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EFFECT OF TEMPERATURE VARIATION ON REPEATABILITY POSITIONING OF A ROBOT WHEN ASSEMBLING PARTS WITH CYLINDRICAL SURFACES

WPŁYW ZMIAN TEMPERATURY NA POWTARZALNOŚĆ POZYCJONOWANIA ROBOTA PRZY MONTAŻU CZĘŚCI O POWIERZCHNIACH CYLINDRYCZNYCH*

In this paper, the effect of the errors induced by temperature changes on the repeatability positioning error of an industrial robot is analysed. It has been shown that after the stabilization of the thermal conditions, these errors can be identified with the systematic errors. It has also been shown that if the ambient temperature cannot be sufficiently stabilized, the temperature errors can be described using a normal or uniform probability distribution. Depending on the choice of a point in the robot's workspace and temperature fluctuations, these errors can comprise a small share of the total error of the robot. Then the total repeatability positioning error can be approximated with sufficient accuracy by a normal probability distribution or it can comprise the dominant component of this error. In this case, the total error can be approximated using a flat normal distribution. It has been shown that, depending on the choice of location in the workspace in which the assembly operation is carried out, it is possible to obtain both different probabilities of assembling the parts correctly and a different effect of errors caused by slight temperature changes on the value of those probabilities. The results found indicate the potential possibility of increasing the reliability of the process by proposing the selection of the location in the robot workspace which has the lowest sensitivity to errors ascribed to changes in temperature.

Keywords: repeatability positioning, robot's workspace, temperature, probability of parts joining.

W niniejszej pracy przeanalizowano wpływ zmiany temperatury na błąd powtarzalności pozycjonowania robota przemysłowego. Wykazano, że po ustabilizowaniu się warunków termicznych błędy te można sklasyfikować jako błędy systematyczne. Wykazano również, że jeżeli w trakcie eksploatacji zrobotyzowanego stanowiska montażowego temperatura otoczenia nie może być wystarczająco ustabilizowana, błędy temperatury można opisać za pomocą jednostajnego rozkładu prawdopodobieństwa i w ten sposób uwzględnić w strukturze całkowitego błędu powtarzalności pozycjonowania. Błędy te na ogół stanowią niewielki udział w całkowitym błędzie robota, wówczas całkowity błąd powtarzalności pozycjonowania robota z dostateczną dokładnością można aproksymować normalnym rozkładem prawdopodobieństwa. W przeciwnym przypadku błąd ten może być przybliżony przy użyciu rozkładu płasko-normalnego. Wykazano że w zależności od wyboru miejsca realizacji zabiegu montażowego w przestrzeni stanowiska można uzyskać zarówno odmienne wartości prawdopodobieństwa połączenia części jak i różny wpływ błędów wywołanych niewielkimi zmianami temperatury na wartość tego prawdopodobieństwa. Uzyskane wyniki badań wskazują na potencjalne możliwości zwiększenia niezawodności procesu poprzez wybór miejsca w przestrzeni roboczej stanowiska charakteryzującego się najniższą wrażliwością na błędy spowodowane zmianami temperatury.

Słowa kluczowe: powtarzalność pozycjonowania, przestrzeń robocza robota, temperatura, prawdopodobieństwo połączenia części.

1. Introduction

An important issue in the field of robotic assembly workstations is the problem of ensuring the required probability of the parts being joined, and thus ensuring the required capability of the assembly process. The usability of the elements to be joined into the assembly

units is a certain feature depending on the construction of the element, the method of joining and the construction of the assembly station. This feature can be referred to as mountability. The basic condition for achieving high reliability of the assembly work station is the fulfilment of the mounting condition for all parts that are being connected

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie www.ein.org.pl

[14]. In fact, due to the occurrence of accidental and systematic errors, these conditions cannot always be fully fulfilled. Therefore, the ratio of the number of connections assembled to the number of all connections can be taken as the probability of joining parts. From the point of view of work station reliability, it is advantageous to ensure a high probability of joining the parts, because this results in a reduction in the cost of its operation due to a lowering of the downtime [31]. Industrial robots, which are the main pieces of equipment on the stands, are delivered to the user with a very small amount of information regarding their accuracy. Usually the technical documentation only gives information on positioning repeatability.

In general, position error is the result of inaccuracies in the whole robotic system, typically categorized as [2]:

- controller errors due to the resolution of the axis encoder devices,
- algorithmic interpolation errors that take place throughout the movement of the robotic arm,
- kinematic errors, which mainly derive from inaccuracies of the kinematic robot model,
- dynamic errors related to the servo systems, friction, and inertia whilst moving,
- mechanical errors owing to manufacturing imprecision, joint wear, bearing wear, and temperature and flexibility deviations,
- errors due to flexibility of links.

The robots are usually constructed of beam-like links with open kinematic chains. As the number of robot links increases, the structure of the robot is more susceptible to generating errors related to positioning accuracy due to the inaccuracy of the kinematic system, inertia and change in the temperature of the environment. In conditions of unstable operating temperatures, the length of the robot links has a significant impact on the accuracy of the robot. A minimisation of the thermal effect can be obtained by using materials which have low thermal expansion in the robot's construction or by implementing an empirical model of error correction based on the signals from several temperature sensors placed inside the robot arm [30]. To avoid the problem of thermal expansion of the robot links, manufacturers use thermally stable materials, such as fibre-reinforced plastics or use isolated heat sources [24]. The process of manufacturing robot links and kinematic pairs introduces some differences in dimensions. In practice, the actual physical zero position and the physical zero position read by the robot controller is affected by errors.

In many cases, a significant proportion of the factors affecting robot error is subjected to constant change, sometimes accidental, which leads to differences between the mathematical models and real characteristics [12, 15]. Works can be found in the literature which are devoted to reducing robot errors by calibration, using laser and vision sensors placed at the end of the grip of the robot [7, 13, 32], selecting the optimal location in the robot workspace [17] and choosing the proper direction to get to the nominal position [16].

The robot calibration procedure consists of four stages [4]: modelling, measurement, identification and compensation. Slamani et al. [28] performed the positioning analysis of an ABB IRB1600 robot using the FARO laser tracker. The results presented by the authors showed that calibration allows one to reduce the robot error by a factor of three. Zhenhua et al. [33] present an attempt to calibrate a 6-DOF robot, using the MDH model (Modified Denavit-Hartenberg model). The measurements were carried out using a Leica AT901 B tracker. The maximum positioning deviation before calibration was about 2500 μm , while after calibration it was reduced to below 1000 μm . Płaczek and Piszczek [23] indicated that laser trackers (Faro Vantage tracker) form an effective method of determining the accuracy and repeatability of the mapping of the trajectory by the robot being examined. The idea of identifying accuracy errors and their calibration using CCD cameras is presented by Abderrahim et al. [1]. The

robot positioning deviation measured before this calibration was 3250 μm , but after calibration it was reduced to 290 μm .

Positioning repeatability is a measure of the robot's ability to return to the same position [18], while accuracy is defined as the robot's ability to move precisely to the desired position in three-dimensional space [18]. Procedures for assessing repeatability and accuracy are set out in international standard EN ISO 9283:2003 [10].

Although many researchers have investigated methods for compensating for the geometrical errors of robots, the error related to thermal deformation has not been discussed in detail in the literature, as was noted by Eastwood and Webb in 2009 [8], and Li and Zhao in 2016 [20]. The precision of robots and machine tools is constantly growing, which requires the taking into account of an increasing number of factors affecting their accuracy. In real production conditions, it is difficult to ensure stable environmental conditions during the operation of a robotic assembly station. Small changes in ambient conditions, in particular temperature changes, are often accidental. Therefore, compensation for their impact requires the use of vision or laser measurement systems, which increases the cost of the station [30]. This article analyses the impact of temperature-induced errors on the structure of the total repeatability positioning error of an industrial robot. Based on the experiments conducted, a methodology of error summation was proposed, which was used to determine the effect of temperature changes on the probability of joining machine parts with cylindrical surfaces and the capability of the process.

2. Kinematic error of robot

During assembly processes, the robot's gripper at any moment should occupy a precise position in space set by programmed joint coordinate values q_i . Any characteristic position of the M point of the gripper (Fig. 1) can be determined, in an accepted stationary coordinate arrangement, by a certain function of the joint coordinates [6]:

$$\begin{aligned} x &= x(q_1, q_2, \dots, q_n), \\ y &= y(q_1, q_2, \dots, q_n), \\ z &= z(q_1, q_2, \dots, q_n) \end{aligned} \quad (1)$$

In reality, the values of the joint coordinates have certain errors Δq_i ($i = 1, 2, \dots, n$), which result in deviation of positioning of the piece from the programmed one (e.g., [27]). The measure of the position dispersion or the measure of the real orientation, obtained by the n -fold repetition of motion in the same direction as the position of the set task, is referred to as the repeatability positioning [3].

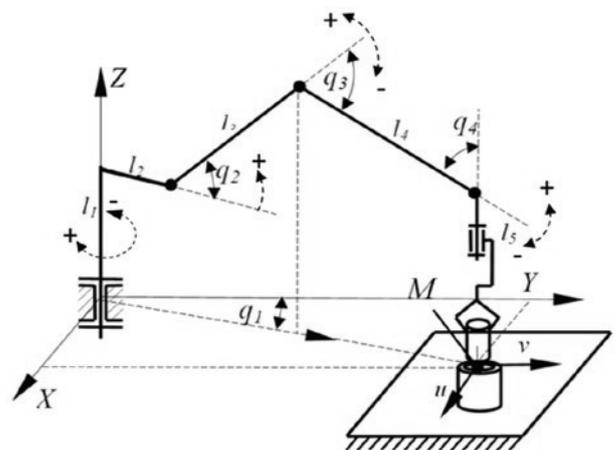


Fig. 1. The kinematic scheme of the industrial robot-making assembly treatment [17]

If we assume that the errors Δq_i of variable stochastic independence q_i relative to their nominal values have a certain given normal distribution and that they are statistically independent, then the repeatability positioning will be a 2-D variable norm, which is a deviation vector of the actual position from the nominal position of the determined parameters in the following manner [3]:

$$\sigma_{xk} = \sqrt{\left(\frac{\partial X}{\partial q_1}\right)^2 \sigma_{q_1}^2 + \left(\frac{\partial X}{\partial q_2}\right)^2 \sigma_{q_2}^2 + \dots + \left(\frac{\partial X}{\partial q_n}\right)^2 \sigma_{q_n}^2} \quad (2)$$

$$\sigma_{yk} = \sqrt{\left(\frac{\partial Y}{\partial q_1}\right)^2 \sigma_{q_1}^2 + \left(\frac{\partial Y}{\partial q_2}\right)^2 \sigma_{q_2}^2 + \dots + \left(\frac{\partial Y}{\partial q_n}\right)^2 \sigma_{q_n}^2} \quad (3)$$

$$\sigma_{zk} = \sqrt{\left(\frac{\partial Z}{\partial q_1}\right)^2 \sigma_{q_1}^2 + \left(\frac{\partial Z}{\partial q_2}\right)^2 \sigma_{q_2}^2 + \dots + \left(\frac{\partial Z}{\partial q_n}\right)^2 \sigma_{q_n}^2} \quad (4)$$

The method and the results of the analysis of the Mitsubishi RV-M2 industrial robot were presented by Kluz and Trzepieciński [17]. The conducted experiments showed that the repeatability positioning error of the robot on the X, Y, and Z axes of the Cartesian coordinate system can be described using a random variable subject to a normal probability distribution with the expected value of 0 and standard deviation $\sigma_k (N(0, \sigma_k))$.

To determine the repeatability positioning error of the assembly robot, a special measurement stand was used, on which the position of the measuring block, mounted in the pneumatic gripper of the robot, is measured by the measuring head equipped with six inductive sensors. These sensors take readings on three surfaces of a hexahedron, perpendicular in regard to each other (Fig. 2).

This setup makes possible the unequivocal determination of displacement of the center of the test block, and in connection the angular and linear errors of the robot. The experiments were carried out by inductive displacement sensors, of GT61 type, from TESSA Company, with a measuring range of ± 5 mm, hysteresis error of $0.2 \mu\text{m}$ and coefficient of linear expansion of $0.09 \mu\text{m}/^\circ\text{C}$.

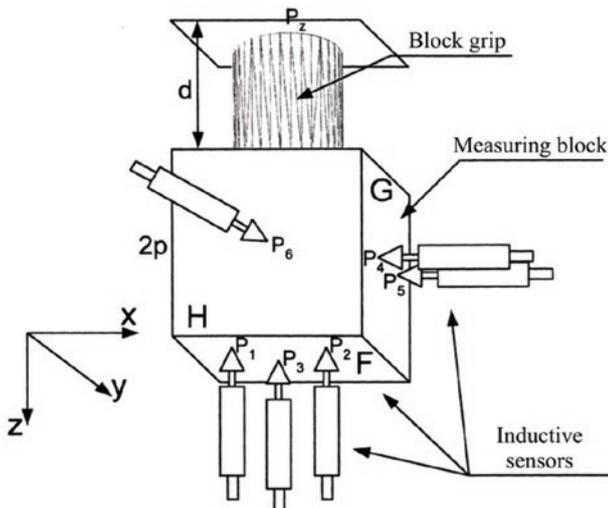


Fig. 2. System of inductive sensors of measuring head: P1-P6 – measuring points; F, G, H – measuring surfaces

The results of calculations and measurements of the errors for two sample points in the robot’s workspace are presented in Table 1 and Fig. 3a,b. The statistical tests conducted at the significance level of $\alpha = 0.05$ showed that the error in positioning repeatability can be

described with a 2-D random variable subject to a Gaussian distribution.

Repeatability positioning tests were carried out in laboratory conditions that ensure the required ambient conditions both by the manufacturer and the EN ISO 9283: 2003 standard. In order not to collide the measuring box with the measuring head and sensors, the robot moved between the points in a straight line - linear interpolation. During the research, it was noticed that the change of the effector’s speed from 10.7% to 65.1% of the maximum speed does not significantly affect the positioning error. The increase of the effector’s speed from 65.1% to 100%, i.e. to the maximum speed, resulted in a significant increase in error (at about 30%), therefore the tests were carried out at maximum speed and maximum load (in accordance with the requirements of EN ISO 9283: 2003), which for the Mitsubishi RV robot -M2 is 2 kg. Such an approach also provides the possibility to compare the results obtained with the maximum error value given in the robot’s instructions (± 0.1 mm).

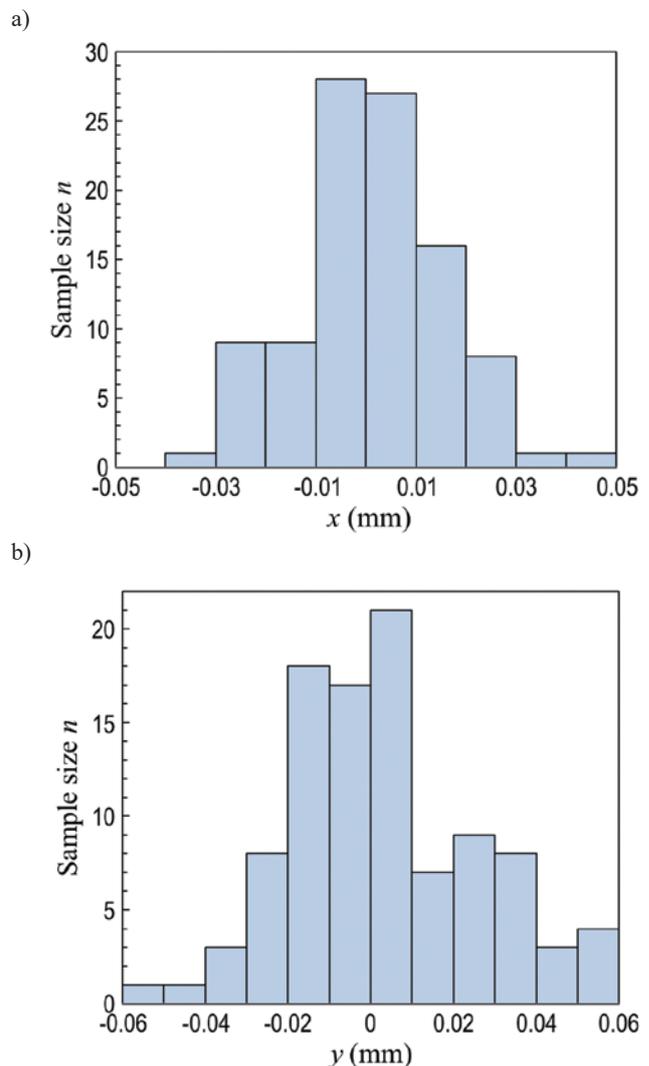


Fig. 3. Histograms of linear errors of the Mitsubishi RV-M2 robot at the point of the workspace described by a set of generalized coordinates given in Table 1 for the x-axis (a) and y-axis (b) directions

3. Robot error induced by temperature change

According to the requirements of the EN ISO 9283:2003 standard [10] on performance criteria and related test methods for robots used for manipulation operations in industrial processes, the investigations have to be carried out under stable temperature conditions. Under real

Table 1. The values of the random variable parameters of the Mitsubishi RV-M2 robot error

Joint coordinate (rad)	The experimentally determined values of parameters of normal distribution of probability density of the robot's error				
	Random variable	Minimum (mm)	Maximum (mm)	Standard deviation (mm)	Skewness
$q_1 = 0.5235$ $q_2 = 0.8726$ $q_3 = -1.3962$ $q_4 = -1.0471$	Distribution parameters evaluated theoretically: $\sigma_{xk} = 0.016$ mm, $\sigma_{yk} = 0.017$ mm, $\sigma_{zk} = 0.017$ mm				
	x	-0.032	0.043	0.014	0.067
	y	-0.054	0.057	0.018	0.287
	z	-0.051	0.050	0.017	0.075
$q_1 = 1.3963$ $q_2 = 0.3839$ $q_3 = -1.2217$ $q_4 = -0.7330$	Distribution parameters evaluated theoretically: $\sigma_{xk} = 0.021$ mm, $\sigma_{yk} = 0.016$ mm, $\sigma_{zk} = 0.016$ mm				
	x	-0.046	0.068	0.023	0.225
	y	-0.048	0.027	0.016	-0.406
	z	-0.053	0.059	0.018	0.045

production conditions, the ambient temperature can change, which has a direct impact on the repeatability positioning of the robot.

In order to determine the robot's error resulting from ambient temperature changes in the established place of the workspace, the schema of the robot's kinematic structure should be modified and some assumptions should be made. The angular dimensions of the robot set point (the values of joint coordinate q_i) should be taken as constant, while the linear dimensions, which in the robot RV-M2, for example, cannot change their values, should be modelled as kinematic reciprocating pairs that are positioned in the position l_i with the deviation Δl_i . The extension of the robot's arm unit with the length of l_i caused by temperature changes should be adopted as the deviation Δl_i . Therefore, the deviation Δl_i takes the form of a function dependent on the temperature (T), the linear expansion coefficient (χ), and the length of the robot's units (l_i).

$$\Delta l_i = f(\chi_i, l_i, \Delta T) \tag{5}$$

If the setting of each kinematic pair is burdened with a certain error Δl_i then the actual position of the robot end tip will be shifted in relation to the desired nominal position of the vector p . The coordinates of this vector end can be written as:

$$\Delta x = \sum_{i=1}^n \frac{\partial X}{\partial l_i} \cdot \Delta l_i \tag{6}$$

$$\Delta y = \sum_{i=1}^n \frac{\partial Y}{\partial l_i} \cdot \Delta l_i \tag{7}$$

$$\Delta z = \sum_{i=1}^n \frac{\partial Z}{\partial l_i} \cdot \Delta l_i \tag{8}$$

For the Mitsubishi RV-M2 industrial robot that is the object of the investigations, these equations (6–8) take the following form:

$$\Delta x = \sin(q_1)\Delta l_2 + \sin(q_1)\cos(q_2)\Delta l_3 + \sin(q_1)\cos(q_2+q_3)\Delta l_4 + \sin(q_1)\cos(q_2+q_3+q_4)\Delta l_5 \tag{9}$$

$$\Delta y = \cos(q_1)\Delta l_2 + \cos(q_1)\cos(q_2)\Delta l_3 + \cos(q_1)\cos(q_2+q_3)\Delta l_4 + \cos(q_1)\cos(q_2+q_3+q_4)\Delta l_5 \tag{10}$$

$$\Delta z = \Delta l_1 + \sin(q_2)\Delta l_3 + \sin(q_2 + q_3)\Delta l_4 + \sin(q_2 + q_3 + q_4)\Delta l_5 \tag{11}$$

Knowing the value of the deviations Δl_i , it is possible to determine the value of the robot error caused by temperature changes. Determination of deviations Δl_i based on direct measurement requires the use of specialized measuring equipment. Furthermore, gear clearances that cause errors in the setting of joint coordinates Δq_i can partially compensate the temperature-induced increase in the length of each robot unit. The use of the indirect method based on knowledge of the kinematic structure of the robot is preferred in this case. For this purpose, measurements of the robot's error at five different points of its workspace were made. The increment in the temperature ΔT is 3°C with no change in the joint coordinates q_i . Then the values of the deviations Δl_i are determined by solving the system of equations:

$$\begin{cases} \Delta x_k = \sum_{i=1}^n \frac{\partial X}{\partial l_i} \cdot \Delta l_i \\ \Delta z = \sum_{i=1}^n \frac{\partial Z}{\partial l_i} \cdot \Delta l_i \end{cases} \tag{12}$$

where $k = 1, 2, \dots, n - 1$; n is the number of robot links.

It was found that the average deviations Δl_i induced by temperature changes are equal to $\Delta l_1 = 0.014$ mm, $\Delta l_2 = 0.005$ mm, $\Delta l_3 = 0.009$ mm, $\Delta l_4 = 0.007$ mm, and $\Delta l_5 = 0.008$ mm. To determine whether the temperature-induced errors affect the nature of the random variable distribution of the robot's error caused by setting errors of joint coordinates, investigations consisting of linear displacement of the measuring block (Fig. 4) to the desired position in the robot's workspace at two different ambient temperatures, 20 and 23°C, were conducted. Next, the arithmetic mean from the sample, which is a consistent and unbiased estimator of the expected value μ , is determined.

Because the investigations were carried out at points of the workspace for which standard deviations of the robot's error were known, for statistical verification of the investigations the parametric test of significance of average value is used. For each sample the hypothesis about the mean value $H: \mu = \mu_0$ is adopted. This hypothesis states that the average value of the analysed characteristic of the population is equal to the value of μ_0 determined during the static measurement of the error at increased ambient temperature (Table 2), assuming that the analysed population characteristic has a distribution $N(\mu, \sigma)$, while the alternative hypothesis is $K: \mu \neq \mu_0$. To verify this hypothesis, the test statistic U is used, which is defined by the formula:

$$U = \frac{\bar{X} - \mu_0}{\sigma} \sqrt{n} \quad (13)$$

which, assuming the truth of the hypothesis $H: \mu = \mu_0$ (for the number of results $n > 30$ [25]) is a standardized normal random variable with distribution $N(0,1)$. The sample size was $n = 100$.

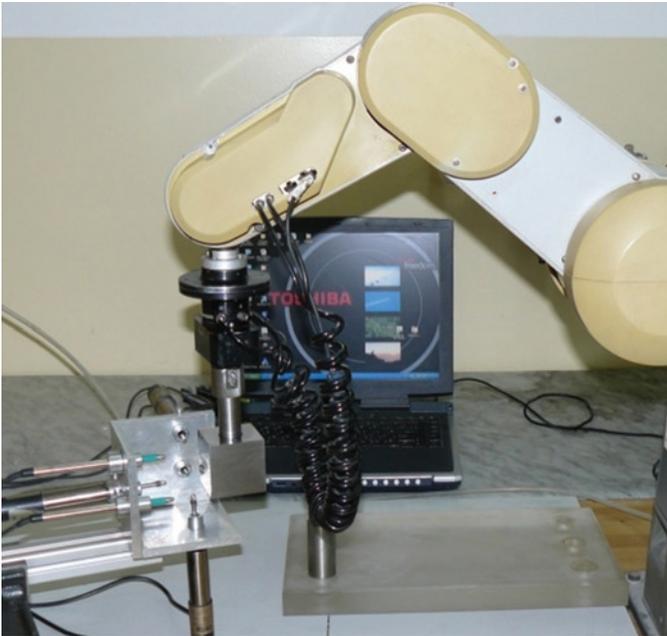


Fig. 4. The view of a measuring head equipped with a measuring block and sensors

In the case in which the measurements were carried out after stabilization of environmental heat conditions, at the level of significance $\alpha = 0.05$, the value of the statistic U (Eq. 13) did not belong to a critical set $\left(-\infty, -u\left(1 - \frac{1}{2}\alpha\right)\right) \cup \left(u\left(1 - \frac{1}{2}\alpha\right), +\infty\right)$, where $u\left(1 - \frac{1}{2}\alpha\right)$ is

the quantile of the order $1 - \frac{1}{2}\alpha$ of the distribution $N(1,0)$. So, there

was no reason to reject the hypothesis H . This means that errors due to temperature changes do not affect the form of the distribution describing the robot's errors but only cause an increase or a decrease in the average value of the obtained results. The results of measurements of the gripper displacement due to temperature change and the mean values of the sample in the two exemplary points in the workspace of the assembly stand are shown in Table 2.

It should be noted that the measurements were carried out after stabilizing the ambient thermal conditions, so that all the arms obtained an equal temperature. In the absence of temperature stabiliza-

tion, it is very difficult to determine the values of the robot's errors, because the lengths of the robot arms do not increase proportionally (the values of statistics U (Eq. 13) belong to the critical set). The robot end tip before the heat stabilization of mechanisms can change its position in relation to the nominal position; that is, it can be displaced within a certain area. To determine this area, the origin of the local reference system is adopted in the nominal position. To investigate how the deviation of the position of the robot end tip from its ideal position changes, it was assumed that the deviation Δl_r in the setting of one of kinematic pairs will change, and the deviation of the setting accuracy of the other kinematic pairs will retain a permanent value. To do this in Eq. 14, which specifies the components of the deviation vector of the working tip from an ideal position, Δl_r should be taken as a variable parameter.

$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} \frac{\partial Y}{\partial l_1} \dots \frac{\partial Y}{\partial l_r} \dots \frac{\partial Y}{\partial l_i} \\ \frac{\partial Z}{\partial l_1} \dots \frac{\partial Z}{\partial l_r} \dots \frac{\partial Z}{\partial l_i} \end{bmatrix} \cdot \begin{bmatrix} \Delta l_1(\chi_1, l_1, \Delta T) \\ \Delta l_r(\chi_r, l_r, \Delta T) \\ \Delta l_i(\chi_i, l_i, \Delta T) \end{bmatrix} \quad (14)$$

The complete matrix of total differentials in Eq. 14 can be treated as a Jacobian matrix of coefficients of sensitivity of the robot to the change in the length of the kinematic chain due to the temperature change. This task entails finding the equation of the family of parallel lines in Cartesian coordinates:

$$\frac{\partial Y}{\partial l_r} z = \frac{\partial Z}{\partial l_r} y + \sum_{i=1}^n \frac{\left| \frac{\partial Y}{\partial l_r} \frac{\partial Y}{\partial l_i} \right|}{\left| \frac{\partial Z}{\partial l_r} \frac{\partial Z}{\partial l_i} \right|} \Delta l_i(\chi_i, l_i, \Delta T) \quad (15)$$

Choosing the appropriate values of the extreme deviations Δl_i caused by temperature changes, we can find two lines from the family of parallel lines defined by Eq. 15. In this way, the polygon of the robot's positioning accuracy taking into account changes in temperature-induced linear dimensions of the robot can be created. Inside the polygon are all the possible vectors of position deviation of the robot's end tip from the desired nominal position. Knowing the most extreme position of a polygon, the largest displacement of the end tip can be found. Figures 5a and 5b show a tolerant polygon of the robot's error caused by temperature changes at two different points of the robot's workspace. An analysis of the figures shows that the maximum error values depend not only on temperature change but also on the choice of the points in the robot's workspace. This makes it possible to reduce the robot's error by choosing (i) the place in its workspace characterized by the lowest error value or (ii) the place with the least sensitivity to the change of the length of the robot's arms due to temperature changes.

Table 2. Values of the Mitsubishi RV-M2 robot error induced by changes in ambient temperature $\Delta T = 3 \text{ }^\circ\text{C}$

Joint coordinate, (rad)	Δx (mm)	Δy (mm)	Δz (mm)	$\bar{\Delta x}$ (mm)	$\bar{\Delta y}$ (mm)	$\bar{\Delta z}$ (mm)
$q_1 = 0.5235$ $q_2 = 0.8726$ $q_3 = -1.3965$ $q_4 = -1.0471$	0.008	0.014	0.009	0.007	0.016	0.011
$q_1 = 1.3963$ $q_2 = 0.3839$ $q_3 = -1.2217$ $q_4 = -0.7330$	0.017	0.003	0.004	0.020	0.003	0.006

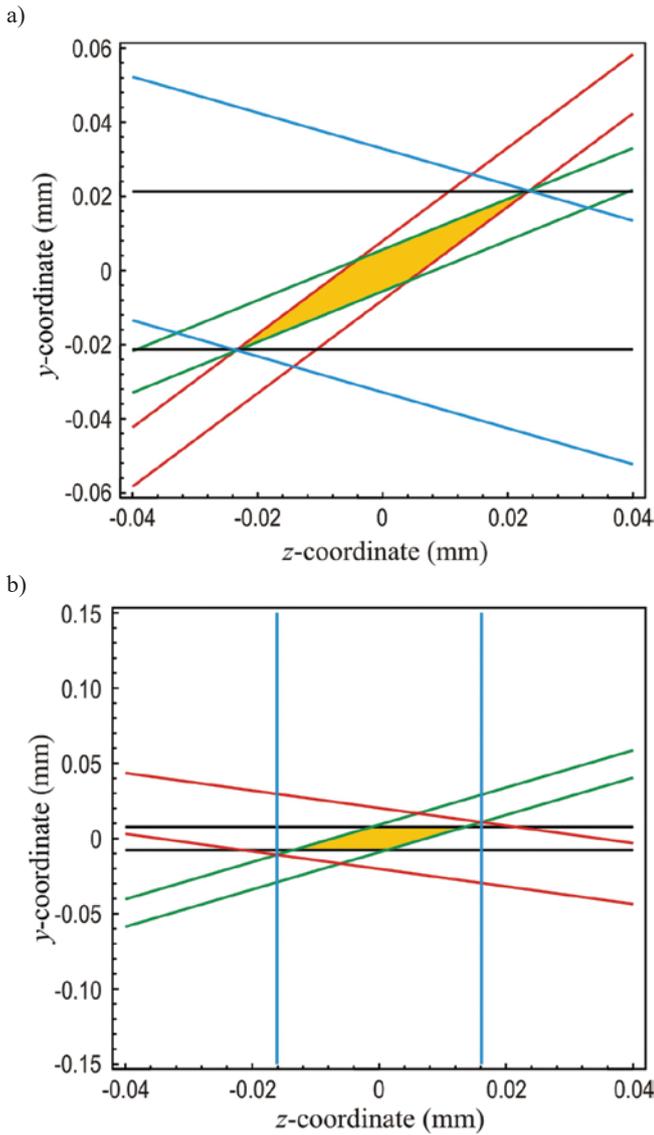


Fig. 5. Polygon of Mitsubishi RV-M2 robot error caused by temperature changes at the point defined by joint coordinates: (a) $q_1 = 0.5235$ rad, $q_2 = 0.6981$ rad, $q_3 = -0.3490$ rad, $q_4 = -1.0471$ rad; (b) $q_1 = 0.5235$ rad, $q_2 = 0.8726$ rad, $q_3 = -1.3962$ rad, $q_4 = -1.0471$ rad

4. Randomization of temperature-induced error of robot

The systematic error resulting from the temperature change can be compensated based on the determined correction [e.g., 4, 9, 21, 29]. However, this requires constant monitoring of the temperature value. If the robot operates in conditions where it is not possible to stabilize the ambient temperature, the correction is unknown. The correction is associated with the error value that can be determined on the basis of the expected range in which the correction is contained. It is connected with the assumption that the systematic error may have a value in the range of $\mu = (0 \pm \delta)$. In this way, the systematic temperature-induced error may be theoretically randomized.

Practical randomization depends on assuring the error dispersion of temperature-induced conditions during the tests, so that in the following measurements the systematic error takes random values from the range $(0 \pm \delta)$ or extreme values. In this case, however, it requires knowledge of the form of the distribution describing the variation of systematic errors. If the temperature in the robotized assembly stand stabilizes at a certain level and the lowest operating temperature occurs as rarely as the highest temperature, the randomization of system-

atic error can be carried out based on the normal distribution, which greatly simplifies the process of assessing the robot's accuracy.

If, however, there is an equal probability of occurrence of both the lowest and the highest temperature, the uniform distribution should be used. Such a situation may take place when total heat stabilization of the manipulator does not occur. Convolution of the normal distribution (repeatability) with density function $f_{Y_k}(y)$ and uniform distribution (temperature error) with density function $f_{Y_t}(y)$ exhibits a flat normal distribution. The probability density function of this distribution is described by the formula:

$$PDF(\eta_y) = f_{Y_k+Y_t}(y) = \int_{-\infty}^{\infty} f_{Y_t}(y) \cdot f_{Y_k}(\zeta - y) \quad (16)$$

Density functions of this distribution are generally characterized by a constant value in the vicinity of the expected value and in slopes described by a Gaussian function (Fig. 6). The range of the stability of the density function depends on the parameter r of the distribution, which determines the ratio of the standard deviation σ_t of its rectangular component to the standard deviation σ_k of its normal component [11, 19]:

$$r = \frac{\sigma_t}{\sigma_k} \quad (17)$$

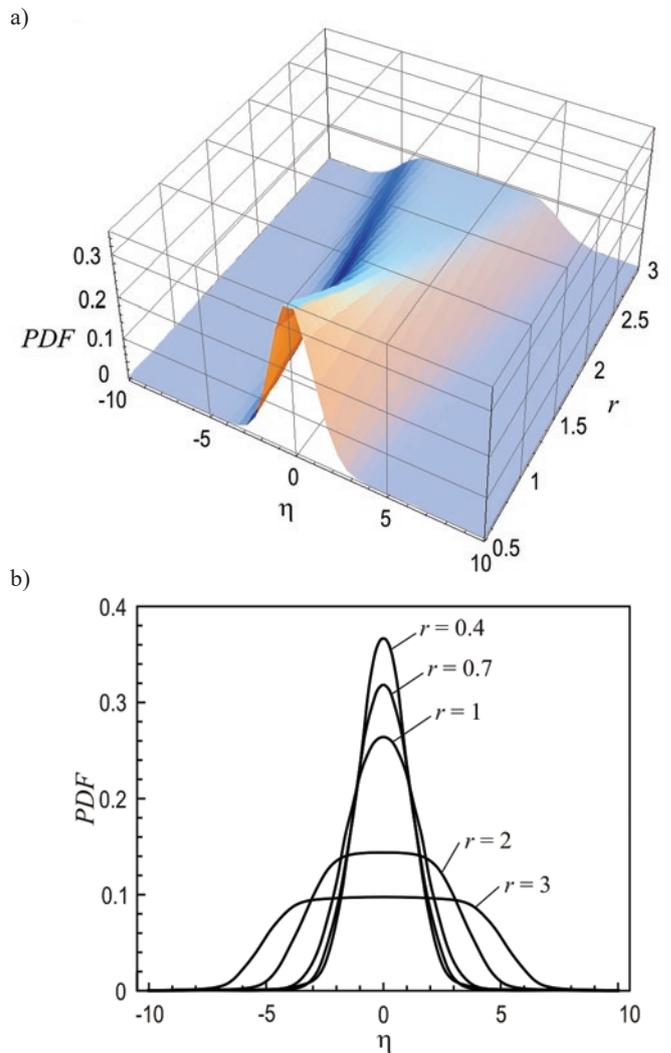


Fig. 6. Distribution of the function of the density of a flat normal distribution depending on the coefficient r (a) and the effect of the r parameter value on the shape of the PDF function (b)

$$PDF(\eta_y) = \frac{1}{2\sqrt{6}\pi r} \int_{\eta-\sqrt{3}^*r}^{\eta+\sqrt{3}^*r} \exp\left[-\frac{\zeta^2}{2}\right] d\zeta \quad (18)$$

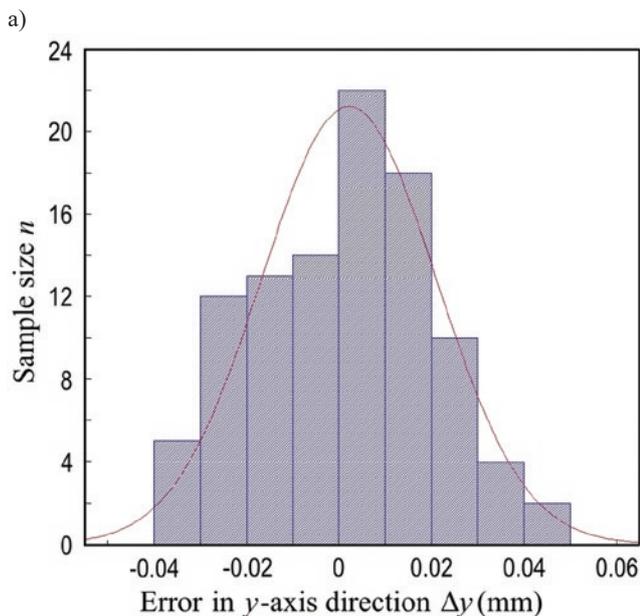
If the standard deviation σ_r of the robot temperature error is less than or equal to the standard deviation σ_k of the kinematic error then the shape of the plane-normal distribution is close to a Gaussian distribution (Fig. 6). Thus, it can be assumed that the total error of positioning repeatability can be approximated in this case in the form of a normal probability density distribution.

For the verification of the abovementioned assumptions, investigations of the repeatability positioning of the robot at the working point defined by the joint coordinates given in row 1 of Table 1 ($\sigma_y = 0.018$ mm) have been carried out. The error values of the Mitsubishi RV-M2 error caused by changes in the ambient temperature at the considered point after the stabilization of the thermal conditions were $\Delta y = \pm 0.016$ mm. Due to the fact that the purpose of the study was to analyse the influence of temperature errors of the robot on the value of the total error of repeatability positioning, the investigations were conducted in a wide range of temperature variation, which assures a significant effect of these errors. During investigations, changes of the ambient temperature were applied in the range of $\pm 3^\circ\text{C}$ without waiting for stabilization of the link temperature. It was also assumed that there is an identical probability of occurrence of the temperature from the considered variation range (uniform distribution). It should be stressed that a similar share of temperature errors can occur with smaller temperature changes, but they occur elsewhere in the workspace or in the case of robots with a greater length of links [2, 7]. It was assumed that the robot error caused by temperature changes is subject to a uniform probability distribution. On this basis, the variance of the randomized distribution of the random variable is determined:

$$\sigma_t^2 = \frac{(\Delta y_{t\max} - \Delta y_{t\min})^2}{12} = 8.1 \cdot 10^{-5} \text{ mm} \quad (19)$$

Then the variance of the resultant distribution:

$$\sigma_{\eta_y}^2 = \sigma_{y_k}^2 + \sigma_{y_t}^2 = \quad (20)$$



is determined.

The results were statistically analysed using the Shapiro-Wilk test [25, 26] to verify the normality of the distribution of a random variable. During the investigations the following hypothesis was formulated: the null hypothesis H_0 that the distribution of the analysed characteristic is normal. For $\alpha = 0.05$ and $n = 100$, the tabulated critical value $W(\alpha, n) = W(0.05, 100) = 0.964$ was less than the calculated value, which meant that there was no basis to reject the hypothesis of the normality of the distribution of the obtained data. A histograms and a graphs of the normal distribution of the obtained results are shown in Figs. 7a and 8a.

An analysis of the results shows that when the standard deviation of a random variable induced by temperature changes (uniform distribution) is equal to 50% of the standard deviation of the robot's kinematic error then the total error of the robot with sufficient accuracy can be approximated by the normal distribution of the random variable. Because the sample size used in the experiment $n = 100$ could be too small to confirm the correctness of the assumptions employed, simulation investigations were carried out.

During the research, 5000 pseudo-random numbers subject to a normal distribution with parameters derived based on the both the measurements (Table 1) and the uniform distribution simulated during the experiment (Fig. 7b) were generated.

Then, the results of a sum of random variables' distributions were investigated to find their consistency with the normal distribution. To carry out this analysis, the Shapiro-Wilk test is also used. The results showed that there were no reasons to reject the hypothesis about the consistency of the results with the normal distribution (Fig. 8b). The results of the simulations confirm, therefore, the results of experimental investigations.

5. The probability of joining parts

An important issue in the operation of a robotic assembly station is the problem of ensuring the required probability of joining the parts involved. The tasks related to the robotisation of assembly can be greatly facilitated by decomposing joints according to the shape of the surface of the assembled parts. From this point of view, the assembly of typical joints can be examined as a typical series of joining parts with flat, cylindrical, conical, spherical, threaded and other sur-

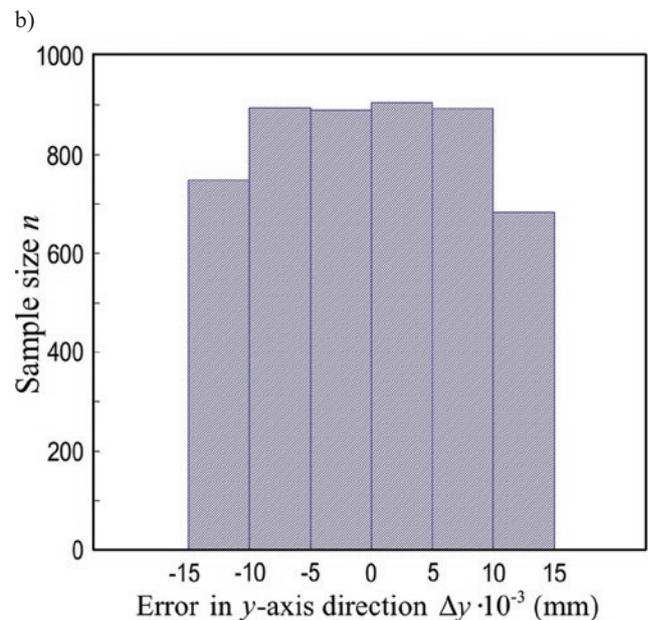


Fig. 7. (a) histogram of the results of the measurement repeatability positioning error of the robot, taking into consideration the temperature errors; and (b) histogram of a random variable of the robot error induced by temperature changes

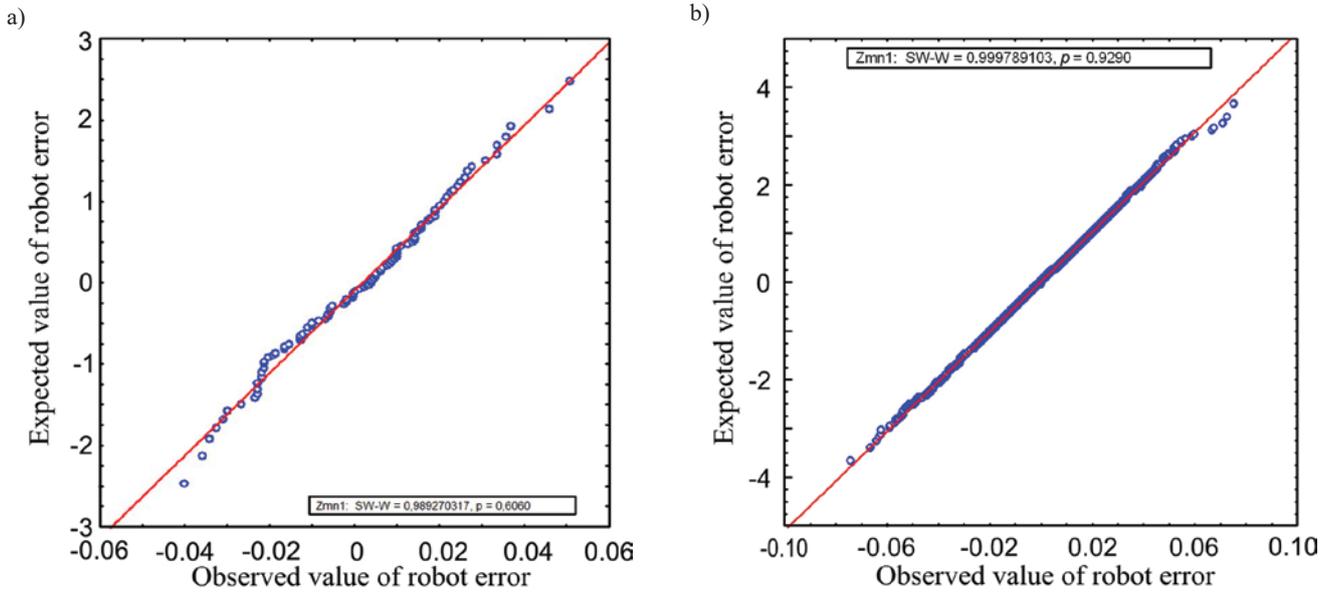


Fig. 8. Diagram of compatibility of the results of measurement (a) and simulation (b) of the robot error with a normal distribution

faces. Among them, joints with cylindrical surfaces constitute about 40% of the total number of connections [5]. Since, in the majority of cases, robots are used to carry out the process of assembly of cylindrical parts with guaranteed clearance [22], the next section of the work is limited to joining parts with cylindrical surfaces. The repeatability positioning error of the robot causing the relative displacement of the axes of the joined parts is a two-dimensional random variable $X = [x, y]^T$ subjected to the normal probability distribution with the covariance matrix A_k and the matrix of expected values μk^T .

$$f(X) = \frac{1}{2\pi\sqrt{|\Lambda_K|}} \exp\left[-\frac{1}{2}(X - \mu_K)^T \Lambda_K^{-1}(X - \mu_K)\right] \quad (21)$$

The elements of the covariance matrix A_k correspond to the boundary standard deviations listed in Table 1. If during the operation of the robotic assembly station there are small temperature changes causing a random displacement of the error mean values, the total positioning repeatability error for $r < 1$ can be described as a two-dimensional normal random variable with the covariance matrix A_η and the matrix of expected values $\mu\eta^T$ (Fig. 9).

The probability of joining cylindrical parts is the probability of an event occurring that the distance between their axes reaches an assumed value of $0.5L$ (η_1, η_2), i.e. the probability of the event that a random variable describing the distance between the axes of the joined parts will be inside a hypothetical cylinder with the centre located in the nominal point N and a diameter corresponding to the clearance of the parts to be joined L . In order to determine the value of the prob-

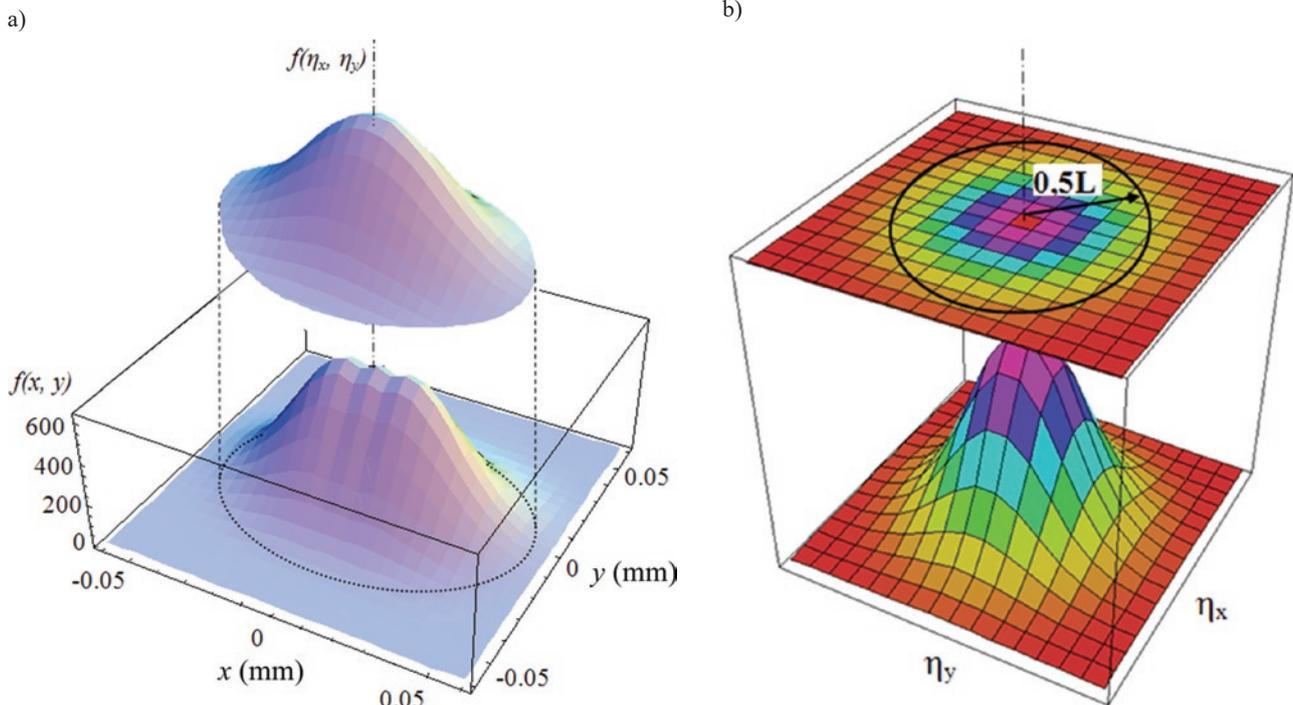


Fig. 9. The method for determining the probability of joining cylindrical parts: a) random variable of the displacement error of the part axes $f(\eta_x, \eta_y)$, b) the area of integration of the random variable $f(\eta_x, \eta_y)$

ability, one should integrate the density function of the relative error distribution of the displacement of the part axis in the area of $O: \{ \eta_x^2 + \eta_y^2 \leq (0,5L)^2 \}$ as follows:

$$P = \iint_{\eta_x^2 + \eta_y^2 \leq (0,5L)^2} \frac{1}{2\pi\sqrt{|\Lambda_\eta|}} \exp\left[-\frac{1}{2}(\eta - \mu_\eta)^T \Lambda_\eta^{-1}(\eta - \mu_\eta)\right] d\eta_x d\eta_y \quad (22)$$

where: Λ_η - covariance matrix of a random variable of error of relative displacement of a part axis, μ_η^T - matrix of expected values of a random variable of relative error of the displacement of part axes.

The occurrence of temperature changes during the assembly process leads to an increase in robot error, which results in a reduction of the probability of joining parts and a reduction of the reliability of the whole process. Depending on the place where the joining is made, a different probability value can be obtained in the work space of the station (Fig. 10). In the case of an assembly operation at a point characterised by the error value given in Table 1 (row 1), a change in temperature causes a reduction in the probability and therefore also reduces the qualitative capability C_p of the process by 17.8% (from 1.33 to 1.1). Ensuring the required level of quality capability of the process ($C_p = 1.33$) requires an increase in the joining clearance by 17.21%.

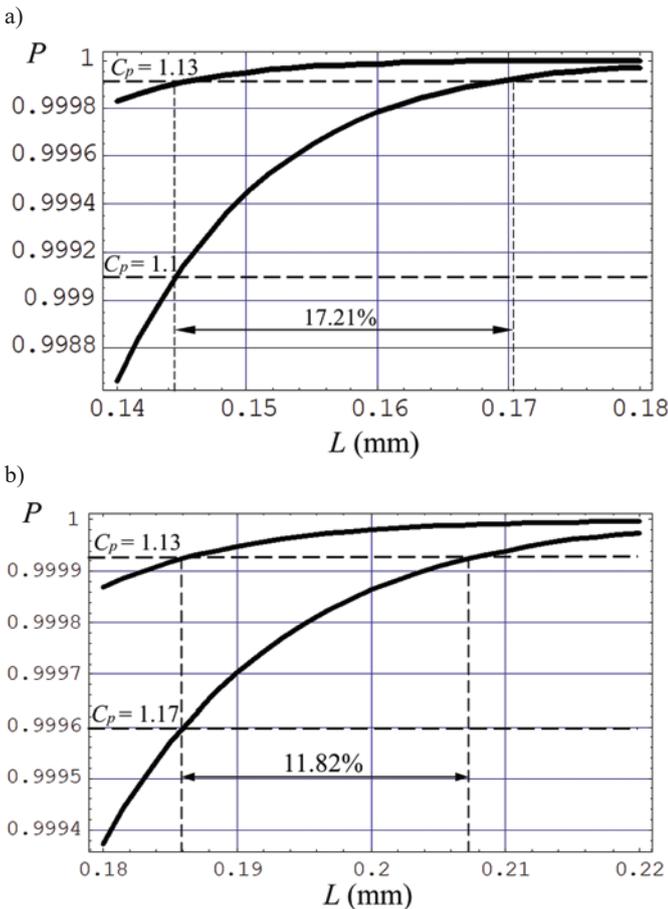


Fig. 10. The influence of temperature changes on the probability of joining parts: a) at the point specified in row 1 of Table 1, b) at the point specified in row 2 of Table 1

At the point in the robot workspace corresponding to the parameters listed in row 2 of Table 1 for a joint clearance of $L = 0.145$ mm, a 24.81% lower quality capability of the process ($C_p = 1$) can

be obtained. This is due to the fact that at this point the robot exhibits a much higher error value of positioning repeatability. A change in the temperature at this point also causes a decrease in the quality capability index of the process from $C_p = 1.33$ to $C_p = 1.17$ and thus by 12%. It is possible to ensure the required probability of joining the part (corresponding to a process quality capability index at a level of $C_p = 1.33$) by increasing the joint clearance by 12%. Thus, depending on the choice of location in the robot workspace where the assembly process is carried out, one can firstly obtain a different level of quality of the capability index of the process, and secondly a different impact of the errors related to the inability to stabilise the ambient temperature on the probability of joining parts.

6. Summary and conclusions

Industrial robots are successfully used in many areas of manufacturing processes such as welding, drilling, and handling. However, in recent years, the interest of many researchers is focused on the possibility of implementing robots in processes that require high precision such as assembly or measurement. Ensuring high reliability of these processes requires consideration of all the factors affecting their repeatability positioning. The most important parameters influencing the repeatability positioning include the error that results from the kinematic errors of settings of programmed joint coordinates' values and the errors caused by changes in ambient temperature.

This paper shows that if the ambient temperature can change within a small range during the operation of a robotic station, it may be very difficult to correct the error associated with it. Adoption of the maximum error value seems to be unjustified, because this error can be partially compensated for by a kinematic error subject to the normal probability distribution. The research conducted has shown that in practice this error can be randomised based on the expected range of error variability in the form of a uniform probability distribution. In the situation where the ratio of the standard deviation of the temperature error σ_t to the standard deviation of the kinematic error σ_i is less than $r < 1$, one can make a sufficiently accurate approximation of the distribution of the total repeatability positioning error using a normal probability distribution. When the value of the coefficient r is greater than 1 this error is subjected to a flat-normal distribution.

The analysis of the results in this study has shown that performing assembly in conditions of unstable ambient temperature results in a lower probability of joining parts and process quality capability. Ensuring the required level of assembly of parts requires an increase in the joint clearance from 12 to 17% depending on the choice of location in the workspace. Otherwise, the quality capacity of the process may be reduced, and thus the reliability of the workstation may be reduced. The results obtained indicate that the required reliability of the process can be obtained by selecting an appropriate location for the joining process characterised by the smallest error value and the smallest sensitivity to changes in the environmental conditions.

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