1. Introduction

MEMS gyroscopes, due to its lower mass, lower power consumption, and smaller volume, have become one of the most promising inertial components, widely used in commercial, military and other fields [13], such as inertial navigation devices, portable electronic products and so on. However, MEMS gyroscopes often undergo high temperature condition, and physical properties of silicon materials would be significantly affected by temperature. How to predict the lifetime of the MEMS gyroscope has become an urgent problem to be solved in engineering [32].

Accelerated degradation test (ADT) collects performance degradation data of products at accelerated stress levels, and uses these data to extrapolate reliability and lifetime of products for the use conditions [20, 23, 33, 34]. Mulloni studied the lifetime prediction of RF-MEMS switches based on ADT [22]. The reliable lifetime of the smart electricity meter was predicted based on two group tests data under high stress level. The results show that the lifetime calculated by the acceleration factor model and mean lifetime under high stress levels is close to the mean lifetime calculated by the linear equation at normal temperature stress.

Keywords: MEMS gyroscope, temperature stress, accelerated degradation test, acceleration factor model, Allan variance.
lifetime under operating conditions [10]. A progressive-stress accelerated degradation test with a non-linear degradation path was proposed to obtain timely information of the product's lifetime distributions for highly reliable products [6]. Chen studied the accelerated degradation reliability modeling and test data statistical analysis of aerospace electrical connectors [5]. Lifetime prediction technology based on ADT has become an inevitable requirement for reliability prediction of MEMS gyroscopes under time and cost constraints [4, 31].

The selection of accelerated stress is the first step in the implementation of ADT. Many scholars have studied the effect of temperature on the performance of MEMS device. Jin-Won Joo studied the global and local deformations of packages caused by temperature, and studied the influence of temperature on frequency shift of MEMS gyroscope [15]. Chun-Lin Lu analyzed the thermal stress for a CMOS-MEMS microphone with various metallization and materials [8]. Saeedivahdat et al. studied the effect of thermal stress on frequency response for a diaphragm in MEMS microphone by numerical model [29]. Therefore, temperature stress may be the most likely accelerated stress of MEMS gyroscope in ADTs.

Extraction of degradation characteristic is the key step to establish degradation model [25, 30]. There are many noise terms in the output voltage of MEMS gyroscope due to its working principle and processing technology, which makes the extraction of the effective degradation characteristics more complicated. Many scholars have studied the noise terms by power spectral density (PSD), time domain analysis methods (autocorrelation function, Gauss Markov process and autoregressive model) and the Kalman filter method. However, these methods have their own inherent shortcomings [16, 19, 36].

Allan variance is an important means to analyze the random noise characteristics of gyroscope, including laser gyroscope, fiber optic gyroscope and MEMS gyroscope. The method is a time domain analysis method proposed by the American National Bureau of Standards, which has been an IEEE standard method for parameter analysis of gyroscopes [9]. Allan variance usually describe the noise of gyroscope as several stochastic characteristics: quantization noise, angular random walk, bias instability, rate random walk and rate ramp. Some stochastic characteristics often directly reflect the performance of gyroscope, such as angular random walk and bias instability [1-3].

Many scholars have studied the performance of laser gyroscope and fiber optic gyroscope based on Allan variance. Lawrence analyzed the performance of ring laser gyroscope based on Allan variance [18]. Pin analyzed the noise characteristics of fiber optic gyroscope based on weighted least squares algorithm and Allan variance [26]. Grantham analyzed the random error of inertial sensors using Allan variance [11]. Chikovani evaluated the performance of laser gyroscopes, and fiber optic gyroscopes from different countries based on Allan variance [7]. However, the research on performance study of MEMS gyroscope is limited.

The rest of this document is organized as follows. In Section 2, we extract the degradation characteristic (bias instability) of MEMS gyroscope based on Allan variance, and analyze the effect of temperature stress on the degradation rate of bias instability. In Section 3, a universal lifetime prediction method of MEMS gyroscope is proposed. In Section 4, a case study is provided to illustrate the application and utility of the proposed method. Finally, the conclusions are presented in Section 5.

2. Theory

2.1. Structure and operating principle of MEMS gyroscope

Almost all MEMS gyroscope use vibrating mechanical elements to sense rotation, “Butterfly” MEMS gyroscope is a typical vibrating MEMS gyroscope based on the Coriolis effect [14]. As shown in Figure 1, it has a four-mass full-differential structure with high sensitivity, good linearity and low cost, and is widely used in commercial fields.

“Butterfly” MEMS gyroscope mainly contains the circuit board and the vacuum packaged resonator, as shown in Figure 1(a). The image of resonator is shown in Figure 1(b), which consists of a single crystal silicon structure and a Pyrex glass base with patterned electrodes [14]. The silicon structure is manufactured by anisotropic wet etching process, which mainly includes four proof masses, a slanted suspension beam, and four trapezoidal cantilever beams, as shown in Figure 2(a). The excitation electrodes and detection electrodes are fabricated on Pyrex glass base under the proof mass, as shown in Figure 2(b), and it is connected with silicon structure by anodic bonding to form driving capacitance and detecting capacitance. The cross section of the slanted suspension beam (A-A) is composed of (100) and (111) crystal planes with an angle of 54.7°, as shown in Figure 2(c). The slanted beam with an asymmetric cross section so that vertical electrostatic forces bend the beams both vertically and horizontally, as shown in Figure 2(c). The excitation mode and the detection mode are two perpendicular degenerate modes of the axially symmetric elastic body. There will be a sinusoidal oscillation in detection axis direction due to the Coriolis Effect when an angular rate inputs.

Fig. 2 Silicon structure and the electrode layout of “Butterfly” MEMS gyroscope
Electrostatic force from the excitation electrodes makes the four proof masses vibrate with stable amplitude and phase on its resonant frequency by automatic gain control methods. There will be an oscillation in detection axis (Z direction) due to the Coriolis Effect if there is an angular rate in the sensitive axis (Y direction), which reflects the value of the input angular rate.

2.2. The principle of Allan variance

Allan Variance is a method of describing the root mean square of random drift error. Sampling the angular rate of MEMS gyroscope with sampling time \( t_0 \) and the total sample size is \( M \). Then \( m = t_0 \times \tau \) data points are made as one cluster, so we can get \( J=[M/m] \) clusters, where \( [x] \) represents the floor of \( x \). \( J \) is the number of sampled groups. The time of each group \( \tau \) is the correlation time, and the average value of the defined angular rate can be expressed as:

\[
\bar{y}_k(m) = \frac{1}{m} \sum_{i=1}^{m} y(k-i)m + 1, \quad k = 1, 2, \ldots, \left[ \frac{M}{2} \right] \tag{1}
\]

The Allan variance defined by the angular rate can be expressed as:

\[
\sigma^2(t) = \frac{1}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{y}_k(m) - \bar{y}_k(m) \right]^2 \tag{2}
\]

where \( \langle y \rangle \) represents the average of \( y \).

Allan variance can usually subdivide the random noise characteristics of the inertial device (MEMS gyroscope) into five typical characteristics, namely quantization noise, angular random walk, bias instability, and rate random walk and rate ramp. It can be expressed as [28]:

\[
\begin{align*}
\sigma_Q^2 &= 3Q^2 / \tau^2 \\
\sigma_N^2 &= N^2 / \tau \\
\sigma_B^2 &= 2 \ln 2 / \pi B_s^2 \\
\sigma_R^2 &= \kappa^2 / 3 \\
\sigma_R^2 &= R^2 / 2
\end{align*}
\]

where \( Q \) is a random drift caused by quantization noise. The initial output of the inertial sensor is analog data, which is quantized into digital data for calculation, thus quantization noise is generated at the output of the inertial device [26]. \( N \) is the white noise generated during the measurement process, which appears as an angular random walk at the output of the MEMS gyro and belongs to high frequency noise [19]. \( B_s \) is a random drift caused by bias instability, which is a kind of low-frequency noise generated by the sensor electronic components or the surrounding environment [26]. \( \kappa \) is a random drift caused by random walk. The source of this error is still unclear. \( R \) is the random drift caused by the rate ramp, which is caused by the monotonic change of the sensor for a long time [26].

It is generally considered that five typical characteristics are independent of each other, so the total variance can be expressed as [24]:

\[
Y = X\beta + e \tag{4}
\]

where:

\[
\begin{bmatrix}
\sigma_1^2 \\
\sigma_2^2 \\
\vdots \\
\sigma_m^2
\end{bmatrix}
, \quad X =
\begin{bmatrix}
\tau_1^2 & \tau_1^{-1} & 1 & \tau_1^{-1} & \tau_1^2 \\
\tau_2^2 & \tau_2^{-1} & 1 & \tau_2^{-1} & \tau_2^2 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\tau_m^2 & \tau_m^{-1} & 1 & \tau_m^{-1} & \tau_m^2
\end{bmatrix}
, \quad \beta = \left[ \begin{array}{c}
3Q^2 \\
N^2 \\
2 \ln 2 / \pi B_s^2 \\
\kappa^2 / 3 \\
R^2 / 2
\end{array} \right]
\]

\[
(2) \quad \text{and, } e = \begin{bmatrix} w_1 \\
w_2 \\
w_m
\end{bmatrix}
\]

\[
\text{Five unknown parameters } Q, N, B_s, \kappa, R \text{ can be obtained by the least squares algorithm:}
\]

\[
\beta = \left( X^T X \right)^{-1} X^T Y \tag{6}
\]

2.3. The effect of temperature stress on the degradation rate of bias instability

Bias instability of MEMS gyroscope in a certain period of time can be calculated by Allan variance. In order to characterize the degradation characteristic changing with time, we calculate the bias instability using daily data, so the bias instability in \( D \) days are \( B_s^1, B_s^2, \ldots, B_s^D \) respectively.

The five characteristics of the MEMS gyroscope under long-term operation (\( D \) days) can be expressed as:

\[
\beta_t = \left( X^T X \right)^{-1} X^T Y_t \quad (t=1,2,3,\ldots,D) \tag{7}
\]

where:

\[
Y_t =
\begin{bmatrix}
\sigma_1^2 \\
\sigma_2^2 \\
\vdots \\
\sigma_m^2
\end{bmatrix}
, \quad X =
\begin{bmatrix}
\tau_1^2 & \tau_1^{-1} & 1 & \tau_1^{-1} & \tau_1^2 \\
\tau_2^2 & \tau_2^{-1} & 1 & \tau_2^{-1} & \tau_2^2 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\tau_m^2 & \tau_m^{-1} & 1 & \tau_m^{-1} & \tau_m^2
\end{bmatrix}
, \quad \beta_t = \left[ \begin{array}{c}
3Q^2 \\
N^2 \\
2 \ln 2 / \pi B_s^2 \\
\kappa^2 / 3 \\
R^2 / 2
\end{array} \right]
\]

where \( \sigma_{2n}^2 \) is the calculated Allan variance of the \( n \)-th day. \( Q_t \) is the quantization noise of the \( n \)-th day, \( N_t \) is the angular random walk of the \( n \)-th day, \( B_s^t \) is the bias instability of the \( n \)-th day, \( \kappa, \) is the rate ramp of the \( n \)-th day, and \( R_t \) is the rate random walk of the \( n \)-th day.

Equation (7) can also be simplified as:

\[
\beta_t = A_{2n} Y_t \quad (t=1,2,3,\ldots,D) \tag{9}
\]

In order to study the bias instability \( B_t \) in the vector \( \beta_t \), it assumed that the elements of the third row in the matrix \( A \) are \( a_{21}, a_{22}, a_{23}, \ldots, a_{2m} \), so equation (9) can be further simplified as:

\[
(2 \ln 2 / \pi B_t (t))^2 = \begin{bmatrix}
a_{21} & a_{22} & a_{23} & \cdots & a_{2m}
\end{bmatrix}
\begin{bmatrix}
\sigma_1^2 \\
\sigma_2^2 \\
\vdots \\
\sigma_m^2
\end{bmatrix}
+ a_{21} \sigma_1^2 + a_{22} \sigma_2^2 + \cdots + a_{2m} \sigma_m^2 \tag{10}
\]

The bias instability change with time can be expressed as (equation (11) is verified in Section 4.3):
\[ B'_t = B_x(t) = Kt + b \]  

(11)

where \( K \) is degradation rate of the bias instability, and \( b \) is the intercept.

Substituting equation (11) into equation (10), equation (10) can also be expressed as:

\[ 0.6643B_x(t) = 0.6643(Kt + b)^2 = a_1\sigma_{11}^2 + a_{12}\sigma_{12}^2 + \cdots + a_{3m}\sigma_{3m}^2 \]

(12)

Partial derivatives for temperature in both sides of equation (12), we can get:

\[ \frac{\partial}{\partial T} \left( 0.6643(Kt + b)^2 \right) = 0.8826(Kt + b) \frac{\partial K}{\partial T} \]

(13)

Equations (7)-(9) show that the third row elements \( a_{31}, a_{32}, a_{33}, \ldots, a_{3m} \) in matrix \( A \) are only related to the correlation time \( \tau \), so the right term of the equation (13) can be written as:

\[ \frac{\partial}{\partial T} \left( a_{31}\sigma_{31}^2 + a_{32}\sigma_{32}^2 + \cdots + a_{3m}\sigma_{3m}^2 \right) = a_{31}\frac{\partial \sigma_{31}^2}{\partial T} + a_{32}\frac{\partial \sigma_{32}^2}{\partial T} + \cdots + a_{3m}\frac{\partial \sigma_{3m}^2}{\partial T} \]

(14)

Substituting equation (2) into equation (14), equation (14) can be expressed as:

\[ a_{31}\frac{\partial \sigma_{31}^2}{\partial T} + a_{32}\frac{\partial \sigma_{32}^2}{\partial T} + \cdots + a_{3m}\frac{\partial \sigma_{3m}^2}{\partial T} = \frac{a_{31}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(1) - \bar{w}_k(1) \right]^2 + \frac{a_{32}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(2) - \bar{w}_k(2) \right]^2 + \cdots + \frac{a_{3m}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(m) - \bar{w}_k(m) \right]^2 \]

(15)

The first term in equation (15) denotes the square of the difference between adjacent arrays when the correlation time \( \tau = 1 \). Substituting equation (1) into the first item of equation (15), we can get:

\[ \frac{a_{31}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(1) - \bar{w}_k(1) \right]^2 = \frac{a_{31}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(1) - \bar{w}_k(1) \right]^2 \]

(16)

It is assumed that all the parameters (electrical parameters and structural parameters) of the MEMS gyroscope do not change during the operating process, only the environment temperature changes from \( T \) to \( T + \Delta T \), resulting in the MEMS gyroscope’s output (angular rate) changing from \( w_k \) to \( w_{k+1} \), so equation (16) can be expressed as:

\[ \frac{a_{31}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(1) - \bar{w}_k(1) \right]^2 = \frac{a_{31}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(1) - \bar{w}_k(1) \right]^2 \]

\[ \frac{a_{31}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(2) - \bar{w}_k(2) \right]^2 \]

(17)

The second term in equation (15) denotes the square of the difference between adjacent arrays when the correlation time \( \tau = 2 \). Substituting equation (1) into the first item of equation (15), we can get:

\[ \frac{a_{32}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(2) - \bar{w}_k(2) \right]^2 = \frac{a_{32}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(2) - \bar{w}_k(2) \right]^2 \]

(18)

Similarly, it is assumed that all the parameters (electrical parameters and structural parameters) of the MEMS gyroscope do not change during the working process, only the environment temperature changes from \( T \) to \( T + \Delta T \), resulting in the MEMS gyroscope’s output (angular rate) changing from \( w_{2k+1} \) to \( w_{2(k+1)+1} \), so equation (18) can be expressed as:

\[ \frac{a_{32}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(2) - \bar{w}_k(2) \right]^2 = \frac{a_{32}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(2) - \bar{w}_k(2) \right]^2 \]

(19)

So the \( m \)-th term in equation (18) can be expressed as:

\[ \frac{a_{3m}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(m) - \bar{w}_k(m) \right]^2 = \frac{a_{3m}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(m) - \bar{w}_k(m) \right]^2 \]

(20)

Substituting equation (17) equation (19) and equation (20) into equation (15), we can get:

\[ \frac{a_{31}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(1) - \bar{w}_k(1) \right]^2 = \frac{a_{31}}{2(J-1)} \sum_{k=1}^{J-1} \left[ \bar{w}_{k+1}(1) - \bar{w}_k(1) \right]^2 \]

(21)

It can be known that the solution of partial derivative is independent of the value of \( k \) from equation (21), without a loss of generality, let \( k = 3 \), we prove that the partial derivative is greater than 0. The proof process is as follows.

Let:
The degradation process of the product can be accelerated by the temperature stress, which can be expressed as:

$$x(T + \Delta T) = \sum_{i=1}^{m} w_{3m+i}(T + \Delta T)$$

$$x(T) = \sum_{i=1}^{m} w_{2m+i}(T)$$

(22)

$$\frac{\partial x(T)}{\partial T} > 0$$ due to $$\frac{\partial \omega}{\partial T} > 0$$ [26], so:

$$\frac{\partial}{\partial T} \left[ x(T + \Delta T) - x(T) \right]^2 = 2 \left[ x(T + \Delta T) - x(T) \right] \frac{\partial x(T + \Delta T) - \partial x(T)}{\partial t} > 0$$

(23)

So we can get:

$$\frac{\partial}{\partial T} \left[ \sum_{i=1}^{m} w_{3m+i}(T + \Delta T) - \sum_{i=1}^{m} w_{2m+i}(T) \right]^2 > 0$$

(24)

Because the correlation time \( \tau \) is positive, the third row elements of matrix \( A \) are all positive, so:

$$0.8826(Kt + b) \frac{\partial K}{\partial T} > 0$$

(25)

Because the bias instability is larger than 0, so:

$$\frac{\partial K}{\partial T} > 0$$

(26)

It indicates that the degradation rate of bias instability would increase with the increase of the temperature. So bias instability can be the degradation characteristic of MEMS gyroscope, and the temperature can be the accelerated stress of MEMS gyroscope in ADTs.

3. A universal lifetime prediction method of MEMS gyroscope

3.1. Model assumptions

The degradation of the product is affected by temperature stress. The degradation process of the product can be accelerated by the temperature stress higher than the use or storage conditions. In ADTs, the temperature level of the accelerated stress is higher than the use or storage conditions. In ADTs, the degradation of the product is affected by temperature stress. It is assumed that the theoretical degradation trajectory and accelerated degradation trajectory of the product satisfy the linear degradation model, which can be expressed as:

$$B_{i,j,k}(t) = K_{i,t} + b_{i} + \epsilon_{i,j,k}, \quad i = 1,2,..,M, j = 1,2,..,n_{i}, k = 1,2,..,l_{j}$$

(27)

where \( B_{i,j,k} \) is the characteristic value (bias instability) of the \( k \)-th measurement of the \( j \)-th specimen under the \( i \)-th stress level, \( t \) is the time, \( \epsilon_{i,j,k} \) is the random measurement error, \( K_{i,t} \) is the degradation rate under the \( i \)-th stress level, and \( b_{i} \) is the intercept under the \( i \)-th stress level.

3.2. The description of the universal method for lifetime prediction of MEMS gyroscope

On the basis of the above model assumptions, a universal method for lifetime prediction of MEMS gyroscope is proposed, as shown in Figure 3. Firstly, a long-term test platform for MEMS gyroscope is constructed, and the ADT scheme is designed. The degradation characteristic (bias instability) of MEMS gyroscope is extracted based on the output voltage of the MEMS gyroscope and Allan variance. In order to characterize the degradation trend of MEMS gyroscope, we propose the following steps:

Step 1, Convert the output voltage of the MEMS gyroscope to angular rate in each day, and then calculate the root mean square value of the angular rate in each day based on equation (2). Step 2, The bias instability of MEMS gyroscope in every day can be calculated by the least squares method, i.e. equation (6). Step 3, The degradation trend of bias instability in \( D \) days can be obtained. Step 4, Fit the degradation trend of bias instability with linear function.

Then the degradation model of MEMS gyroscope can be established. Because the accelerated stress in ADTs is temperature, so we use Arrhenius model as the acceleration model [17, 27, 35], and the degradation rate of the MEMS gyroscope can be expressed by:

$$\frac{dB_{i}(t)}{dr} = K = A\exp(-E / k_{B}T)$$

(28)

where \( k_{B} \) is the Boltzmann constant, which is \( 8.6171 \times 10^{-5} \text{eV/°C} \). \( T \) is the Kelvin temperature with a unit of K, \( A \) is an unknown parameter and \( E \) is the activation energy.

Taking the natural logarithm on both sides of equation (28) results in:

$$\ln \left( \frac{dB_{i}(t)}{dr} \right) = \ln A - E / k_{B}T$$

(29)

So, the activation energy \( E \) can be expressed as:

$$E = -k_{B} \int_{\frac{1}{T}} \ln \left( \frac{dB_{i}(t)}{dr} \right) \frac{1}{T}$$

(30)

Integrating on the both two sides of equation (28) results in:

$$\int_{0}^{T_{F}} dB_{i}(t) = A\exp(-E / k_{B}T)_{0}^{T_{F}}$$

(31)

where \( T_{F} \) is the failure time and \( B_{i}^{cr} \) is failure threshold of bias instability.

So the failure time \( T_{F} \) can be expressed as:

$$T_{F} = \frac{B_{i}^{cr}}{A\exp(-E / k_{B}T)}$$

(32)

The acceleration factor model of MEMS gyroscope can be expressed as:
where $T_{F_0}$ and $T_{F_1}$ are failure time under normal stress and accelerated stress, $T_0$ and $T_1$ are normal temperature level and high temperature level.

As shown in equation (33), the acceleration factor model of MEMS gyroscope contains two unknown parameters, which can be estimated by least square method. Finally, the lifetime under normal stress level can be calculated by acceleration factor model and failure data under high stress level.

$$AF = \frac{T_{F_0}}{T_{F_1}} = \exp\left(-\frac{E}{k_B T_1}\right) \exp\left(-\frac{E}{k_B T_0}\right) = \exp\left(\frac{E}{k_B T_0} \frac{1}{T_1} \frac{1}{T_0}\right)$$ (33)

4. Application Case

4.1. Design of ATD scheme and construction of test platform

According to GJB150.1-86, two accelerated stress levels were selected as $S_1=45℃$, $S_2=60℃$, and the normal temperature stress $S_0=25℃$. The number of specimens under each stress level was 4. Twelve MEMS gyroscopes, numbered 1#, 2#, 3#, ..., 11#, 12#, were selected from the same batch MEMS gyroscopes. 1-4# MEMS gyroscopes were tested at 25℃ for 23 days, 5-8# MEMS gyroscopes were tested at 45℃ for 23 days, and 9-12# MEMS gyroscopes were tested at 60℃ for 23 days.

The test platform consists of E3631A power supply, NI PXLe-1071 data acquisition card, computer, constant temperature and humidity test chamber and multiple data lines, as shown in Figure 4. The E3631A power supply provides 5V voltage for the MEMS gyroscopes. The NI PXLe-1071 acquisition card can collect the output voltage of four MEMS gyroscopes simultaneously. It should be noted that Figure 4 only lists the test environment at normal stress level. The remaining two accelerated stress tests also used the test platform shown in Figure 4, only changing the temperature of the constant temperature and humidity test chamber.

4.2. Analysis of test data

The output voltage of 12 MEMS gyroscopes at three constant stress levels are shown in Figure 5 (a) (b) (c). Allan variance curves at different stress level are shown in Figure 5 (d), all MEMS gyroscopes have line with slopes of $-1/2$ and 0 at lower correlation time, and have line with slope of 1/2 at different correlation time after the line with slope of 0. It indicates that the angular random walk, bias instability and rate random walk are the main characteristics in the output of MEMS gyroscopes.

In addition, as shown in Figure 5(d), no line with slope of $-1$ and $+1$ appear in the Allan variance curve, which indicates that the quantization noise and rate slope are not the main characteristics in the output of MEMS gyroscopes. This is because the MEMS gyroscopes have no digital-to-analog conversion process from the output of capacitance to output of voltage, other scholars have same conclusions [12, 21, 26].

4.3. Degradation model

The degradation model of MEMS gyroscope can be established based on the test data and the new method. Step 1. Convert the output voltage of the 12 MEMS gyroscopes to angular rate, and then calculate the root mean square value of the angular rate every day based on the equation (2). Step 2. The bias instability of MEMS gyroscope in every day can be calculated by the least squares method, i.e. equation (6). Step 3. The degradation trend of bias instability in 23 days can be obtained. Step 4. Fit the degradation trend of bias instability with linear function. As shown in Figure 6, bias instability of all MEMS gyroscope increase approximately linearly with time at different temperature stress level. The degradation rates of each gyroscope under different temperature stress levels are shown in Table 1.

So the degradation model of the MEMS gyroscope based on Figure 4 and Table 1 can be expressed as:
where \( b \) is the interception of the fitting line.

In addition, Table 1 shows that the degradation rate of bias instability is larger in higher temperature stress. This is consistent with the theoretical analysis results in Section 2.3. The degradation model of the MEMS gyroscope under different stress levels can be established by the mean values of degradation rate and intercept, which can be expressed as respectively:

\[
B_i(t) = Kt + b
\]  

\[
B_i(t) = 0.0200t + 4.3694
\]  

\[
B_i(t) = 0.0299t + 1.3204
\]  

\[
B_i(t) = 0.0499t + 2.1060
\]

4.4. Lifetime prediction

The literature shows that the failure threshold \( B_{i}^{\text{THR}} \) of the MEMS gyroscope is 25°/h [1-3]. The lifetime of 12 MEMS gyroscopes can be calculated based on equation (34) and Table 1, as shown in Table 2.

So the interception and degradation of the Arrhenius equation can be expressed as:

\[
B_i(t) = 0.0200t + 4.3694
\]

\[
B_i(t) = 0.0299t + 1.3204
\]

\[
B_i(t) = 0.0499t + 2.1060
\]

### Table 1. Degradation rate of MEMS gyroscopes’ bias instability at different temperature stress levels

<table>
<thead>
<tr>
<th>Stress level</th>
<th>1#-12#MEMS</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_0 )</td>
<td>interception 4.2401</td>
<td>4.2174</td>
</tr>
<tr>
<td></td>
<td>degradation rate 0.0066</td>
<td>0.0132</td>
</tr>
<tr>
<td>( S_1 )</td>
<td>interception 1.6059</td>
<td>1.4122</td>
</tr>
<tr>
<td></td>
<td>degradation rate 0.0521</td>
<td>0.0137</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>interception 1.5055</td>
<td>2.1970</td>
</tr>
<tr>
<td></td>
<td>degradation rate 0.0406</td>
<td>0.0241</td>
</tr>
</tbody>
</table>

### Table 2. The lifetime of 1#-12# MEMS gyroscope

<table>
<thead>
<tr>
<th>Stress level</th>
<th>MEMS gyroscope 1#</th>
<th>2#</th>
<th>3#</th>
<th>4#</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_0 )</td>
<td>lifetime 3145.4</td>
<td>1574.4</td>
<td>484.8</td>
<td>1147.5</td>
<td>1588.0</td>
</tr>
<tr>
<td>( S_1 )</td>
<td>MEMS gyroscope 5#</td>
<td>6#</td>
<td>7#</td>
<td>8#</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>lifetime 449.0</td>
<td>1721.7</td>
<td>1087.3</td>
<td>746.8</td>
<td>1001.2</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>MEMS gyroscope 9#</td>
<td>10#</td>
<td>11#</td>
<td>12#</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>lifetime 578.7</td>
<td>946.2</td>
<td>280.1</td>
<td>429.1</td>
<td>558.5</td>
</tr>
</tbody>
</table>

![Fig. 5. Voltage output and Allan variance curves of 12 MEMS gyroscopes under different temperature stresses](image-url)
Fig. 6. Bias instability of 12 MEMS gyroscopes changes with time under different temperature stresses
So the acceleration factor model between the stress level \( S_0 \) and \( S_1 \) can be expressed as:

\[
AF_{S0}^{F02} = \frac{TF_0}{TF_2} = \exp\left(\frac{E}{E / kT_2} - \frac{E}{E / kT_0}\right) = \exp\left(\frac{E}{kT_0} - \frac{1}{T_2}\right)
\]

\[
= \exp\left(\frac{0.2234}{8.6171 \times 10^{-5}} \times \frac{1}{25 + 273} - \frac{1}{60 + 273}\right) = 2.4950
\]

The lifetime of MEMS gyroscope under normal stress level can be calculated based on the mean lifetime at stress level \( S_1 \) and equation (40):

\[
TF_{S0} = TF_{S1} \times AF_{S0}^{F02} = 558.5 \times 2.4950 = 1393.5
\]

So the acceleration factor model between the stress level \( S_0 \) and \( S_1 \) can be expressed as:

\[
AF_{S0}^{F01} = \frac{TF_0}{TF_1} = \exp\left(\frac{E}{E / kT_1}\right) = \exp\left(\frac{E}{kT_0} - \frac{1}{T_1}\right)
\]

\[
= \exp\left(\frac{0.1642}{8.6171 \times 10^{-5}} \times \frac{1}{25 + 273} - \frac{1}{45 + 273}\right) = 1.4950
\]

The lifetime of MEMS gyroscope under normal stress level can be calculated based on the mean lifetime at stress level \( S_1 \) and equation (40):

\[
TF_{S0} = TF_{S1} \times AF = 1001.2 \times 1.4950 = 1496.8
\]

5. Conclusions

This paper establishes the degradation model and acceleration factor model of MEMS gyroscope based on Allan variance and ADTs, and proposes a universal lifetime prediction method for MEMS gyroscope. The main conclusions are as follows:

- The degradation characteristic (bias instability) of MEMS gyroscope is extracted based on Allan variance, and the effect of temperature stress on the degradation rate of bias instability is analyzed. The theoretical analysis shows that the degradation rate of bias instability would increase with the increase of the temperature.

- ADTs of MEMS gyroscope are designed and conducted, and the experimental results indicate that bias instability of MEMS gyroscopes increase approximately linearly with time at different temperature stress levels, and the degradation rate of bias instability is larger in higher temperature stress.

- Based on the theoretical analysis and experimental results above, the acceleration factor model of MEMS gyroscope under temperature stress is established, and the lifetime of MEMS gyroscope is predicted. The presented methodology for lifetime prediction of MEMS gyroscopes could reduce the testing duration and expense prominently. It may also be used as a reference for lifetime prediction for other MEMS devices.

Acknowledgement

The authors would like to acknowledge the financial supports provided by the National Natural Science Foundation of China under Grant 51875570, the National Key Research and Development Program of China under Grant No. 2017YFC0806302, and the National Natural Science Foundation of China under Grant 51705526.

References


Yao LIU
Yashun WANG
Zhengwei FAN
Laboratory of Science and Technology on Integrated Logistics Support
College of Intelligence Science and Technology
National University of Defense Technology
Yanwachi str., Changsha, 410073, Hunan, China

Zhanqiang HOU
College of Intelligence Science and Technology
National University of Defense Technology
Yanwachi str., Changsha, 410073, Hunan, China

Shufeng ZHANG
Xun CHEN
Laboratory of Science and Technology on Integrated Logistics Support
College of Intelligence Science and Technology
National University of Defense Technology
Yanwachi str., Changsha, 410073, Hunan, China

E-mails: yaoliu133@126.com, wangyashun@nudt.edu.cn,
fanzhengwei15@nudt.edu.cn, houzhanqiang@nudt.edu.cn,
sfzhang@nudt.edu.cn, chenxun@nudt.edu.cn