in its life cycle.

In many studies [10, 14, 28, 32], the competing risks were treated as independent. However, the natural degradation processes of a system are usually affected by the shock loads. That is, the same shock arriving at a system will influence the degradation process of each component. Hence, the assumption of independence between competing risks is not reasonable, and it may cause underestimation or overestimation of the system reliability. It is very necessary to consider the dependence relationship between multiple degradation processes and a shock process when establishing the reliability model for a system.

Recently, some researchers [5, 15] have taken the dependence relationship into account to develop the reliability model of a system. Peng and Feng [20] built a reliability model for the system subject to multiple dependent competing risks, where dependent competing risks referred to soft failure and hard failure. Soft failure was caused by continuous natural degradation and additional SDI due to random shocks. Hard failure was induced by fatal shock loads from the shock process. Jiang and Feng [11] proposed a reliability model for a sys-

Table 1. Comparisons with existing reliability models

Models	DBDS[1]	SDIs[2]	CL[3]	DRA[4]	NDP[5]	ATS[6]
Peng, Feng and Coit (2010)	Y	Y			1	HPP
Guo, Wang and Guo (2013)	Y	Y			2	HPP
Song, Coit, and Feng (2014)	Y	Y			*m*	HPP
Jiang, Feng, and Coit (2015)	Y	Y	Y		1	HPP
Wang and Pham (2012)	Y	Y		Y	*m*	HPP
An and Sun (2017)	Y	Y	Y		*m*	HPP
Huynh, Castro, and Barros (2012)	N				1	NHPP
Bocchetti, Giorgio, and Guida (2009)	N				1	NHPP
Proposed model	Y	Y	Y	Y	*m*	NHPP

[1] *Dependence between degradation and shocks.*
[2] *Sudden degradation increments.*
[3] *Certain level.*
[4] *Degradation rate acceleration.*
[5] *Number of degradation processes.*
[6] *Arrival time of shocks.*

tem subject to multiple dependent competing risks. In their research, the shock threshold may shift due to exposure to various shock patterns. Guo and Wang [8] developed a joint copula reliability model for systems subject to two degradation processes and a random shock process. Song and Coit [24] developed a reliability model for a multi-component system. They thought a system may fail due to any soft failure process or hard failure process. Meanwhile, the soft failure processes of different components in a system were mutually competing. Besides, Song and Coit [25] also established a reliability model for multi-component systems subject to dependent competing risks of natural degradation and random shocks, in which the shocks were categorized according to their sizes, function, etc.

Previous researches have mainly investigated the dependence relationship between multiple degradation processes and a shock process based on an assumption that the arrival of each shock only causes SDIs. Nevertheless, the assumption is not always reasonable for systems with high reliability and long life because they have the ability to resist small shock loads. The research by Jiang and Feng [12] manifested that small shock loads had no effect on the degradation process, which was supposed to be a gamma process. In addition, Wang and Pham [29] considered two types of shocks when evaluating the system reliability: fatal shock loads leading to the hard failure of systems and general shock loads increasing the system degradation level. Moreover, a novel reliability model was proposed by An and Sun [1] for highly reliable systems experiencing multiple dependent competing risks. They assumed that only shock loads above a certain level can affect degradation processes. In their study, shock loads were separated into three parts by the shock threshold and the certain level. The first part of shocks, which were above the hard failure threshold, were fatal shocks causing sudden failure. The shocks in the second part were general shocks, which were between the certain level and shock threshold. Only general shocks can cause SDIs. The rest of shocks were small shocks, which were supposed to have no effect on degradation processes. However, almost all the researchers were used to considering the SDIs of degradation processes and ignoring the impact of general shock loads on the degradation rate. What is more, in most previous studies about the random shock process, the arrival of shocks was assumed to be a homogeneous Poisson process (HPP) [30]. But this assumption is not always appropriate for systems which work in the convoluted environment. Under a complex circumstance, the occurrence rate of shocks is a variable rather than a constant. Therefore, the assumption that the arrival of shocks is a NHPP may be more reasonable [2, 9].

Motivated by the above, we propose a new reliability model to evaluate the reliability of the system subject to multiple dependent competing risks. In the model, the general shock loads can bring about DRA in degradation processes, besides SDIs. In addition, only shock loads above a certain level can impact the degradation processes. The dependence between a shock process and multiple degradation processes is dealt with by conditional probability. In addition, the dependence relationship between multiple degradation processes is solved by a copula method. The arrival of shocks is assumed to be a NHPP. A summary of the comparison with existing reliability models is presented in Table 1. The remainder of this paper is organized as follows: a system subject to multiple dependent competing risks is described in Section 2. In Section 3, we develop a new reliability model in which DRA is considered into the degradation path. In Section 4, a numerical example is presented to validate the proposed reliability model. Finally, some concluding remarks are given in Section 5.

2. System description

In fact, systems with multiple components used in industrial applications always experience two types of failures: soft failure and hard failure. Either of them may lead to the failure of systems. Assume that there are i components in a system and every component experiences a degradation process. Once a shock arriving at the system, it will affect all the degradation processes. If there are no random shocks, the ith natural degradation process corresponding to the ith component of the system is shown in Figure 1, which is a continuous increasing process [3]. $D_i(t)$ represents the wear volume of the ith component of the system at t. The threshold of soft failure of the ith component is denoted by $l^{(i)}$.

As shown in Figure 2, any random shock arriving at the system, whose load is above the threshold of hard failure W_U, may cause the failure of components in the system. Meanwhile, general shocks, whose loads are between the failure threshold W_U and a certain level W_L, could give rise to the SDI of every component once they arrive at the system. Actually, the sudden degradation increments on different components caused by the same shock may be different due to the different material properties of components. In addition, small shocks, whose loads are below the certain level W_L, have no effect on degradation processes of components due to the system's good performance. It indicates that the degradation process and the shock process are dependent because every degradation process is affected by the same shock process.

From Figure 3, we can see that the total degradation of the ith component in a system is composed of natural degradation and SDIs, which is denoted by Y_{ij} (i=1, 2, …, m; j=1, 2, …, ∞). Y_{ij} represents the SDIs of the ith component of a system caused by the jth shock arriving at the system. The ith component occurs soft failure once the cumulative degradation of any component exceeds its soft failure threshold $l^{(i)}$. The total degradation $M^{(i)}(t)$ (i=1, 2, …, m) of the ith component exceeds the degradation threshold $l^{(i)}$ or the magnitude of a shock exceeds the shock threshold W_U, which will result in the failure of the component.

With the development of science and technology, the reliability of modern products has been greatly improved. The failure occurs rarely in a long working time. However, many products degrade over time before they fail or break down. Thus, in many engineering reliability experiments, the measure of degradation can be observed over a period of time before failure occurs to provide additional information of systems. There are many studies to overcome this kind of difficulty. For highly-reliable or long-life modern products, it often takes much more time to obtain lifetime and degradation data under usual use conditions. One solution is to use accelerated degradation tests to collect the performance degradation data at greater environmental stress levels so that the degradation data can be analyzed earlier before any specimens "fail" [6, 16, 17, 19, 21, 22, and 31]. For the convenience

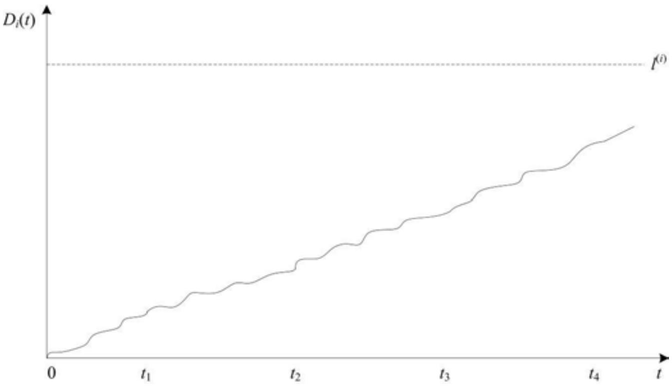

Fig. 1. Continuous degradation of the ith component

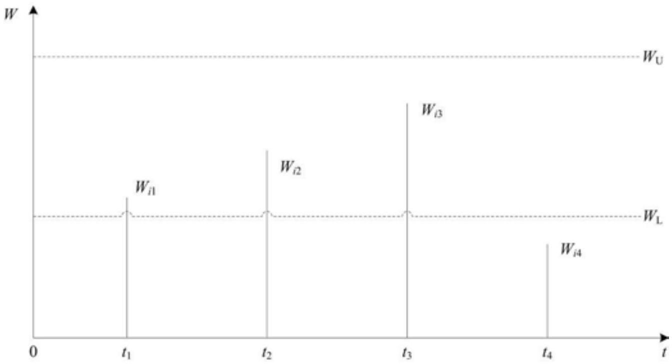

Fig. 2. Magnitude of every shock arriving at the system imposed on the ith component

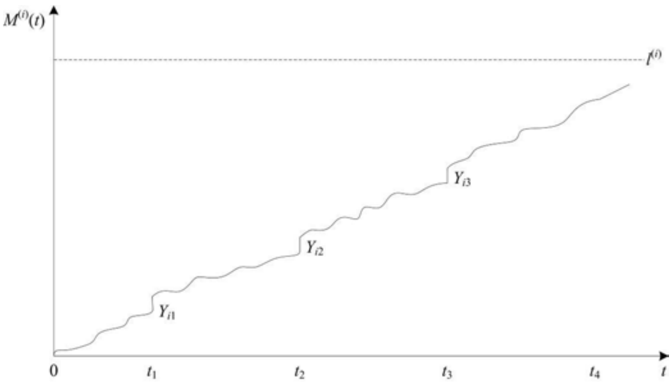

Fig. 3. Total degradation of the ith component

to exhibit reliability model, a summary of the main notations used in this paper is presented in Table 2.

Table 2. List of symbols and definitions

Symbol	Definition
$N_1(t)$	Number of general shocks at time t
$N_2(t)$	Number of fatal shocks at time t
W_U	Hard failure threshold for the shock process
W_L	A certain level only above which the sudden degradation increments occurs
W_{ij}	Magnitude of the jth general shock on the ith component
$\lambda(t)$	Intensity function of non-homogeneous Poisson process at time t
p_1	Probability of general shock
p_2	Probability of fatal shock
$D_i(t)$	Natural degradation at time t for the ith component
$S_i(t)$	Cumulative sudden degradation increments of the ith component caused by general shocks at time t
$M^{(i)}(t)$	Cumulative degradation of the ith component at time t
Y_{ij}	Sudden degradation increments of the ith component caused by the jth general shock
$\gamma_1^{(i)}$	Coefficient of general shock number on the ith component
$\gamma_2^{(i)}$	Coefficient of cumulative sudden degradation increments on the ith component
$l^{(i)}$	Soft failure threshold of the ith component
$T^{(i)}$	Failure time of the ith component
$R_M^{(i)}(t)$	Marginal reliability function of the ith component at time t
$R(t)$	System reliability function at time t
C	Copula function

3. Proposed reliability model with degradation rate acceleration

Recently, many reliability models have been developed to estimate the reliability of systems which are subject to multiple dependent competing risks. Song and David [24-25] considered that any shock load on the system increases the amount of degradation. However, the research by Tanner and Walraven [27] showed that no SDIs in micro-engine when shock loads are below a certain level. Thus, for systems with high reliability and long life, only those shocks, whose loads are above a certain level, can increase systems' degradation. However, general shocks may result in an increase in degradation rate as well as SDIs. Based on the analysis above, a new reliability model is proposed to contribute to the improvement of reliability evaluation. In the reliability model, DRA caused by general shock loads is paid attention to. Random shocks are divided into three parts: fatal shocks, general shocks and small shocks. In addition, the arrival time of random shocks is assumed to follow a NHPP instead of HPP.

3.1. Reliability analysis for shock process

Random shocks are likely to be introduced from the external environment. Most shocks are harmful to systems, decreasing residual useful life of systems. Even, some shocks may immediately cause the failure of the system.

As shown in Figure 2, for a system with m components, any shock arriving at the system will affect all the m components. Take the ith component for example, the magnitude of shock load imposed on it is denoted by W_{ij} (i=1, 2, …, m; j=1, 2, …, ∞), which is caused by jth shock reaching the system. The component fails once W_{ij} exceeds the shock threshold W_U. Assume W_{ij} imposed on the ith component of the system is independent and identically distributed ($i.i.d$) random variable. Then the cumulative distribution function of W_{ij} can be denoted by $F_{W_{ij}}(w)$, then the probability that the hard failure of the ith component does not occur under the jth shock is:

$$P(W_{ij} < W_U) = F_{W_{ij}}(W_U),\ i = 1,2,\ldots,m;\ j = 1,2,\ldots,\infty \tag{1}$$

To simplify the calculation process, W_{ij} for the ith component is assumed to follow a normal distribution $N(\mu_{W_i},\sigma_{W_i}^2)$. Then Equation (1) can be expressed by:

$$P(W_{ij} < W_U) = \phi(\frac{W_U - \mu_{W_i}}{\sigma_{W_i}}),\ i = 1,2,\ldots,m;\ j = 1,2,\ldots,\infty \tag{2}$$

3.2. Reliability analysis for degradation process

As is shown in Figure 3, the soft failure of the ith degradation process happens when total degradation $M^{(i)}(t)$ exceeds its degradation threshold $l^{(i)}$. The total degradation $M^{(i)}(t)$ includes continuous natural degradation amount and SDIs.

3.2.1. Analysis of SDI

It is assumed that the probabilities of shocks occurring at different time intervals are independent. Simultaneously, these random shocks are supposed to occur in a NHPP with an intensity function:

$$\lambda(t) = re^{ct},\ r \in (0,\infty),\ c \in (-\infty,+\infty) \tag{3}$$

Let $N(t)$ represent the number of random shocks until t. Then the numbers of general shocks and fatal shocks can be denoted by $N_1(t)$ and $N_2(t)$, respectively. The probability of general shocks is $p_1=P(W_L<W_{ij}<W_U)$. And the probability of fatal shocks can be calculated as $p_2=P(W_{ij}>W_U)$. According to the decomposition method of Poisson process, the arrival time of general shocks follows a NHPP with an intensity function:

$$\lambda_1(t) = p_1\lambda(t) = p_1 re^{ct},\ r \in (0,\infty),\ c \in (-\infty,+\infty) \tag{4}$$

Similarly, the arrival time of fatal shocks also follows a NHPP with an intensity function:

$$\lambda_2(t) = p_2\lambda(t) = p_2 re^{ct},\ r \in (0,\infty),\ c \in (-\infty,+\infty) \tag{5}$$

Then the numbers of arrivals for general shocks and fatal shocks at time t are given by:

$$W_1(t) = E\left[N_1(t)\right] = \int_0^t p_1 re^{cs}ds = \begin{cases} p_1 \cdot \frac{r}{c}(e^{ct} - 1),\ c \neq 0 \\ p_1 \cdot rt,\ c = 0 \end{cases} \tag{6}$$

$$W_2(t) = E\left[N_2(t)\right] = \int_0^t p_2 re^{cs}ds = \begin{cases} p_2 \cdot \frac{r}{c}(e^{ct} - 1),\ c \neq 0 \\ p_2 \cdot rt,\ c = 0 \end{cases} \tag{7}$$

Therefore, the probability of n general shocks and n fatal shocks reaching the system are calculated by Equation (8) and Equation (9), respectively:

$$P(N_1(t) = n) = \frac{(W_1(t))^n}{n!}e^{-W_1(t)} \tag{8}$$

$$P(N_2(t) = n) = \frac{(W_2(t))^n}{n!}e^{-W_2(t)} \tag{9}$$

General shocks usually cause additional damages to degradation processes. Here, we utilize Y_{ij} (i=1, 2, …, m; j=1, 2, …, ∞) to represent the SDIs in ith degradation process caused by the jth general shock arriving at the system. What is more, Y_{ij} greatly depends on the magnitude of shock load W_{ij} (i=1, 2, …, m; j=1, 2, …, ∞). Y_{ij} is described by a linear function, which reflects the difference between the magnitude of shock load W_{ij} and the certain level W_L, namely:

$$Y_{ij} = b(W_{ij} - W_L) \tag{10}$$

where b is a constant, which indicates the SDIs in degradation process caused by a unit change in the magnitude of shock load. Then, the cumulative degradation caused by general shocks at time t can be written as:

$$S_i(t) = \begin{cases} \sum_{j=1}^{N_1(t)} Y_{ij},\ N_1(t) > 0 \\ 0,\ N_1(t) = 0 \end{cases} \tag{11}$$

3.2.2. Analysis of continuous natural degradation

Consider a system with unknown soft failure threshold, which is a fixed quantity. The system is placed under an environment with accelerated stress levels, which is steadily increased until the failure of the system. Assume the increasing loads are converted to discrete values so the stress is incremented by small, discrete amounts until the system fails. And each small increment of stress causes a nonnegative damage amount, which is a random variable and denoted by X. According to [7, 13, 23], the cumulative damage after n+1 increments of stress is denoted by:

$$D_{n+1} = D_n + X_n \cdot \eta(D_n) \tag{12}$$

where $\eta(\mu)$ is the damage model function. For example, $\eta(\mu)$=1 gives an additive damage model, whereas $\eta(\mu)=\mu$ gives a multiplicative damage model. In fact, the stress is incremented by continuous amounts, so the cumulative damage of the system at t should be:

$$D(t) = X \cdot \eta(t) \tag{13}$$

In this paper, a multiplicative path function is used to reflect the item-to-item variation. Then the degradation model of the ith degradation process for a series system with m components is given by:

$$D_i(t) = X_i \cdot \eta_i(t) \tag{14}$$

where X_i is a random variable, $\eta_i(t)$ represents the mean degradation path in the ith degradation process, which is either monotonically decreasing or monotonically increasing.

The cumulative degradation $M^{(i)}(t)$ at time t for the ith degradation process consists of natural degradation and the SDIs caused by the general shocks, which is written as:

$$M^{(i)}(t) = D_i(t) + S_i(t) \tag{15}$$

The first term $D_i(t)$ refers to the continuous natural degradation of the ith degradation process. And the second term $S_i(t)$ shown in Equation (11) reflects the SDIs in the ith degradation process.

In this section, DRA is considered into the degradation model because the general shocks may accelerate the internal clock of systems. To embody the effect of DRA, a time-scaled covariate factor is used. A new term $G(t,\gamma^{(i)})$ is introduced into $D_i(t)$ for the ith degradation process through the time-scaled model of accelerated life testing. Here, the ith degradation path $D_i(t)$ is scaled by an accelerated factor from t to $te^{G(t,\gamma(i))}$. Thus, Equation (15) can be rewritten as:

$$M^{(i)}(t) = X_i\eta_i(te^{G(t,\gamma^{(i)})}) + \sum_{j=1}^{N_1(t)} Y_{ij} \tag{16}$$

where $G(t,\gamma^{(i)}) = \gamma_1^{(i)}N_1(t) + \gamma_2^{(i)}\sum_{j=1}^{N_1(t)} Y_{ij}$, and the vector parameters $\gamma^{(i)}$ are unknown. Note that the first term in function $G(t,\gamma^{(i)})$ embodies the effect from the number of general shocks towards the ith degradation process. In general, we have $\gamma^{(i)}\geq 0$, and the first term reflects the fact that the degradation rate likely increases with the increase of the number of general shocks. If $\gamma_1^{(i)} = 0$, it signifies that the degradation rate does not be affected by the number of general shocks. The second term is developed to present the situation that the cumulative SDIs may give rise to an accelerated degradation rate of systems. Likewise, it indicates that the cumulative SDIs have no effect on the degradation rate of systems if $\gamma_2^{(i)} = 0$.

3.3. Reliability modeling for systems with a shock process and multiple degradation processes

3.3.1. System reliability model

For a series system with m components, which is subject to a shock process and m degradation processes, the measurements of m degradation processes at time t are denoted by $M(t)=\{M^{(1)}(t), M^{(2)}(t), \ldots, M^{(m)}(t)\}$. The system fails once any degradation process reaches its soft failure threshold. The soft failure thresholds corresponding to the m degradation processes are denoted by $\boldsymbol{L}=\{l^{(1)}, l^{(2)}, \ldots, l^{(m)}\}$. Meanwhile, the hard failure occurs once any fatal shock arrives at the system. Hence, only when there is no fatal shock and the degradation amount of each degradation process keeps below its soft failure threshold, the system is in the working state. Let $T^{(i)}$ be the time to failure for the ith degradation process. Then, the reliability of the series system subject to a shock process and m degradation processes at time t can be expressed by:

$$\begin{aligned}R(t) &= P[T^{(1)} > t, T^{(2)} > t, \ldots, T^{(m)} > t] \times P[N_2(t) = 0] \\ &= P[M^{(1)} < l^{(1)}, M^{(2)} < l^{(2)}, \ldots, M^{(m)} < l^{(m)}] \times P[N_2(t) = 0]\end{aligned} \tag{17}$$

If the m degradation processes are assumed to be independent, the reliability of the system presented in Equation (17) can be rewritten as:

$$R(t) = R_M^{(1)}(t) \times R_M^{(2)}(t) \times \ldots \times R_M^{(m)}(t) \times P[N_2(t) = 0] \tag{18}$$

where $R_M^{(m)}(t)$ indicates the marginal reliability of the mth degradation process at t. However, Equation (18) can't provide precise estimation of system reliability because these degradation processes are not independent with each other. Therefore, the dependence relationship between any two degradation processes should be paid attention to.

Similarly, for a parallel system with m components, which is subject to m degradation processes and a shock process, the reliability of this system is calculated as:

$$\begin{aligned}R(t) &= 1 - [1 - P(T^{(1)} > t)] \times [1 - P(T^{(2)} > t)] \times \ldots \times [1 - P(T^{(m)} > t)] \times P[N_2(t) = 0] \\ &= 1 - [1 - P(M^{(1)} < l^{(1)})] \times [1 - P(M^{(2)} < l^{(2)})] \times \ldots \times [1 - P(M^{(m)} < l^{(m)})] \times P[N_2(t) = 0]\end{aligned} \tag{19}$$

From Equation (19), it is found that there is a dependence relationship between multiple degradation processes. Obviously, the reliabilities of other systems of any structure functions can be easily calculated according to their structures through the definition of reliability.

As we know, copula method is a powerful statistical tool to specify joint distribution if the known marginal distributions are complex. Thus, a copula method is utilized to develop the dependent structure among multiple degradation processes. In Sections 3.3.2 and 3.3.3, we use a series system as a representative system to represent the copula method.

3.3.2. Copula method for marginal reliability function of degradation processes

In probability theory and statistic, a copula is a multivariate probability distribution for which the marginal probability distribution of each variable is uniform. Copulas are used to describe the dependence structure between random variables. Any multivariate joint distribution can be written in terms of univariate marginal distribution functions [18]. Consider a random vector $(X_1, X_2, \ldots, X_m)$. Suppose its marginals are continuous, and the marginal CDFs $F_i(x)=P(X_i\leq x)$ are continuous functions. Sklar's theorem states that every multivariate cumulative distribution function $H(x_1, x_2, \ldots, x_m)=P(X_1\leq x_1, \ldots, X_m\leq x_m)$ of the random vector $(X_1, X_2, \ldots, X_m)$ can be expressed in terms of its marginals $F_i(x)=P(X_i\leq x_i)$ and a copula C:

$$H(x_1, x_2, \ldots, x_m) = C(F_1(x_1), F_2(x_2), \ldots, F_m(x_m)) \tag{20}$$

The copula C contains all information on the dependence structure between the components of $(X_1, X_2, \ldots, X_m)$, whereas the marginal cumulative distribution function $F_i(x_i)$ contains all information of the marginal distribution.

The CDF of the time to failure for the m degradation processes in a series system can be expressed as $F_i(t)=1-R_i(t)$ $(i=1, 2, \ldots, m)$. The joint CDF of $T_1, T_2, \ldots, T_m$ is written as:

$$P(T_1 \leq t_1, T_2 \leq t_2, \ldots, T_m \leq t_m) = H(t_1, t_2, \ldots, t_m) = C(F_1(t_1), F_2(t_2), \ldots, F_m(t_m)) \tag{21}$$

Correspondingly, the marginal reliability for the system is expressed as:

$$P(T_1 > t_1, T_2 > t_2, \ldots, T_m > t_m) = \overline{H}(t_1, t_2, \ldots, t_m) = C(R_1(t_1), R_2(t_2), \ldots, R_m(t_m)) \tag{22}$$

The relationship between $C(F_1(t_1), F_2(t_2), \ldots, F_m(t_m))$ and $C(R_1(t_1), R_2(t_2), \ldots, R_m(t_m))$ is represented as:

$$C(R_1(t_1), R_2(t_2), \ldots, R_m(t_m)) = 1 - \sum_{i=1}^{m} F_i(t_i) + \sum_{1 \le i < h \le m} C(F_i(t_i), F_h(t_h), \ldots)$$
$$- \sum_{1 \le i \le h < k \le m} C(F_i(t_i), F_h(t_h), F_k(t_k), \ldots) + (-1)^m C(F_1(t_1), F_2(t_2), \ldots, F_m(t_m)) \quad (23)$$

Therefore, the marginal reliability of the series system subject to m degradation processes at time t is expressed as:

$$\begin{aligned} R(t) &= P(T_1 > t, T_2 > t, \ldots, T_m > t) \\ &= P(M^{(1)}(t) < l^{(1)}, M^{(2)}(t) < l^{(2)}, \ldots, M^{(m)}(t) < l^{(m)}) \\ &= C(R_M^{(1)}(t), R_M^{(2)}(t), \ldots, R_M^{(i)}(t)) \end{aligned} \quad (24)$$

where $R_M^{(i)}(t)$ denotes the marginal reliability function for the ith degradation process at time t.

In particular, based on Equation (23) and Equation (24), the marginal reliability function for a system subject to two degradation processes is computed by:

$$\begin{aligned} P(M^{(1)}(t) < l^{(1)}, M^{(2)}(t) < l^{(2)}) &= C(R_M^{(1)}(t), R_M^{(2)}(t)) \\ &= 1 - F_M^{(1)}(t) - F_M^{(2)}(t) + C(F_M^{(1)}(t), F_M^{(2)}(t)) \\ &= R_M^{(1)}(t) + R_M^{(2)}(t) - 1 + C(u,v) \end{aligned} \quad (25)$$

where $u = F_M^{(1)}(t) = 1 - R_M^{(1)}(t)$, $v = F_M^{(2)}(t) = 1 - R_M^{(2)}(t)$.

3.3.3. Calculation steps for the system reliability

In this paper, a maximum likelihood estimation (MLE) is introduced to complete the statistical inference for copula. In stage 1, we should calculate marginal reliability function for the ith degradation process. The system does not fail due to the ith degradation process until t only if the cumulative degradation is below its soft failure threshold $l^{(i)}$ conditioned on the event that there is no fatal shock. Hence, the marginal reliability function for the ith degradation process at time t is given by:

$$R_M^{(i)}(t) = P(M^{(i)}(t) < l^{(i)}) = \sum_{n=0}^{\infty} P(X_i \cdot \eta_i(te^{G(t,\gamma^{(i)})}) + S_i(t) < l^{(i)} | N_1(t) = n) P(N_1(t) = n) \quad (26)$$

Suppose that random variables W_{ij} (i=1, 2, …, m; j=1, 2, …, ∞) are $i.i.$d. Based on Equation (10) and (11), these random variables Y_{ij} (i=1, 2, …, m; j=1, 2, …, ∞) are also $i.i.$d. In this section, Y_{ij} (i=1, 2, …, m; j=1, 2, …, ∞) are assumed to follow a common distribution $Q_i(x)$ (i=1, 2, …, m) in the ith degradation process. Let $Q_i^{(j)}(z) = P(Y_{i1} + Y_{i2} + \ldots + Y_{ij} \le z)$ for j=1, 2, …, ∞. The marginal reliability function for the ith degradation process at time t can be rewritten as:

$$\begin{aligned} R_M^{(i)}(t) &= P(X_i \cdot \eta_i(t) < l^{(i)}) P(N_1(t) = 0) + \sum_{n=1}^{\infty} P(N_1(t) = n) \times \int_{z=0}^{l^{(i)}} P(X_i \cdot \eta_i(te^{\gamma_1^{(i)} n + \gamma_2^{(i)} z}) + z < l^{(i)}) dQ_i^{(n)}(z) \\ &= \exp(-\int_0^t \lambda_1(s) ds) \times F_{X_i}\left(\frac{l^{(i)}}{\eta_i(t)}\right) + \sum_{n=1}^{\infty} \frac{\exp(-\int_0^t \lambda_1(s) ds) \times (\int_0^t \lambda_1(s) ds)^n}{n!} \times \int_{z=0}^{l^{(i)}} F_{X_i}\left(\frac{l^{(i)} - z}{\eta_1(te^{\gamma_1^{(i)} n + \gamma_2^{(i)} z})}\right) dQ_i^{(n)}(z) \end{aligned} \quad (27)$$

where the number of general shocks is denoted by n, and $S_i(t)$ is replaced by z. Assume $Q_i(x)$ follows a normal distribution with mean μ_{Yi} and variance $\sigma_{Y_i}^2$. Then, $Q_i^{(n)}(z)$ also follows a normal distribution $N(n\mu_{Y_i}, n\sigma_{Y_i}^2)$. Then, the marginal reliability probability $R_M^{(i)}(t)$ of the ith degradation process at discrete time points can be calculated by Equation (27). When m=2, we can obtain a group of data of u and v at the corresponding time t.

In stage 2, we firstly estimate parameters in the copula $C(F_1(t_1), F_2(t_2), \ldots, F_m(t_m))$ using the values of u and v through the Copulafit function provided by MATLAB. A summary of the common copulas for two variates is presented in Table 3. Then, a likelihood criterion is used to check the goodness of fitting and specify the optimal copula. The likelihood function of the bivariate copula is given by:

$$L = \sum_{k=1}^{m_0} \ln c(F_M^{(1)}(t_k), F_M^{(2)}(t_k) | \alpha) \quad (28)$$

where m_0 represents the number of discrete time points, and α is the parameter in copulas. The copula with the biggest likelihood value is the most suitable copula, which can be used to describe the dependence between any two degradation processes.

Finally, the probability that the system will not fail due to degradation processes is calculated according to Equation (24). Thus, the specific formulation of the system reliability becomes:

$$R(t) = C(R_M^{(1)}(t), R_M^{(2)}(t), \ldots, R_M^{(m)}(t)) P[N_2(t) = 0] \quad (29)$$

If m=2, the reliability of the system at time t is derived as:

$$R(t) = [R_M^{(1)}(t) + R_M^{(2)}(t) - 1 + C(u,v)] \exp(-\int_0^t \lambda_2(s) ds) \quad (30)$$

where $\lambda_2(t)$ is shown in Equation (5). Thus, the simplified steps to calculate the system reliability with copula approach are shown in Table 4.

4. Numerical example

Microelectromechanical systems (MEMS) oscillators are timing devices that generate highly stable reference frequencies to sequence electronic systems, manage data transfer and measure elapsed time. MEMS oscillators vibrate at their natural resonant frequency. Due to the working loss of operation, the mass of MEMS oscillators decreases after a period of time. The decrease of mass may cause an increase in the frequency of vibration, which is an obvious common phenomenon that exists in MEMS oscillators. On the other hand, thermal shock, jitter and other vibration from the environment can bring about the change of system frequency. If these shocks or vibration are large enough, hard failure will occur to the MEMS oscillators. MEMS oscillators are particularly interesting and typical systems which are subject to multiple degradation processes and a random shock process [26]. Besides, these processes are dependent and compete with each other.

4.1. Reliability modeling

In this example, the reliability model of microelectromechanical systems (MEMS) will be developed. Assume MEMS oscillators subject to two degradation processes and a shock process operate in unstable environment. The arrival time of shocks follows a NHPP with an intensity function $\lambda(t)$. Meanwhile, for the first degradation path,

Table 3. Commonly used copulas

Copula	C (u, v|α)	α∈Ω
Gauss	$\int_{-\infty}^{\phi^{-1}(u)}\int_{-\infty}^{\phi^{-1}(v)}\frac{1}{2\pi\sqrt{1-a^2}}\exp\left(\frac{2asw-s^2-w^2}{2(1-a^2)}\right)dsdw$	α∈[-1,1]
t	$\int_{-\infty}^{T_v^{-1}(u)}\int_{-\infty}^{T_v^{-1}(v)}\frac{1}{2\pi\sqrt{1-a^2}}\left(1+\frac{s^2+w^2-2asw}{v(1-a^2)}\right)^{\left(-\frac{v+2}{2}\right)}dsdw$	α∈[-1,1]
Frank	$-\frac{1}{\alpha}\ln\left(1+\frac{(e^{-au}-1)(e^{-av}-1)}{e^{-a}-1}\right)$	α∈(-∞, ∞)/{0}
Gumbel	$\exp\left\{-[(-\ln u)^{1/\alpha}+(-\ln v)^{1/\alpha}]^{\alpha}\right\}$	α∈(0, ∞)
Clayton	$(u^{-a}+v^{-a}-1)^{-1/\alpha}$	α∈(0, ∞)

Table 4. The calculation steps of system reliability

I Marginal Reliability Function for Degradation Processes

1. Calculate the marginal reliability function for each degradation process.
2. Calculate the marginal reliability probability for each degradation process at discrete time points.

II Joint Reliability Function for Degradation Processes

3. Estimate the parameters in different copulas based on the marginal reliability probability.
4. Find out the most suitable copula through MLE.
5. Calculate the joint copula $C(u, v)$, and then calculate the reliability of the system subject to two degradation processes.

Table 5. Parameters for MEMS oscillators subject to a shock process and two degradation processes

Parameter	Value	Source
μ_{Wi}	5	Wang et al. (2012)
σ_{Wi}	3	Wang et al. (2012)
$\gamma_1^{(1)}$	0.05	Wang et al. (2012)
$\gamma_2^{(1)}$	0.008	Wang et al. (2012)
$\gamma_1^{(2)}$	0.01	Wang et al. (2012)
$\gamma_2^{(2)}$	0.005	Wang et al. (2012)
$l^{(1)}$	100	Wang et al. (2012)
$l^{(2)}$	3.5	Wang et al. (2012)
W_U	8	Wang et al. (2012)
W_L	2	Assumption
b	0.6	Assumption
c	0.01	Guo et al. (2013)
r	0.1	Guo et al. (2013)
μ	0.8	Wang et al. (2012)
k	1	Wang et al. (2012)
θ	4.8	Wang et al. (2012)
α	20	Assumption
β	0.01	Wang et al. (2012)

we set $D_1(t; X_1) = X_1 t$, in which X_1 is Weibull distributed with a CDF $F_{X_1}(x) = 1-\exp[-(x/\mu)^k]$, μ, k>0. For the second degradation path, suppose that $D_2(t; X_2)=X_2\log[\theta_1+t]$, θ_1>0, in which X_2 is gamma distributed with a pdf $f_{X_2}(x) = x^{\alpha-1}e^{-(x/\beta)}/\beta^{\alpha}\Gamma(\alpha)$, x≥0. The soft failure threshold of the two degradation processes are denoted by $l^{(1)}$ and $l^{(2)}$. The system fails once the total degradation of any degradation process reaches its soft failure threshold. In addition, the shock loads towards the two degradation processes are assumed to follow a normal distribution with mean μ_{W_i} and variance $\sigma_{W_i}^2$. The model parameters are presented in Table 5.

Substituting these parameters listed in Table 5 into Equation (27), we can obtain the marginal reliability functions of two degradation processes. Then, the two functions are plotted in Figure 4.

As shown in Figure 4, in the early stage of system operation, the marginal reliability of the system under the second degradation process decreases faster compared to the first degradation process. In the later period of the system operation, the marginal reliability of the system under the first degradation process is slowly approaching 0, while the marginal reliability of the system under the second degradation process drops rapidly to 0. The marginal reliability for the first degradation process becomes close to zero when the running time of the system is about 150 hours, while the marginal reliability for the second degradation process is almost 0 when the system works for 75 hours. The values of marginal reliability for two degradation processes at some discrete time points are given in Table 6.

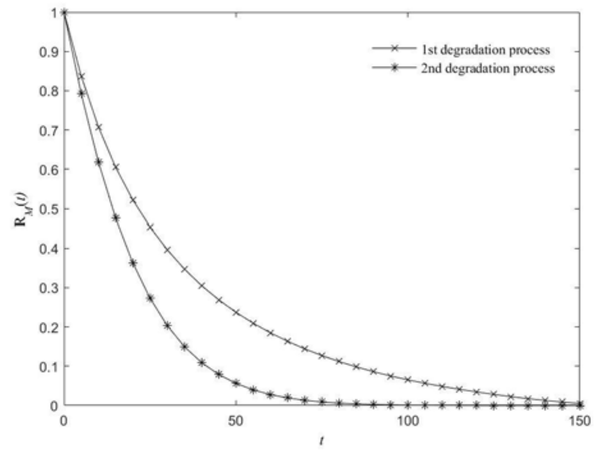

Fig. 4. Marginal reliability function for two degradation processes

According to the properties of copula function, we can utilize the marginal reliability function to obtain the joint reliability function for degradation processes by using Copulafit function provided by

Table 6. Simulated marginal reliability and values of u and v for two degradation processes

t	0	5	10	15	20	25	30	35	40
$R_M^{(1)}(t)$	1	0.8373	0.7078	0.6050	0.5222	0.4540	0.3965	0.3475	0.3054
$R_M^{(2)}(t)$	1	0.7923	0.6185	0.4767	0.3631	0.2734	0.2036	0.1500	0.1093
u	0	0.1627	0.2922	0.3950	0.4778	0.5460	0.6035	0.6525	0.6946
v	0	0.2077	0.3815	0.5233	0.6369	0.7266	0.7964	0.8500	0.8907

Table 7. Likelihood values and correlated parameters under five copulas

Copula	Likelihood value	Parameter
t	4.0099	α=0.9934, v=1.9724
Gaussian	5.9222	α=0.9930
Gumbel	1.1569	α=17.9540
Clayton	7.4402	α=13.7277
Frank	29.28	α=54.2398

Table 8. Values of C(u, v) at discrete time points under five copulas

Copula	$C(u,v)$ at t								
	0	5	10	15	20	25	30	35	40
t	0	0.1611	0.2907	0.3939	0.4771	0.5455	0.6032	0.6523	0.6945
Gaussian	0	0.1616	0.2919	0.3950	0.4778	0.5460	0.6035	0.6525	0.6946
Gumbel	0	0.1615	0.2920	0.3950	0.4778	0.5460	0.6035	0.6525	0.6946
Clayton	0	0.1623	0.2917	0.3944	0.4771	0.5452	0.6026	0.6514	0.6933
Frank	0	0.1612	0.2921	0.3950	0.4778	0.5460	0.6035	0.6525	0.6946

Table 9. Comparisons of system reliability under five copulas and independence assumption

Copula	$R(t)$								
	0	5	10	15	20	25	30	35	40
t	1	0.7289	0.5222	0.3679	0.2550	0.1739	0.1167	0.0770	0.0500
Gaussian	1	0.7294	0.5232	0.3687	0.2555	0.1742	0.1168	0.0771	0.0501
Gumbel	1	0.7293	0.5233	0.3687	0.2555	0.1742	0.1168	0.0771	0.0501
Clayton	1	0.7301	0.5230	0.3683	0.2550	0.1737	0.1163	0.0766	0.0495
Frank	1	0.7290	0.5234	0.3687	0.2555	0.1742	0.1168	0.0771	0.0501
Independence	1	0.6634	0.4378	0.2884	0.1896	0.1241	0.0807	0.0521	0.0334

MATLAB. In this article, t-copula, Gaussian copula, Gumbel copula, Clayton copula and Frank copula are used to fit the joint reliability function for two degradation processes. MLE is employed to estimate the parameters of different copulas based on the values of marginal failure functions, namely, u and v listed in Table 6. Thus, the likelihood values and parameter values under five kinds of copulas are listed in Table 7.

Through MATLAB, the values of five copula functions based on u and v in Table 6 are obtained and listed in Table 8. Then the reliabilities of the system shown in Table 9 at different discrete time points are calculated according to Equation (30). And the reliabilities of the system at some discrete time points when the two degradation processes are assumed to be independent are also listed in Table 9. From Table 9, it is noted that the system reliabilities at different time points under those five copulas are similar, while they are bigger than that when the two degradation processes are assumed to be independent. It indicates that the reliability of the series system will be underestimated if not considering the independence between multiple degradation processes.

We find from the five likelihood values in Table 7 that Frank copula is the most appropriate copula for fitting joint reliability for degradation processes of the system because it has the maximal likelihood estimation value 29.28. Therefore, the reliability of the system should be calculated under Frank copula. The system reliability functions based on dependence and independence assumption are plotted in Figure 5. From Figure 5, the system reliability decreases quickly when two degradation processes are assumed to be independent. An interesting finding is that the system reliability is relatively high under the assumption that the two degradation processes are dependent. It also indicates that the reliability of the series system will be underestimated when the two degradation processes are assumed to be independent. The finding has been clearly stated and explained in [4].

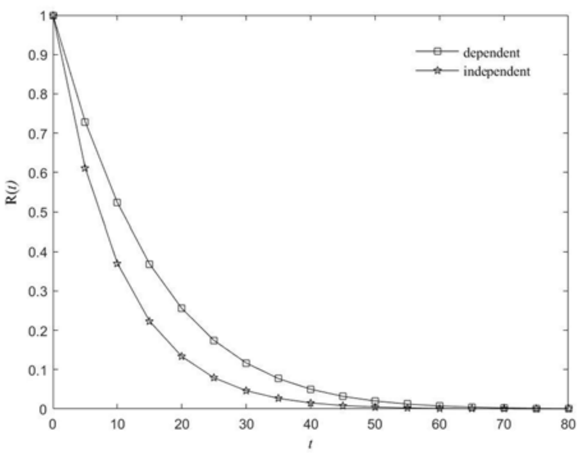

Fig. 5. System reliability function R(t)

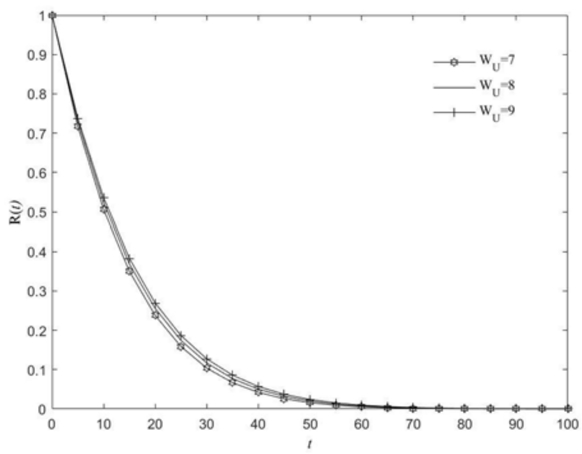

Fig. 6. Sensitivity analysis of R(t) on WU

Therefore, it is very essential and important for reliability engineers to take the dependence among multiple degradation processes into account.

4.2. Sensitivity analysis

In this section, sensitivity analysis is performed to estimate the effects of the model parameters W_U and W_L on the system reliability $R(t)$. The results are exhibited in Figure 6 and Figure 7. As shown in Figure 6, we can find that $R(t)$ is sensitive to the hard failure threshold W_U. When the hard failure threshold W_U increases from 7 to 9, the system reliability $R(t)$ shifts to right, which embodies a better reliability performance for a larger value of W_U. It indicates that the system has the capacity to resist external shocks as the improvement of system reliability. Moreover, as shown in Figure 7, the system reliability $R(t)$ shifts to the right when the certain level W_L increases from 1 to 3. It indicates that the bigger the certain level W_L, the better the performance of systems. The reason is that if the performance of the system is good enough, small shocks will not have any impact on the system. It is obvious that a system with a certain level 3 is more reliable than systems with a certain level 1 or 2. From the viewpoint of sensitivity analysis, we know that the accuracy of the hard failure threshold and the certain level is very important for the reliability of systems.

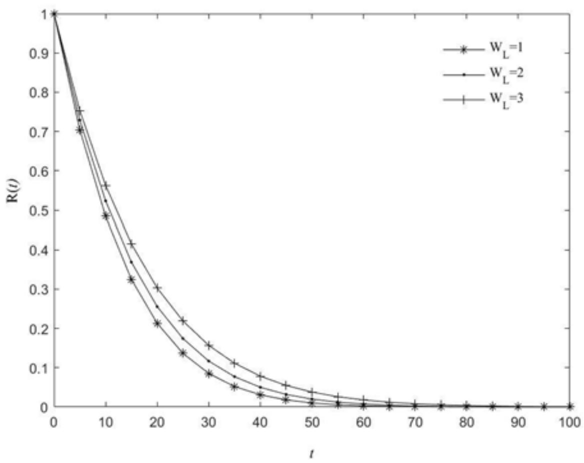

Fig. 7. Sensitivity analysis of R(t) on WL

5. Conclusions

A new reliability model for systems subject to multiple dependent competing risks has been proposed. The DRA caused by random shocks into each degradation process is considered in the novel reliability model. The dependence relationship among different degradation processes has been dealt with by a copula method. Meanwhile, the shock process is assumed to be a NHPP instead of HPP. A numerical example is presented to verify the feasibility of the proposed model. Furthermore, a detailed analysis about model parameters has been done through sensitivity analysis. Based on the results of the numerical example, it can be concluded that the proposed reliability model is very suitable for complex systems subject to multiple dependent competing risks, and it can assess the system reliability more factually. In addition, three interesting research findings are obtained through the analysis of the reliability model:

(1) The reliabilities of series systems will be underestimated when multiple degradation processes are assumed to be independent. Therefore, it is very necessary to consider the dependence among different degradation processes.

(2) The assumption that the arrival time of random shocks follows a NHPP is more reasonable when a system works in complex environment.

(3) The series system reliability changes with the values of the hard failure threshold W_U and the certain level W_L. The greater the values of W_U and W_L, the higher the system reliability.

In the numerical example, for convenience, multiple degradation processes is simplified to two degradation processes. However, a system always experiences multiple different degradation processes when it is in a working state. Therefore, a challenging work is to calculate the reliability of systems subject to multiple degradation processes and a shock process. Furthermore, the aim of reliability assessment is to monitor systems real-timely and ensure the system reliability at a predetermined level by adopting some maintenance policies. Hence, the future work can be extended to develop a condition-based maintenance policy for systems subject to multiple competing risks.

Acknowledgements

The research work was supported by grants from the China Scholarship Council (201706840122), the National Natural Science Foundation of China (NSFC 71471088, 71702072, 71871119), and the Natural Science Foundation for Jiangsu Institutions (BK20170810), which are gratefully acknowledged.

References

1. An Z, Sun D. Reliability modeling for systems subject to multiple dependent competing failure processes with shock loads above a certain level. Reliability Engineering and System Safety 2017; 157: 129-138, https://doi.org/10.1016/j.ress.2016.08.025.
2. Bocchetti D, Giorgio M, Guida M. A competing risk model for the reliability of cylinder liners in marine diesel engines. Reliability Engineering and System Safety 2009; 94(8): 1299-1307, https://doi.org/10.1016/j.ress.2009.01.010.
3. Bae S J, Kuo W, Kvam P H. Degradation models and implied lifetime distributions. Reliability Engineering and System Safety 2007; 92(5), 601-608, https://doi.org/10.1016/j.ress.2006.02.002.
4. Barlow R E, Proschan F. Statistical theory of reliability and life testing: probability models. Florida: Florida State University Tallahassee, 1975.
5. Cha J H, Pulcini G. A dependent competing risks model for technological units subject to degradation phenomena and catastrophic failures. Quality and Reliability Engineering International 2016; 32(2), 505-517, https://doi.org/10.1002/qre.1767.
6. Durham S D, Padgett W J. Cumulative damage models for system failure with application to carbon fibers and composites. Technometrics 1997; 39(1): 34-44, https://doi.org/10.1080/00401706.1997.10485437.
7. Desmond A. Stochastic models of failure in random environments. Canadian Journal of Statistics 1985; 13(3): 171-183, https://doi.org/10.2307/3315148.
8. Guo C, Wang W, Guo B. Maintenance optimization for systems with dependent competing risks using a copula function. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2013; 15(1): 9-17.
9. Huynh K T, Castro I T, Barros A, Berenguer C. Modeling age-based maintenance strategies with minimal repairs for systems subject to competing failure modes due to degradation and shocks. European Journal of Operational Research 2012; 218(1): 140-151, https://doi.org/10.1016/j.ejor.2011.10.025.
10. Jiang R. Discrete competing risk model with application to modeling bus-motor failure data. Reliability Engineering and System Safety 2010; 95(9): 981-988, https://doi.org/10.1016/j.ress.2010.04.009.
11. Jiang L, Feng Q, Coit D W. Reliability and maintenance modeling for dependent competing failure processes with shifting failure thresholds. IEEE Transactions on Reliability 2012; 61(4): 932-948, https://doi.org/10.1109/TR.2012.2221016.
12. Jiang L, Feng Q, Coit D W. Modeling zoned shock effects on stochastic degradation in dependent failure processes. IIE Transactions 2015; 47(5): 460-470, https://doi.org/10.1080/0740817X.2014.955152.
13. Jiang C, Wang D. A sort of multiple life model under dependent causes of decrement. Statistical and Application 2015; 4(3): 169-175, https://doi.org/10.12677/SA.2015.43019.
14. Li W, Pham H. An inspection-maintenance model for systems with multiple competing processes. IEEE Transactions on Reliability 2005; 54(2): 318-327, https://doi.org/10.1109/TR.2005.847264.
15. Liu Z, Ma X, Shen L, Zhao Y. Degradation-shock-based reliability models for fault-tolerant systems. Quality and Reliability Engineering International 2016; 32(3), 949-955, https://doi.org/10.1002/qre.1805.
16. Mann N R, Schafer R E, and Singpurwalla N D. Methods for Statistical Analysis of Reliability and Life Data. New York: John Wiley & Sones, 1974.
17. Meeker W Q, Escobar L A. Statistical Methods for Reliability Data. New York: John Wiley & Sons, 1998.
18. Nelsen R B. An introduction to copulas. New York: Springer Science & Business Media, 2006.
19. Onar A, Padgett W J. Inverse Gaussian accelerated test models based on cumulative damage. Journal of Statistical Computation and Simulation 2000; 66(3): 233-247, https://doi.org/10.1080/00949650008812024.
20. Peng H, Feng Q, Coit D W. Reliability and maintenance modeling for systems subject to multiple dependent competing failure processes. IIE Transactions 2010; 43(1): 12-22, https://doi.org/10.1080/0740817X.2010.491502.
21. Padgett W J, Durham S D, and Mason A M. Weibull analysis of the strength of carbon fibers using linear and power law models for the length effect. Journal of Composite Materials 1995; 29(14): 1873-1884, https://doi.org/10.1177/002199839502901405.
22. Park C, Padgett W J. Stochastic degradation models with several accelerating variables. IEEE Transactions on Reliability 2006; 55(2): 379-390, https://doi.org/10.1109/TR.2006.874937.
23. Park C, Padgett W J. New cumulative damage models for failure using stochastic processes as initial damage. IEEE Transactions on Reliability 2005; 54(3): 530-540, https://doi.org/10.1109/TR.2005.853278.
24. Song S, Coit D W, Feng Q, Peng H. Reliability analysis for multi-component systems subject to multiple dependent competing failure processes. IEEE Transactions on Reliability 2014; 63(1): 331-345, https://doi.org/10.1109/TR.2014.2299693.
25. Song S, Coit D W, Feng Q. Reliability for systems of degrading components with distinct component shock sets. Reliability Engineering and System Safety 2014; 132: 115-124, https://doi.org/10.1016/j.ress.2014.06.020.
26. Song S, Coit D W, Feng Q. Reliability analysis of multiple-component series systems subject to hard and soft failures with dependent shock effects. IIE Transactions 2016; 48(8), 720-735, https://doi.org/10.1080/0740817X.2016.1140922.
27. Tanner D M, Walraven J A, Helgesen K, Irwin L W, Brown F, Smith N F, Masters N. MEMS reliability in shock environments. Proceedings of IEEE International Reliability Physics Symposium 2000; 129-138, https://doi.org/10.1109/RELPHY.2000.843903.
28. Wang Z, Huang H Z, Li Y. An approach to reliability assessment under degradation and shock process. IEEE Transactions on Reliability 2011; 60(4): 852-863, https://doi.org/10.1109/TR.2011.2170254.
29. Wang Y, Pham H. A multi-objective optimization of imperfect preventive maintenance policy for dependent competing risk systems with hidden failure. IEEE Transactions on Reliability 2011; 60(4): 770-781, https://doi.org/10.1109/TR.2011.2167779.
30. Wang Y, Pham H. Modeling the dependent competing risks with multiple degradation processes and random shock using time-varying copulas. IEEE Transactions on Reliability 2012; 61(1):13-22, https://doi.org/10.1109/TR.2011.2170253.
31. W. Nelson. Accelerated Testing: Statistical Models, Test Plans, and Data Analyses. New York: John Wiley & Sons, 1990, https://doi.org/10.1002/9780470316795.
32. Zhu Y, Elsayed E A, Liao H. Availability optimization of systems subject to competing risk. European Journal of Operational Research 2010; 202(3): 781-788, https://doi.org/10.1016/j.ejor.2009.06.008.

Yanjing ZHANG
School of Economics and Management
Nanjing University of Science and Technology
No. 200 Xiaolingwei, Xuanwu District, Nanjing, Jiangsu Province, China

Yizhong MA
School of Economics and Management
Nanjing University of Science and Technology
No. 200 Xiaolingwei, Xuanwu District, Nanjing, Jiangsu Province, China

Linhan OUYANG
College of Economics and Management
Nanjing University of Aeronautics and Astronautics
No. 29 General Avenue, Jiangning District, Nanjing, Jiangsu Province, China

Lijun LIU
School of Economics and Management
Nanjing University of Science and Technology
No. 200 Xiaolingwei, Xuanwu District, Nanjing, Jiangsu Province, China

E-mails: zhangyanjing_1211@163.com,yzma-2004@163.com, oylh871109@163.com, lijun_liu06@163.com