Article citation info:
LI J, DUAN R. Dynamic diagnostic strategy based on reliability analysis and distance-based VIKOR with heterogeneous information. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2018; 20 (4): 610–620, http://dx.doi.org/10.17531/ein.2018.4.12.

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DYNAMIC DIAGNOSTIC STRATEGY BASED ON RELIABILITY ANALYSIS AND DISTANCE-BASED VIKOR WITH HETEROGENEOUS INFORMATION

DYNAMICZNA STRATEGIA DIAGNOSTYCZNA Z WYKORZYSTANIEM INFORMACJI HETEROGENICZNYCH BAZUJĄCA NA ANALIZIE NIEZAWODNOŚCI I OPARTYM NA ODLEGŁOŚCIACH ALGORYTMIE VIKOR

This paper presents a dynamic diagnostic strategy based on reliability analysis and distance-based VIKOR with heterogeneous information. Specifically, the proposed method uses a dynamic fault tree (DFT) to describe the dynamic fault characteristics and evaluates the failure rate of components using interval numbers to deal with the epistemic uncertainty. Furthermore, DFT is mapped into a dynamic evidential network (DEN) to calculate some reliability parameters and these parameters together with test cost constitute a decision matrix. In addition, a dynamic diagnostic strategy is developed based on an improved VIKOR algorithm and the previous diagnosis result. This diagnosis algorithm determines the weights of attributes based on the Entropy concept to avoid experts' subjectivity and obtains the optimal ranking directly on the original heterogeneous information without a transformation process, which can improve diagnosis efficiency and reduce information loss. Finally, the performance of the proposed method is evaluated by applying it to a train-ground wireless communication system. The results of simulation analysis show the feasibility and effectiveness of this methodology.

Keywords: *reliability analysis, dynamic evidential network, VIKOR, decision matrix, heterogeneous information.*

W artykule przedstawiono dynamiczną strategię diagnostyczną, w której wykorzystuje się oryginalne informacje heterogeniczne. Metoda ta bazuje na analizie niezawodności i opartym na odległościach algorytmie VIKOR. Dokładniej, przedstawiona strategia polega na wykorzystaniu dynamicznego drzewa błędów (DFT) do opisu dynamicznych charakterystyk błędów oraz ocenie intensywności uszkodzeń komponentów przy użyciu liczb przedziałowych, co pozwala rozwiązać problem niepewności epistemicznej. Ponadto, w proponowanej metodzie, DFT zostaje odwzorowane w dynamiczną sieć dowodową (DEN) w celu obliczenia niektórych parametrów niezawodności, a parametry te wraz z kosztem badań diagnostycznych tworzą matrycę decyzyjną. Opracowana dynamiczna strategia diagnostyczna opiera się na udoskonalonym algorytmie diagnostycznym VIKOR oraz wynikach wcześniejszej diagnostyki. Algorytm VIKOR określa wagi atrybutów w oparciu o koncepcję Entropii, co pozwala wyeliminować subiektywność oceny eksperckiej i ustalić optymalną kolejność działań diagnostycznych bazując bezpośrednio na oryginalnych informacjach heterogenicznych bez konieczności ich transformacji, co może poprawić efektywność diagnozy i zmniejszyć utratę informacji. Działanie proponowanej metody oceniano poprzez zastosowanie jej do diagnostyki systemu łączności radiowej pociąg–ziemia. Wyniki analizy symulacyjnej wskazują na możliwość praktycznego wykorzystania i skuteczność omawianej metodologii.

Słowa kluczowe: *analiza niezawodności, dynamiczna sieć dowodowa, VIKOR, matryca decyzyjna, informacja heterogeniczna.*

1. Introduction

The application of high technology to the engineering system has significantly improved the performance of modern systems and at the same time greatly increased the complexity of the systems structure. Manufacture cost of these systems is too high. Once these systems fail, it will cause a great loss. Therefore, it will be extremely important to establish a fault diagnosis model based on their unique fault characteristics and develop a dynamic diagnosis strategy which can locate the fault component quickly and reduce the maintenance cost when these systems break down. Usually, fault diagnosis requires a large amount of historical fault data. However, in engineering practice, application of redundant technologies have improved the reliability of these systems, which raises some challenges in fault diagnosis. For one thing, the behaviours of components in these systems, such as failure priority, functional dependent failures, and sequentially dependent failures should be taken into account. For another, high reliability makes it extremely difficult to obtain complete fault data because these systems may still be in the early life cycle, which results in the epistemic uncertainty. Aiming at these challenges, many researchers have put forward a large number of efficient fault diagnosis methods over the last few decades. Johnson presented a sequential diagnostic method based on heuristic information search [10], which constructed a sequential test procedure to locate the failure using information theory. However, the diagnostic result was not satisfied. A novel diagnosis strategy for multi-value attribute system was proposed based on rollout algorithm, and it obtained an optimal diagnostic sequence [9]. Based on these researches, Tian et al. proposed a fault diagnostic strategy of multivalued attribute system based on growing algorithm, which chose failure states and found an appropriate test set for these states [21]. This growing algorithm could avoid the backtracking approach of traditional algorithms and obtained good diagnostic results with a high efficiency. A real-time fault diagnosis approach was presented based on reliability analysis and Bayesian networks (BN) [6]. BN was used to calculate the system reliability, and the real-time system reliability was monitored and compared with the previous values. If the deviations exceeded the preset threshold, a heuristic algorithm was used to locate the failed component which had the greatest changes between the prior probability and posterior probability. In the literature [3], a real-time fault diagnosis method for complex systems using object-oriented BN was proposed. It included an off-line BN construction phase and an on-line fault diagnosis phase. Nevertheless, these methods constructed the BN model based on the parameter learning algorithm, which needed a large amount of fault data and could not handle epistemic uncertainty. Chiremsel et al. proposed a probabilistic fault diagnosis method of safety instrumentation system using the fault tree and BN [4]. A static fault tree was used to construct the fault model of safety instrument system and was mapped into BN to calculate the importance measure which was used to design the diagnosis algorithm. Nevertheless, this method is unable to model the dynamic fault behaviours and deal with epistemic uncertainty.

For dynamic fault characteristics, Dugan introduced a DFT to model the dynamic fault behaviours and used diagnostic importance factor (DIF) to determine the diagnostic sequence [1-2]. However, this method calculated DIF based on Markov chains which had a state space explosion problem and determined the diagnosis sequence only by components' DIF which is a single attribute decision making problem, thereby influencing the diagnosis efficiency. Besides, it assumed that the failure rates of the components are expressed in defined values describing their reliability characteristics and failed to cope with the epistemic uncertainty. Although some researchers put forward interval analysis [24], the possibility theory [19, 22], imprecise probability [12], fuzzy set theory [5, 11] and evidence theory [25], these theories were only used for the reliability analysis and risk assessment and were not further applied to the fault diagnosis. Therefore, Duan et al. presented a novel fault diagnosis method based on fuzzy set and DFT analysis [8]. The fuzzy information obtained by fuzzy set theory and domain expert was transformed into quantitative information to obtain the fuzzy failure rates of components. Discrete time Bayesian Networks was used to calculate some reliability results, and an efficient diagnosis algorithm was developed based on qualitative structural information and quantitative parameters. However, it is usually difficult to determine the corresponding membership function of each language value, and this diagnosis algorithm was also a single attribute decision making problem. To overcome these limitations, multiple attributes decision-making was used in [7, 20]. However, these methods usually used the attributes with defined values and could not make decisions under uncertainty. Besides, the proposed methods dealt with the decision problems regarding one particular type of values. It was more reasonable to express the different attributes in their appropriate data types. Only a few work took into consideration the heterogeneous information [13, 23]. However, there was litter work connected with the diagnostic strategy for complex systems. Furthermore, diagnostic algorithms failed to update the diagnostic decision table according to the previous diagnosis result.

Motivated by the problems mentioned above, this paper proposes a dynamic diagnostic strategy based on reliability analysis and distance-based VIKOR, a multi-criteria decision analysis method, with heterogeneous information considering epistemic uncertainty shown in Fig. 1. A DFT is used to establish the system fault model to describe the dynamic fault characteristics. Interval numbers are used to describe the failure rate of components to deal with epistemic uncertainty. Furthermore, a DFT is converted into a DEN to obtain the reliability parameters such as DIF and risk achievement worth (RAW). In addition, DIF, RAW, test cost and previous diagnosis result are taken into account comprehensively to obtain the optimal diagnostic ranking order using a distance-based VIKOR with heterogeneous information. Finally, a train-ground wireless communication system is given to demonstrate the efficiency of this proposed method.

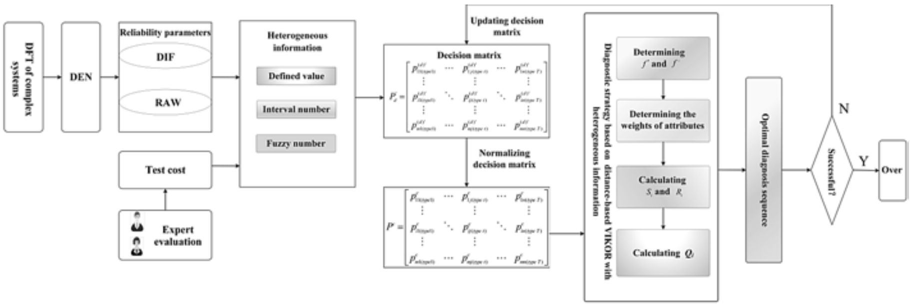

Fig. 1. A dynamic diagnostic framework based on reliability analysis and distance-based VIKOR with heterogeneous information.

The remainder of this article is organized as follows. Section 2 presents the DFT model construction and quantitative analysis of DFT based on DEN. A novel dynamic diagnostic strategy based on reliability analysis and distance-based VIKOR with heterogeneous information considering the epistemic uncertainty is given in Section 3. Section 4 is devoted to a simple illustration example of the proposed approach. Some conclusions are given in the final section.

2. DFT analysis

2.1. Model Construction of DFT

Fault tree is a deductive method to decide the potential causes that may cause the occurrence of a predefined undesired event, generally denoted as the top event. DFT extends a static fault tree to describe the dynamic failure behaviours such as priorities of failure events, spares, and sequence-dependent events. Dynamic gates in DFT include the priority AND gate, the functional dependency gate (FDEP), the sequence enforcing gate, the cold, hot, and warm spare gates. The model construction of the fault tree usually requires an in depth knowledge of the system and its components. It includes the construction of a network topology and the failure rates estimation of components. The former can resort to fault mode and effect analysis and the latter needs to obtain lots of fault data, which is almost impossible to estimate precisely the failure rates of the basic events in the practical engineering application. In this paper, interval numbers are used to describe the failure rates of the basic events based on the expert elicitation and some data sheet at the design stage.

2.2. Quantitative analysis of DFT based on DEN

Traditional DFT assumes that the failure rates of the components are expressed in defined values is inadequate to deal with epistemic uncertainty. To this end, the failure rates of the basic events in DFT are considered as interval numbers in this paper and a new DFT solution is proposed to calculate the reliability results by mapping a DFT into a DEN.

In evidence theory, $\Theta = \{W_i, F_i\}$ is the knowledge framework of the component i and the focal elements are defined by:

$$2^{\Theta} = \{\{\varnothing\}, \{W_i\}, \{F_i\}, \{W_i, F_i\}\} \tag{1}$$

where $\{W_i\}$ and $\{F_i\}$ denote the working state and failure state respectively. The state of $\{W_i, F_i\}$ corresponds to the epistemic uncertainty.

Belief measure *(Bel)* defines the lower bound of the probabilities that the focal element exists, and plausibility measure *(Pl)* defines the upper bound of the probabilities that the focal element exists. The basic belief assignment on the system state expresses an epistemic uncertainty, where *Bel* and *Pl* measures are not equal and bound the system reliability. Therefore, the basic probability assignment (BPA) of component *i* can be computed as:

$$\begin{cases} m(\{W_i\}) = Bel(\{W_i\}) \\ m(\{F_i\}) = 1 - Pl(\{W_i\}) \\ m(\{W_i, F_i\}) = Pl(\{W_i\}) - Bel(\{F_i\}) \end{cases} \tag{2}$$

If a component *i* follows the exponential distribution with the interval failure rate $[\underline{\lambda}, \overline{\lambda}]$, the interval failure probability of the component at a mission time *T* can be calculated as follows:

$$[\underline{P_i(x)}, \overline{P_i(x)}] = 1 - exp([\underline{\lambda}, \overline{\lambda}]T) \tag{3}$$

where $\underline{P_i(x)}$ and $\overline{P_i(x)}$ denote respectively the lower failure probability of the component and the corresponding upper failure probability.

Presumably, the upper and lower bounds of the component's failure probability is equivalent to the BPA of component *i* in the DEN:

$$\begin{cases} m(\{W_i\}) = 1 - \overline{P_i(x)} \\ m(\{F_i\}) = \underline{P_i(x)} \\ m(\{W_i, F_i\}) = \overline{P_i(x)} - \underline{P_i(x)} \end{cases} \tag{4}$$

where $Bel(\{F_i\}) = \underline{P_i(x)}$ and $Pl(\{F_i\}) = \overline{P_i(x)}$.

2.2.1. Mapping a static logic gate into an DEN

Static logic gates mainly include three gates, AND gate, OR gate and voting gate. This section takes an OR gate as an example and provides the schemes to map an OR gate into a DEN. When any of the input components X_i (i=1,…, n) of an OR gate fails, the output of the gate fails too. Fig. 2 shows an OR gate and the equivalent DEN. Table 1 gives the conditional probabilities of node *A (T+ΔT)* in the DEN. Equation (5) gives the conditional probabilities of output node *E (T+ΔT)*. A more detailed description of this work can be found in [16].

Table 1. The conditional probabilities of node A (T+ΔT)

A(T)	A(T+ΔT)		
	{W}	{F}	{W,F}
{W}	m_A(W)	m_A(F)	m_A(W,F)
{F}	0	1	0
{W,F}	0	m_A(F)	1− m_A(F)

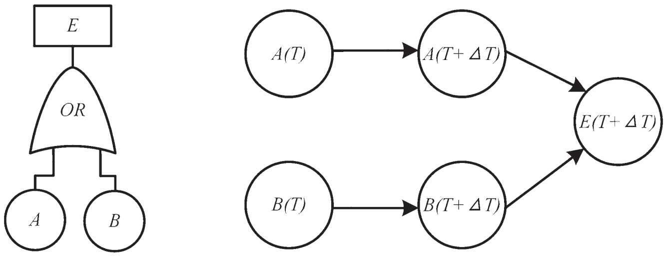

Fig. 2 An OR gate and the equivalent DEN

$$\begin{cases} P(E=1 \mid A(T+\Delta T)=0, B(T+\Delta T)=1)=1 \\ P(E=1 \mid A(T+\Delta T)=1, B(T+\Delta T)=0)=1 \\ P(E=1 \mid A(T+\Delta T)=1, B(T+\Delta T)=1)=1 \\ P(E=1 \mid A(T+\Delta T)=1, B(T+\Delta T)=\{0,1\})=1 \\ P(E=1 \mid A(T+\Delta T)=\{0,1\}, B(T+\Delta T)=1)=1 \\ P(E=\{0,1\} \mid A(T+\Delta T)=0, B(T+\Delta T)=\{0,1\})=1 \\ P(E=\{0,1\} \mid A(T+\Delta T)=\{0,1\}, B(T+\Delta T)=0)=1 \\ P(E=\{0,1\} \mid A(T+\Delta T)=\{0,1\}, B(T+\Delta T)=\{0,1\})=1 \\ P(E=1 \mid A(T+\Delta T)=0, B(T+\Delta T)=0)=0 \end{cases} \tag{5}$$

2.2.2. Mapping a dynamic logic gate into a DEN

Some dynamic logic gates are introduced to model the functional and sequential in the DFT. These logic gates include priority AND gate, the sequence enforcing gate, FDEP and spare gates. An FDEP gate will be used to describe how the dynamic logic gates are mapped into DEN. An FDEP gate includes a trigger event and some dependent basic events. The trigger event can be a basic event or an output of another gate in the DFT. The occurrence of a trigger event will force all basic events to occur, which means all basic events functionally depend upon the trigger event. Fig. 3 shows an FDEP gate and the equivalent DEN. Table 2 and Table 3 show the conditional probabilities of the node *A*(*T*+*ΔT*) and *E*(*T*+*ΔT*) respectively.

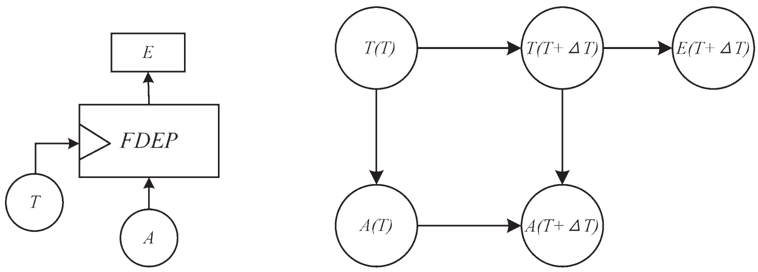

Fig. 3 An FDEP gate and the equivalent DEN.

Table 2. The conditional probabilities of the node A (T+ΔT)

T(T+ΔT)	A(T)	A(T+ΔT)		
		{W}	{F}	{W,F}
{W}	{W}	m_A(W)	m_A(F)	m_A(W,F)
{W}	{F}	0	1	0
{W}	{W,F}	0	0	1
{F}	{W}	0	1	0
{F}	{F}	0	1	0
{F}	{W,F}	0	1	0
{W,F}	{W}	0	0	1
{W,F}	{F}	0	1	0
{W,F}	{W,F}	0	0	1

Table 3. The conditional probabilities of the node $E(T+\Delta T)$

$T(T+\Delta T)$	$E(T+\Delta T)$		
	$\{W\}$	$\{F\}$	$\{W,F\}$
$\{W\}$	1	0	0
$\{F\}$	0	1	0
$\{W,F\}$	0	0	1

2.2.3. Calculating reliability results

(1) DIF

DIF is usually defined as the probability that a basic event has occurred given that the top event has also occurred [2]. The DIF of a component i is given by:

$$DIF_i = P(i \mid S) = [Bel(\{F_{i|S}\}), Pl(\{F_{i|S}\})] \tag{6}$$

where i is a component in the system S; $P(i \mid S)$ is the probability that the basic event i has occurred given the top event has occurred.

(2) RAW

RAW, one of the most widely used importance measures, is defined as the ratio of the system unreliability if a component has failed over the system unreliability [17]. Traditionally, the definition of RAW does not take the uncertainties into account. An extension of RAW is introduced which allows us to deal with epistemic uncertainty. The interval RAW of a component i can be defined as follows under uncertainties.

$$I_{X_i}^{RAW} = \frac{P(S=1 \mid X_i=1)}{P(S=1)} = \frac{[Bel(\{F_{S=1|X_i=1}\}), Pl(\{F_{S=1|X_i=1}\})]}{[Bel(\{F_{S=1}\}), Pl(\{F_{S=1}\})]} = \frac{[\underline{Q_{S=1|X_i=1}}, \overline{Q_{S=1|X_i=1}}]}{[\underline{Q_{S=1}}, \overline{Q_{S=1}}]} \tag{7}$$

where $I_{X_i}^{RAW}$ is the RAW for the event X_i, $Bel(\{F_{S=1|X_i=1}\})$ and $Pl(\{F_{S=1|X_i=1}\})$ respectively denote the belief and plausibility measures that the system is in a failed state given that the component i has failed.

3. Dynamic diagnosis algorithm based on heterogeneous information

3.1. Multi-attribute decision-making problem description in the fault diagnosis

If a fault tree has m root nodes, each root node represents a diagnostic scheme. All diagnostic schemes can be expressed in root node set $X=\{X_1,X_2,\cdots,X_m\}$ and each root node has n attributes to evaluate the performance. Evaluation attributes are expressed in attribute set $v=\{v_1,v_2,\cdots,v_n\}$. Different attributes may have different weights and the weights vector is expressed in $\omega=\{\omega_1,\omega_2,\cdots,\omega_n\}$, $\sum_{j=1}^{n}\omega_j=1,\ 0<\omega_j<1$. As for the complexity of decision problem in fault diagnosis and uncertainty, the evaluations for each attribute may be described in different types of values. For example, for precise information, defined value is used; otherwise, due to the epistemic uncertainty, some parameters can be evaluated by some experts. In this situation, the interval number, fuzzy number and linguistic term are more reasonable. In this paper, attribute values are expressed with defined value v^n, interval value v^i and triangle fuzzy number v^f, where $v^n=\{v_1,v_2,\cdots v_{n_1}\}$, $v^i=\{v_{n_1+1},v_{n_2+2},\cdots v_{n_2}\}$, $v^f=\{v_{n_2+1},v_{n_2+2},\cdots v_n\}$ and $v^n \cup v^i \cup v^f = v$; $N_1=\{1,2,\cdots,n_1\}$, $N_2=\{n_1+1,n_1+2,\cdots n_2\}$, $N_3=\{n_2+1,n_2+2,\cdots n\}$.

3.2. Distance measure for heterogeneous information [18]

3.2.1. Interval numbers

Definition 1 Let $A=[a^-,a^+]$ and $B=[b^-,b^+]$ be two interval numbers, the distance between A and B is defined as in 1-norm concept:

$$d(A,B)=\|A-B\|=|\underline{a}-\underline{b}|+|\overline{a}-\overline{b}| \tag{8}$$

The larger the distance $d(A,B)$, the greater the degree of separation will be. In particular, when $d(A,B)$ is 0, it means that A and B are equal.

3.2.2. Triangular fuzzy numbers

A triangular fuzzy number is usually given in the form $A=(a,b,c)$, where b is the median value, a is the left distribution of the confidence interval and c is the right distribution of the confidence interval of the fuzzy number A. The membership function of A which associated with a real number in the interval [0, 1] can be defined as:

$$\mu(x)=\begin{cases}(x-a)/(b-a), a\le x\le b\\ (c-x)/(c-b), b\le x\le c\\ 0 \qquad ,others\end{cases} \tag{9}$$

Definition 2 Let $A=(a_1,b_1,c_1)$ and $B=(a_2,b_2,c_2)$ be two triangular fuzzy numbers, the distance between them is defined as in 1-norm concept:

$$d(A,B)=\|A-B\|=|a_1-a_2|+|b_1-b_2|+|c_1-c_2| \tag{10}$$

Similarly, the larger the distance $d(A,B)$, the greater the degree of separation will be. In particular, if $d(A,B)$ is 0, it means that A and B are equal.

3.3. VIKOR algorithm based on generalized distance aggregation function

3.3.1. Generalized distance aggregation function

In the decision making situations where the evaluation values are represented by more than two values types, it is necessary to deal with the heterogeneous information to make full use of this information as much as possible. The base for VIKOR approach is an aggregation function which measures the distance for multi-attributes to compromise ranking. Due to the different types of values for each attribute, a generalized distance aggregation function, $G\text{-}L_P$ [15, 23], is used and is defined as follows:

$$G\text{-}L_{p,i}=\left\{\sum_{j=1}^{J}[\omega_j \cdot d(f_j^*,f_{ij})/d(f_j^*,f_j^-)]^p\right\}^{1/p} \tag{11}$$

where $1 \le p \le \infty$; $i = 1, 2, \cdots I$. $d(x, y)$ is generalized distance measure function; I is the number of candidate alternatives and J is the number of attributes; ω_j is the weight of j^{th} attribute; For an alternative X_i, its rating on j^{th} attribute is represented as f_{ij} ; the positive and negative ideal solution on j^{th} attribute is represented as f_j^* and f_j^- respectively;

$L_{1,i}$ (represented as R_i) is represented as majority rule to satisfy a maximum group utility, while $L_{\infty,i}$ (represented as S_i) is interpreted as a rule to satisfy minimum individual regret [23]. S_i and R_i are used to compromise ranking in group decision and they are calculated by the following equations:

$$S_i = \sum_{j=1}^{J} \omega_j (f_j^* - f_{ij}) \Big/ (f_j^* - f_j^-) \tag{12}$$

$$R_i = \max_j [\omega_j (f_j^* - f_{ij}) \Big/ (f_j^* - f_j^-)] \tag{13}$$

The generalized distance aggregation function is used to eliminate the units of different attribute functions. Because $d(x, y)$ is precise real number belonging to the interval [0,1], VIKOR algorithm with heterogeneous information is similar to the idea of traditional VIKOR.

3.3.2. Determine the best value f_j^* and the worst value f_j^- of all attributes

Dynamic diagnostic strategy is essentially an optimization decision process. For fault diagnosis of systems with heterogeneous information, the first task is to build a decision matrix $F = [f_{ij}]_{m \times n}$. And then the positive ideal solution f_j^* and the negative ideal solution f_j^- of all attributes are calculated as follows according to the different types of values.

If the value type of the attributes is a defined number, the positive ideal solution f_j^* and the negative ideal solution f_j^- can be solved by the following equations:

$$f_j^* = \begin{cases} \max\limits_{1 \le i \le m} \{f_{ij}\}, \text{ if the } j_{th} \text{ attribute is a benefit attribute;} \\ \min\limits_{1 \le i \le m} \{f_{ij}\}, \text{ if the } j_{th} \text{ attribute is a cost attribute.} \end{cases} \tag{14}$$

$$f_j^- = \begin{cases} \min\limits_{1 \le i \le m} \{f_{ij}\}, \text{ if the } j_{th} \text{ attribute is a benefit attribute;} \\ \max\limits_{1 \le i \le m} \{f_{ij}\}, \text{ if the } j_{th} \text{ attribute is a cost attribute.} \end{cases} \tag{15}$$

If the value type of the attributes is an interval number, the positive ideal solution f_j^* and the negative ideal solution f_j^- can be calculated by the following equations:

$$f_j^* = \begin{cases} [\max\limits_{1 \le i \le m} \{f_{ij}^L\}, \max\limits_{1 \le i \le m} \{f_{ij}^U\}], \text{ if the } j_{th} \text{ attribute is a benefit attribute;} \\ [\min\limits_{1 \le i \le m} \{f_{ij}^L\}, \min\limits_{1 \le i \le m} \{f_{ij}^U\}], \text{ if the } j_{th} \text{ attribute is a cost attribute.} \end{cases} \tag{16}$$

$$f_j^- = \begin{cases} [\min\limits_{1 \le i \le m} \{f_{ij}^L\}, \min\limits_{1 \le i \le m} \{f_{ij}^U\}], \text{ if the } j_{th} \text{ attribute is a benefit attribute;} \\ [\max\limits_{1 \le i \le m} \{f_{ij}^L\}, \max\limits_{1 \le i \le m} \{f_{ij}^U\}], \text{ if the } j_{th} \text{ attribute is a cost attribute.} \end{cases} \tag{17}$$

where $[f_{ij}^L, f_{ij}^U]$ is **an interval evaluation value of the i^{th} alternative on the j^{th} attribute.**

If the value type of the attributes is a triangular fuzzy number, the positive ideal solution f_j^* and the negative ideal solution f_j^- can be calculated by the following equations:

$$f_j^* = \begin{cases} (\max\limits_{1 \le i \le m} \{f_{ij}^L\}, \max\limits_{1 \le i \le m} \{f_{ij}^M\}, \max\limits_{1 \le i \le m} \{f_{ij}^U\}), \text{ if the } j_{th} \text{ attribute is a benefit attribute;} \\ (\min\limits_{1 \le i \le m} \{f_{ij}^L\}, \min\limits_{1 \le i \le m} \{f_{ij}^M\}, \min\limits_{1 \le i \le m} \{f_{ij}^U\}), \text{ if the } j_{th} \text{ attribute is a cost attribute.} \end{cases} \tag{18}$$

$$f_j^- = \begin{cases} (\min\limits_{1 \le i \le m} \{f_{ij}^L\}, \min\limits_{1 \le i \le m} \{f_{ij}^M\}, \min\limits_{1 \le i \le m} \{f_{ij}^U\}), \text{ if the } j_{th} \text{ attribute is a benefit attribute;} \\ (\max\limits_{1 \le i \le m} \{f_{ij}^L\}, \max\limits_{1 \le i \le m} \{f_{ij}^M\}, \max\limits_{1 \le i \le m} \{f_{ij}^U\}), \text{ if the } j_{th} \text{ attribute is a cost attribute.} \end{cases} \tag{19}$$

where $(f_{ij}^L, f_{ij}^M, f_{ij}^U)$ is a triangular fuzzy value of the i^{th} alternative on the j^{th} attribute given by domain experts; $\|f_j^* - f_j^-\| \ne 0$.

3.3.3. Normalize the decision matrix

Usually, evaluation values of different attributes have different dimensions, which are not directly comparable. So the decision matrix $F = [f_{ij}]_{m \times n}$ for heterogeneous information with different dimensions should be normalized. A normalized decision matrix $P^C = [p_{ij}]_{m \times n}$ is obtained based on the 1-norm concept using the following equation:

$$p_{ij} = \frac{\| f_j^* - f_{ij} \|}{\| f_j^* - f_j^- \|} = \begin{cases} \dfrac{| f_j^* - f_{ij} |}{| f_j^* - f_j^- |}, i \in M, j \in N_1 \\ \dfrac{| f_j^{L*} - f_{ij}^L | + | f_j^{U*} - f_{ij}^U |}{| f_j^{L*} - f_j^{L-} | + | f_j^{U*} - f_j^{U-} |}, i \in M, j \in N_2 \\ \dfrac{| f_j^{L*} - f_{ij}^L | + | f_j^{M*} - f_{ij}^M | + | f_j^{U*} - f_{ij}^U |}{| f_j^{L*} - f_j^{L-} | + | f_j^{M*} - f_j^{M-} | + | f_j^{U*} - f_j^{U-} |}, i \in M, j \in N_3 \end{cases} \tag{20}$$

3.3.4. Calculate the weights of attributes based on the Entropy concept

There are several attributes in the multi-attribute decision making, and their weights may be unknown. Subjective evaluation method and objective evaluation method can be used to determine the weights of attributes. However, the former usually uses the subjective judgment of the decision maker to determine the weights of attributes and it has subjectivity and arbitrariness to a certain degree. Objective evaluation method uses some algorithms to calculate the weights of attributes according to the attributes information and it is more scientific. Entropy weight method [14] is widely used to determine the weights in practical engineering. Shannon Entropy is a measure of information uncertainty based on probability theory. It is very suitable for measuring the relative contrast intensities of attributes to represent the average intrinsic information transmitted to the decision makers. The smaller the entropy value of evaluation attribute v_j is, the more the value of this attribute plays in the decision. That is to say, its weight is larger. The steps for determining the weights of attributes based on the entropy weight method are as follows:

Step 1: Normalize the normalized decision matrix and calculate the weighted proportion of the i^{th} alternative on the j^{th} attribute using the following equation.

$$P_{ij} = \frac{p_{ij}}{\sum_{i=1}^{n} p_{ij}} \tag{21}$$

Step 2: Calculate the entropy H_j value of the j^{th} attribute as follows:

$$H_j = -K\sum_{i=1}^{n} p_{ij} \ln p_{ij} \tag{22}$$

where $K = 1/\ln n$ $(K > 0, 0 \le p_{ij} \le 1)$ and assume $p_{ij} \ln p_{ij} = 0$ if p_{ij} is 0.

Step 3: Calculate the value of α_j defined as follows:

$$\alpha_j = 1 - H_j \tag{23}$$

where α_j is the divergence degree of the intrinsic information of the j^{th} attribute. The greater the value of α_j, the more important the attribute is in the decision making process.

Step 4: Calculate the weights of attributes using the following equation:

$$\omega_j = \frac{\alpha_j}{\sum_{j=1}^{m} \alpha_j} \tag{24}$$

where $\sum_{j=1}^{n} \omega_j = 1, 0 \le \omega_j \le 1$.

3.3.5. Calculate the values S_i, R_i and Q_i

A generalized distance aggregation function is used to obtain the optimal ranking in the decision making according to VIKOR algorithm. The optimal ranking should satisfy the maximum group utility and satisfy the minimum individual regret. S_i, R_i and Q_i are defined as follows.

$$S_i = \sum_{j=1}^{n} \omega_j \cdot p_{ij} \tag{25}$$

$$R_i = \max_{1 \le j \le n} \{\omega_j \cdot p_{ij}\} \tag{26}$$

$$Q_i = v\frac{S_i - S^+}{S^- - S^+} + (1-v)\frac{R_i - R^+}{R^- - R^+} \tag{27}$$

where $S^+ = \min_i S_i$, $S^- = \max_i S_i$, $R^+ = \min_i R_i$ $R^- = \max_i R_i$ and v is introduced as the weight for the strategy of maximum group utility, whereas 1-v is the weight of the individual regret. If v>0.5, it means that a decision making is based on the conditions agreed by the vast majority of policy makers. If v<0.5, the decision making is based on the circumstances refused by the vast majority of policy makers. Usually, v can take any value from 0 to 1 and the value of v is set to 0.5 in the paper. Finally, we can obtain the optimal diagnosis ranking by the value Q_i in ascending order.

3.3.6. Updating the decision matrix using the previous diagnosis result

The component with a smaller Q_i value should be diagnosed first. This assures a reduced number of system checks while bringing the system back to life. Nevertheless, this approach fails to update the reliability parameters in order to optimize the diagnosis process using the previous diagnosis result. That is to say, DIF and RAW are not updated by the previous diagnosis result, thereby having a significant effect on the diagnosis efficiency. When the component diagnosed at the present time works we should feed this evidence information to a DEN and obtain the updating DIF and RAW. In addition, the decision matrix should be updated too and the corresponding value of Q_i can be calculated to determine the next optimal ranking. And so on, the final optimal diagnostic ranking can be obtained.

4. A case study

Train-ground wireless communication system, a vital subsystem of urban rail transit, is responsible for data transmission between vehicle equipment and ground equipment. To ensure safe operation, application of high technologies has been used to improve its reliability greatly. Once train-ground wireless communication system breaks down, it may decrease the operation performance and even causes a great loss. Therefore, an efficient diagnosis strategy should be taken to bring it back to life as soon as possible when it fails. Fig.4 shows the DFT model of a train-ground wireless communication system. It is assumed that all components have the exponential distribution and failure rates of components expressed in interval values are shown in Table 4.

Table 4. Interval failure rates of components

Components	Interval failure rates	Components	Interval failure rates
X1	[4.22e-6, 5.28e-6]	X8,X9	[5.49e-6, 6.71e-6]
X2	[5.94e-6, 7.26e-6]	X10,X11	[3.15e-5, 3.85e-5]
X3	[4.86e-5, 5.94e-5]	X12,X13	[6.12e-5, 7.48e-5]
X4,X5	[3.78e-5, 4.62e-5]	X14	[5.04e-5, 6.11e-5]
X6,X7	[6.48e-5, 7.92e-5]	X15	[5.04e-5, 6.11e-5]

Table 5. DIF of all components

Components	DIF of components	Components	DIF of components
X1	[0.0508,0.0518]	X8,X9	[0.0857,0.0939]
X2	[0.0709,0.0722]	X10,X11	[0.0708,0.0756]
X3	[0.5681,0.5727]	X12,X13	[0.2012,0.2156]
X4,X5	[0.0751,0.0822]	X14	[0.1788,0.1914]
X6,X7	[0.1963,0.2148]	X15	[0.1788,0.1914]

The DFT is mapped into a corresponding DEN for quantitative analysis using the method mentioned above. Assuming the task time T=1000 h, the probability of system failure can be obtained using the inference algorithm, and it is [0.08293, 0.10714]. In addition, the DIF and RAW of all components can be calculated shown in Table 5 and

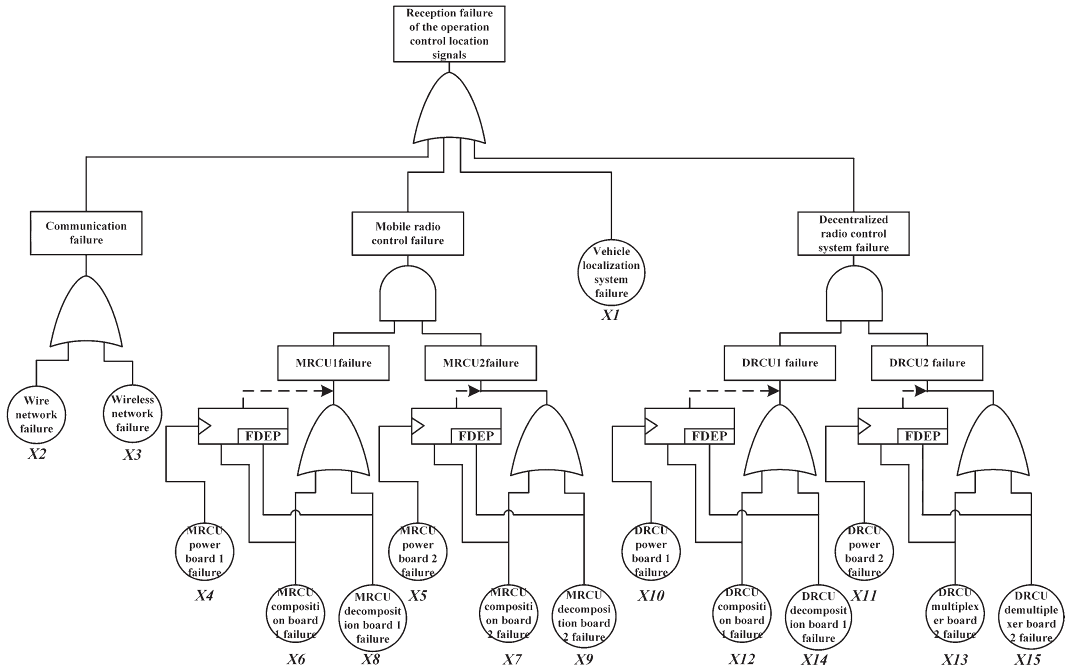

Fig. 4. DFT model of train-ground wireless communication system

Table 6. RAW of all components

Components	$P(S=1\mid X_i=1)$	$I_{X_i}^{RAW}$
X1	1	[9.3337,12.0694]
X2	[0.99214,1]	[9.2603,12.0694]
X3	[0.99214,1]	[9.2603,12.0694]
X4	[0.16768, 0.205466]	[1.5651,2.4799]
X5	[0.16768, 0.205466]	[1.5651,2.4799]
X6	[0.16768, 0.205466]	[1.5651,2.4799]
X7	[0.16768, 0.205466]	[1.5651,2.4799]
X8	[0.16768, 0.205466]	[1.5651,2.4799]
X9	[0.16768, 0.205466]	[1.5651,2.4799]
X10	[0.18919, 0.230323]	[1.7658,2.7799]
X11	[0.18919, 0.230323]	[1.7658,2.7799]
X12	[0.18919, 0.230323]	[1.7658,2.7799]
X13	[0.18919, 0.230323]	[1.7658,2.7799]
X14	[0.18919, 0.230323]	[1.7658,2.7799]
X15	[0.18919, 0.230323]	[1.7658,2.7799]

Table 7. Linguistic assessment of Components' test cost

Components	test cost
X1	High
X2	Moderate
X3	Very High
X4,X5	Very Low
X6,X7	Low
X8,X9	Low
X10,X11	Very Low
X12,X13	Low
X14,X15	Low

Table 8. Evaluation standards of the test cost

Linguistic expression for test cost	Fuzzy numbers
Very High	(0.7, 0.9, 1)
High	(0.5, 0.7, 0.9)
Moderate	(0.3, 0.5, 0.7)
Low	(0.1, 0.3, 0.5)
Very Low	(0.1, 0.2, 0.3)

Table 9. A decision matrix with heterogeneous information

Components	DIF	RAW	Test cost
X1	[0.0508,0.0518]	[9.3337,12.0694]	(0.5,0.7,0.9)
X2	[0.0709,0.0722]	[9.2603,12.0694]	(0.3,0.5,0.7)
X3	[0.5681,0.5727]	[9.2603,12.0694]	(0.7,0.9,1)
X4	[0.0751,0.0822]	[1.5651,2.4799]	(0.1,0.2,0.3)
X5	[0.0751,0.0822]	[1.5651,2.4799]	(0.1,0.2,0.3)
X6	[0.1963,0.2148]	[1.5651,2.4799]	(0.1,0.3,0.5)
X7	[0.1963,0.2148]	[1.5651,2.4799]	(0.1,0.3,0.5)
X8	[0.0857,0.0939]	[1.5651,2.4799]	(0.1,0.3,0.5)
X9	[0.0857,0.0939]	[1.5651,2.4799]	(0.1,0.3,0.5)
X10	[0.0708,0.0756]	[1.7658,2.7799]	(0.1,0.2,0.3)
X11	[0.0708,0.0756]	[1.7658,2.7799]	(0.1,0.2,0.3)
X12	[0.2012,0.2156]	[1.7658,2.7799]	(0.1,0.3,0.5)
X13	[0.2012,0.2156]	[1.7658,2.7799]	(0.1,0.3,0.5)
X14	[0.1788,0.1914]	[1.7658,2.7799]	(0.1,0.3,0.5)
X15	[0.1788,0.1914]	[1.7658,2.7799]	(0.1,0.3,0.5)

Table 10. A normalized decision matrix with heterogeneous information

components	DIF	RAW	Test cost
X1	1.0000	0.0000	0.2500
X2	0.9610	0.0042	0.5500
X3	0.0000	0.0042	0.0000
X4	0.9473	1.0000	1.0000
X5	0.9473	1.0000	1.0000
X6	0.7029	1.0000	0.8500
X7	0.7029	1.0000	0.8500
X8	0.9258	1.0000	0.8500
X9	0.9258	1.0000	0.8500
X10	0.9578	0.9712	1.0000
X11	0.9578	0.9712	1.0000
X12	0.6974	0.9712	0.8500
X13	0.6974	0.9712	0.8500
X14	0.7422	0.9712	0.8500
X15	0.7422	0.9712	0.8500

Table 11. The positive and negative ideal solutions

Attributes	Positive ideal solutions	Negative ideal solutions
DIF	[0.5681，0.5727]	[0.0508，0.0518]
RAW	[9.3337，12.0694]	[1.5651，2.4799]
Test cost	(0.1，0.2，0.3)	(0.7，0.9，1)

Table 6 respectively. DIF enables us to discriminate between components by their importance from a diagnostic point of view. RAW is defined as the ratio of the system unreliability if a component has failed over the system unreliability and it plays an important role in the diagnostic sequence. Furthermore, test cost of the components has a significant impact on diagnostic strategy. However, test cost of all components is usually very difficult to express as defined values because of uncertainties. So the linguistic assessments are used for generating criteria and alternative ratings, which are transformed into triangular fuzzy numbers to describe test cost of all components. Table 7 and Table 8 show the linguistic assessment of the test cost and alternative ratings of all components.

DIF, RAW and test cost are used to build a decision matrix. The former two, expressed in interval numbers, belong to the benefit attributes. The latter belongs to the cost attribute, which is expressed in a triangular fuzzy number. Table 9 and table 10 show the deci-

Table 12. Values of S_i, R_i and Q_i for all components

Components	S_i	R_i	Q_i
X1	0.5782	0.2905	0.4156
X2	0.4532	0.2792	0.1275
X3	0.3850	0.3836	0.5000
X4	0.6010	0.3259	0.6275
X5	0.6010	0.3259	0.6275
X6	0.5876	0.3259	0.6023
X7	0.5876	0.3259	0.6023
X8	0.6524	0.3259	0.7234
X9	0.6524	0.3259	0.7234
X10	0.5947	0.3165	0.5706
X11	0.5947	0.3165	0.5706
X12	0.5766	0.3165	0.5367
X13	0.5766	0.3165	0.5367
X14	0.5896	0.3165	0.5611
X15	0.5896	0.3165	0.5611

Table 13. An updating decision matrix with the previous diagnosis result

Components	DIF	RAW	Test cost
X1	[0.0544,0.0554]	[9.9374,12.9137]	(0.5,0.7,0.9)
X3	[0.6078,0.6120]	[9.8593,12.9137]	(0.7,0.9,1.0)
X4	[0.0778,0.0850]	[1.6174,2.5785]	(0.1,0.2,0.3)
X5	[0.0778,0.0850]	[1.6174,2.5785]	(0.1,0.2,0.3)
X6	[0.2039,0.2222]	[1.6174,2.5785]	(0.1,0.3,0.5)
X7	[0.2039,0.2222]	[1.6174,2.5785]	(0.1,0.3,0.5)
X8	[0.0890,0.0971]	[1.6174,2.5785]	(0.1,0.3,0.5)
X9	[0.0890,0.0971]	[1.6174,2.5785]	(0.1,0.3,0.5)
X10	[0.0739,0.0790]	[1.8336,2.9019]	(0.1,0.2,0.3)
X11	[0.0739,0.0790]	[1.8336,2.9019]	(0.1,0.2,0.3)
X12	[0.2100,0.2242]	[1.8336,2.9019]	(0.1,0.3,0.5)
X13	[0.2100,0.2242]	[1.8336,2.9019]	(0.1,0.3,0.5)
X14	[0.1866,0.1990]	[1.8336,2.9019]	(0.1,0.3,0.5)
X15	[0.1866,0.1990]	[1.8336,2.9019]	(0.1,0.3,0.5)

Table 14. Revised values of S_i, R_i and Q_i for all components

Components	S_i	R_i	Q_i
X1	0.5369	0.2746	0.2790
X3	0.3512	0.3496	0.3709
X4	0.6372	0.3757	0.9298
X5	0.6372	0.3757	0.9298
X6	0.6245	0.3757	0.9107
X7	0.6245	0.3757	0.9107
X8	0.6839	0.3757	1
X9	0.6839	0.3757	1
X10	0.6289	0.3649	0.8636
X11	0.6289	0.3649	0.8636
X12	0.6117	0.3649	0.8377
X13	0.6117	0.3649	0.8377
X14	0.6237	0.3649	0.8557
X15	0.6237	0.3649	0.8557

sion matrix with heterogeneous information and normalized decision matrix respectively. The positive and negative ideal solutions can be obtained shown in table 11. Based on the entropy methodology, the weights of the three attributes, ω_1=0.2905，ω_2=0.3259，ω_3=0.3836 are obtained using the Eq. (21) - (24). Table 12 presents the values of S_i, R_i and Q_i for all components. The optimal diagnosis sequence is as follows according to the corresponding Q_i in ascending order.

X2≻X1≻X3≻X12(X13)≻X10(X11)≻X14(X15)≻X6(X7)≻X4(X5)≻X8(X9)

If a train-ground wireless communication system broke down, we should diagnose X2 firstly. If X2 fails, then diagnosis is over. Otherwise, we should feed this evidence information (X2 works) to the DEN and recalculate DIF and RAW. An updating decision matrix with the previous diagnosis result is shown in table 13. Similarly, the revised values of S_i, R_i and Q_i for all components can be obtained shown in table 14. The updating optimal diagnosis sequence is as follows.

X1≻X3≻X12(X13)≻X14(X15)≻X10(X11)≻X6(X7)≻X4(X5)≻X8(X9)

So we can draw a conclusion that the next component diagnosed is X1. If X1 fails, then diagnosis is over. Otherwise, we input this evidence information to the DEN and update the decision matrix again. These steps are repeated several times, and the final optimal diagnostic ranking can be obtained as follows.

X2≻X1≻X3≻X12(X13)≻X10(X11)≻X4(X5)≻X14(X15)≻X6(X7)≻X8(X9)

Obviously, the diagnostic strategy which takes the previous diagnosis result into account is more reasonable and efficient because it can update the decision matrix dynamically. To avoid subjectivity and arbitrariness, the proposed method determines the weights of attributes based on the Entropy concept. Besides, the optimal ranking is obtained directly based on the original heterogeneous information without a transformation process using a generalized distance-based function, which can improve diagnosis efficiency and reduce information loss.

5. Conclusion

In this paper, a novel dynamic diagnostic strategy for complex systems is proposed based on reliability analysis and distance-based VIKOR with heterogeneous information, which aims to deal with two important issues that arise in engineering applications, such as failure dependency and epistemic uncertainty. For the challenge of the failure dependency, a DFT is used to describe the dynamic fault behaviours. For the challenge of the epistemic uncertainty, the failure rates of components in complex systems are expressed in interval numbers. Furthermore, DFT is converted into a DEN to calculate some reliability results and these parameters together with test cost constitute a decision matrix. In addition, a dynamic diagnostic strategy is developed based on an improved VIKOR algorithm and the previous diagnosis result. This diagnosis algorithm determines the weights of attributes based on the Entropy concept to avoid experts' subjectivity and obtains the optimal ranking directly on the original heterogeneous information without a transformation process, which can improve diagnosis efficiency and reduce information loss. Finally, a train-ground wireless communication system is given to demonstrate the efficiency of the proposed method. This method takes full advantages of DFT for modelling, DEN for the uncertainty inference and VIKOR for dynamic decision making, which is especially suitable to diagnose complex systems.

Acknowledgement

This research was funded by National Natural Science Foundation of China [71461021].

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