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DYNAMIC RELIABILITY ANALYSIS OF A MULTI-STATE MANUFACTURING SYSTEM

ANALIZA DYNAMICZNEJ NIEZAWODNOŚCI WIELOSTANOWEGO SYSTEMU PRODUKCYJNEGO

Dynamic reliability analysis of binary systems has been widely studied in case of homogeneous continuous time Markov process assumption in the literature. In this study, we evaluate dynamic performance of a multi-state rotor line of electric motors manufacturing system under non-homogeneous continuous time Markov process (NHCTMP) degradation by using lifetime distributions of seven workstations within the system. By means of this degradation process assumption we capture the effect of age on the state change of components in the analysis by means of time dependent transition rates between states of the workstations. Essentially this is typical of many systems and more practical to use in real life applications. The working principle is based on a three state structure. If all the machines within each workstation work, the workstation is defined as working with the full performance. Whenever at least one machine fails within each workstation, then the workstation is defined as working with partial performance. If all the machines in the workstation fail then the workstation is defined as failed. The lifetime properties of the workstations under NHCTMP assumption have been studied for this three-state structure of the workstations. The workstations are all working independently and nonidentically from each other and they are connected in series within the system.We especially performed an extensive application study based on the lifetime data regarding the seven workstations within a manufacturing system. Dynamic reliability results are also discussed for the system structure. Some performance characteristics are developed for both workstations and the system as well. Numerical results for the performance characteristics of those workstations and the system are provided and supported with some graphical illustrations.

Keywords: *multi-state systems, non-homogeneous continuous time markov process, dynamic reliability measure, manufacturing system, transition rate.*

W literaturze przedmiotu, niezawodność dynamiczną układów binarnych analizuje się szeroko przy założeniu, że badane procesy stanowią jednorodne procesy Markowa z czasem ciągłym. W niniejszym artykule dokonano oceny dynamiki pracy wielostanowej linii do produkcji wirników będącej częścią systemu produkcji silników elektrycznych. Badania prowadzono przy założeniu, że degradacja stanowi niejednorodny proces Markowa z czasem ciągłym (NHCTMP). Do badań wykorzystano rozkłady cyklu życia siedmiu stanowisk wchodzących w skład systemu. Dzięki założeniu dotyczącemu procesu degradacji, udało się uchwycić wpływ wieku komponentów na zmianę ich stanu wykorzystując w analizie zależne od czasu szybkości przejścia między stanami badanych stanowisk. Ujęte w ten sposób zjawisko degradacji jest typowe dla wielu systemów, co oznacza, że proponowana metoda lepiej niż inne metody sprawdzi się w rzeczywistych zastosowaniach. W metodzie przyjmuje się, że stanowiska produkcyjne charakteryzuje struktura trójstanowa. Jeśli wszystkie maszyny na danym stanowisku działają prawidłowo, stanowisko określa się jako w pełni sprawne. Gdy co najmniej jedna maszyna na danym stanowisku ulegnie uszkodzeniu, stanowisko określa się jako częściowo sprawne. Jeśli wszystkie maszyny na danym stanowisku ulegną uszkodzeniu, stanowisko określa się jako niesprawne. Właściwości cyklu życia stanowisk produkcyjnych badano przy założeniu NHCTMP oraz trójstanowej struktury stanowisk. Wszystkie stanowiska w omawianym systemie działają niezależnie od siebie w sposób nieidentyczny i tworzą układ szeregowy. W pracy przeprowadzono obszerne badania aplikacyjne w oparciu o dane dotyczące cyklu życia siedmiu stanowisk wykorzystywanych w badanym systemie produkcyjnym. Omówiono także wyniki analizy niezawodności dynamicznej dla struktury systemu. Ponadto opracowano parametry pracy zarówno dla indywidualnych stanowisk jak i systemu jako całości. Wartości liczbowe tych parametrów zestawiono w tabelach oraz przedstawiono w formie graficznej.

Słowa kluczowe: *systemy wielostanowe, niejednorodny proces Markowa z czasem ciągłym, miara niezawodności dynamicznej, system produkcyjny, szybkość przejścia.*

1. Introduction

In reliability analysis of systems, multi-state systems are proposed instead of binary-state systems because they are more practical to use in real life situations. Especially multi-state models are useful in describing many engineering systems such as pipe-line systems, power generating systems or manufacturing systems, etc. Different from binary state systems, they have more than just two levels of working efficiency. A multi-state system and its components can have M (M>1) working states, from perfect functioning state $\varphi(0)=M$ to less efficient states $\varphi(t)\in\{M-1,M-2,\ldots,1,0\}$, where $\varphi(t)$ denotes the state of the system at an arbitrary time point t. To have the general idea behind the multi-state theory and the basic evaluation methods on the reliability of such systems, see the works of Huang et al.[8], Tian et al. [24] and Eryilmaz [4]. Also for a detailed theory of multi-state modelling we refer to Kuo and Zuo [10].

This study examines a manufacturing system containing serially-connected workstations within its structure. Although the literature includes a lot of studies dealing with the reliability analysis of manufacturing systems by using two-state system structures, there are very few reliability studies on multi-state manufacturing systems. The reliability studies on multi-state manufacturing systems usually involve illustrative examples. Niknam and Sawhney [19] proposed a model with reliability analysis in the performance measurement of multi-state manufacturing processes. Khatab et al. [9] carried out the reliability analyses of series and parallel multi-state systems through Kronecker algebra. Lisnianski [14] employed the extended block diagram method in the reliability analysis of a multi-state system. Using failure modes, Lia et al. [12] carried out reliability analysis on a multi-state optical sensor through a method combining modified binary decision diagram and multi-state multi-valued decision diagram models. Abou [1] proposed a reliability-based performance model in a multi-state flotation circuit of mineral processing plant using two failure modes. Qin et al. [20] proposed a method which was a combination of the Markov stochastic process and the universal generating function methods for the reliability analysis of multi-state systems and performed illustrative examples in a power station coal feeding system. Levitin ve Lisnianski [11] employed the universal generating function method in the importance and sensitivity analysis of a multi-state power station coal-feeding system.

We deal with the dynamic reliability analysis of a manufacturing system in this study. In the dynamic reliability analysis, of multi-state systems, a system or the components degradate into any lower state over time. For a detailed theory of dynamic reliability analysis we cite Lisnianski and Levitin [13]. One of the reliability measures used in dynamic reliability analysis of multi-state systems is the probability that the system is in some intermediate state j or above at time t. This definition can be expressed as $R_j(t) = P\{\varphi(t) \geq j\} = P\{T^{\geq j} > t\}$, $\forall j \in [1,\ldots,M]$, where $T^{\geq j}$ denotes the lifetime of the system at state *j* or above. Generally the degradation process of the multi-state systems or the components is assumed to have a homogeneous Markov degradation process[21, 2, 3, 17] or a non-homogeneous Markov degradation process [18, 23, 15, 16]. In this study, we assume a non-homogeneous Markov process for the degradation process of the workstations because according to this process the length of time a workstation stays in a certain state depends not only on the current state, but also on how long the workstation has been in the current state which is more realistic to reflect the performance degradation of multi-state systems. The transition rates to other states may change over the duration of the states, thus they are time dependent.

The dynamic performance evaluation of a multi-state system with seven workstations is considered. The reliability measure which is the lifetime of a multi-state workstation, $T^{\geq j}$, in the state subset {j,j+1,…,M} is considered to evaluate the workstation's performance at state j or above. Each workstation in the system work independently and nonidentically. Because of the difficulty in obtaining the performance evaluation of systems under nonidentical case and three-state systems have been the topic of various reliability papers because of its simplicity [5, 6, 7], we consider that the system or the workstations can be in three-states, perfect functioning ("2"), partial working ("1") and complete failure ("0") . There are different numbers of machines within each workstation. The state of a workstation is determined based on the number of working machines in the workstation at time *t*. If all the machines within each workstation work, the workstation is defined as working with the full performance. Whenever at least one machine fails within the workstation, then the workstation is defined to be partially working. If all the machines in the workstation fail, then the workstation is defined as failed. We also consider the system's performance based on those workstations' performances. The workstations are connected in series. Thus the reliabilities for the three-state system are obtained based on a series structure of the workstations. In summary, we study the dynamic reliability analysis of this three-state manufacturing system and the workstations in case of non-homogeneous continuous time Markov degradation process assumption.

2. Manufacturing structure

In this study, we handled the manufacturing process of a company producing electric motors. Different types of electric motors are produced in the company. The manufacturing process of an electric motor mainly takes place in four basic stages: rotor, stator, cap, and body. Within the scope of the study, we focused on the rotor manufacturing process. This process involves seven workstations including rotor packaging, rotor die casting, shaft removing, rotor shaft grouping, grinding, rotor turning, and rotor balancing. The flow chart of the manufacturing process is shown in Figure 1. Each workstation has a different number of machines. However, we were not interested in the reliability calculations of the machines in the workstations. Instead, we dealt with the work performance of the workstations.

The manufacturing process takes place on three shifts, and each shift involves a different number of machines in each workstation. These machines operate independently of each other. In other words, there is no input-output relationship between machines in a workstation. After an operation ends in a machine in a workstation, it is sent to the next workstation. One part undergoes one operation in one machine in a workstation.

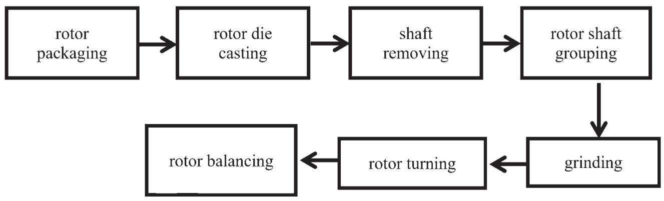

Fig. 1. Rotor manufacturing process

3. Data collection and data analysis

Within the scope of the study, unplanned failure data of machines working in each workstation on three shifts from 08.01.2018 to 26.06.2018 were analyzed. These data included failure mode and repair time data of each machine in the workstations. A machine is in operating state when it is not in an unplanned failure state. We consider that the system or a workstation can be in one of the following three-states: perfect functioning ("2"), partial working ("1"), and complete failure ("0"). The perfect functioning state refers to when all the machines in a workstation are working in a particular time period. For example, if all of the five machines are working on a shift in a workstation at a particular moment t, the performance of the workstation at moment t is expressed as perfect functioning state. This being the case, incessant working time of a workstation without any unplanned failure in any one of five machines is referred to as perfect functioning state time of the workstation. In the event that any machine in the workstation gets into unplanned failure state, the time period when four machines are working in the workstation at the same time is the repair time of the machine in unplanned failure state. This is the partial working state time of the workstation. If another machine gets into unplanned failure state at that moment, this means that 3 machines are working in the workstation at moment t. The performance in this time period is also called partial working. That is, the working of all machines makes the workstation in perfect functioning state, while failure of at least one of them (excepting when all of them fail) puts the workstation into partial working state. The performance

Table 1. Failure Data Collection Scheme

Machine failure datas for rotor die casting workstation					Performans state of rotor die casting workstation	
Date	**Start**	**Finish**	**Machine**	**Repair time (min)**	**perfect functioning (min)**	**partial working (min)**
08.01.2018		08.01.2018 13:16				
08.01.2018	08.01.2018 17:33	08.01.2018 17:38	M1	5	256	5
09.01.2018	09.01.2018 06:37	09.01.2018 06:40	M2	3	779	3

of the workstation is referred to as complete failure when all the machines in it get into unplanned failure state.

Accordingly, perfect functioning (“2”) and partial working (“1”) durations were calculated by using the unplanned failure data of all the machines in rotor line. As shown in Table 1, no unplanned failures occurred in the machines working in the rotor die casting workstation for 256 minutes from 13:16 on 08.01.2018 to 17:33 on 08.01.2018. Thus, the workstation worked in perfect functioning state for 256 minutes. One of the machines in the workstation got into unplanned failure state for 5 minutes from 17:33 on 08.01.2018 to 17:38 on 08.01.2018. Since the other machines were continuing to work then, the work place was in partial working state. Following the end of the repair time of the machine getting into unplanned failure state, the workstation worked in perfect functioning state for 770 minutes from 17:38 on 08.01.2018 to 06:37 on 09.01.2018. Likewise, a machine in the workstation got into unplanned failure state and was under repair for 3 minutes from 06:37 on 09.01.2018 to 06:40 on 09.01.2018, which put the workstation into partial working state.

3.1. Statistical data analysis of the workstations

After arranging the failure data by considering the number of failed machines during the manufacturing process, we then analyze the data. The statistical analysis of the data is important to determine the distribution of the lifetimes of the workstations at each state. First some descriptive statistics are given for each of the workstations in the manufacturing system in Table 2. WS2 is the one functioning mostly(N=442) in perfect state within the system with a mean value of 494 minutes. Coefficient of variation greater than 1, indicates a high variation for the workstation. For each workstation it is greater than 1. However, coefficient of variation is the highest in WS6 in state 2. This indicates a high variation for the lifetimes spent at state 2 for the WS6. When we consider the lifetimes spent at state 1, WS7 functions partially 471 times with a mean value of 5 minutes. However, the coefficient of variation is smaller than 1. The coefficient of variations of the lifetimes spent at state 1 for most of the workstations are lower than 1. This indicates a low variation among the lifetimes spent at this state. When we check the coefficient of skewness and the kurtosis, for

Table 2. Descriptive Statistics related to the workstations

	State 2							
	N	Minimum	Maximum	Mean	Standard Deviation	Coefficient of Variation	Coefficient of Skewness	Coefficient of Kurtosis
WS1	31	32	35778	7439,52	9629,45	1,2944	1,61696	2,0187
WS2	442	25	3988	493,939	669,303	1,3550	2,5083	6,9526
WS3	33	145	36160	7366,03	9919,3	1,3466	1,71520	1,83033
WS4	111	11	12954	2176,05	2633,25	1,2101	2,0964	4,6203
WS5	373	27	5845	610,131	868,974	1,4242	2,75203	9,2664
WS6	313	1	6029	693,594	1072,22	1,5459	2,4592	6,3013
WS7	397	24	3933	528,456	715,543	1,3540	2,3385	5,7032
	State 1							
	N	Minimum	Maximum	Mean	Standard Deviation	Coefficient of Variation	Coefficient of Skewness	Coefficient of Kurtosis
WS1	29	3	148	44,5586	41,0743	0,9218	1,2477	0,8099
WS2	358	10	154	33,4921	27,0047	0,8063	1,88175	3,54769
WS3	34	1	131	24,2515	28,8571	1,1899	2,0889	4,8672
WS4	72	4	47	9,1968	8,2952	0,9020	2,8262	8,4229
WS5	167	6	37	11,8204	6,9816	0,5906	1,7068	2,3492
WS6	301	3	33	6,6142	4,3835	0,6627	2,5054	8,1111
WS7	471	3	13	4,7042	1,6721	0,3554	1,6791	3,1936

WS 1: rotor packaging, WS 2: rotor die casting, WS 3: shaft removing, WS 4: rotor shaft grouping, WS 5: grinding, WS 6: rotor turning, WS 7: rotor balancing

Table 3. The Anderson Darling statistics for TTF data of Workstations related to State "1" and State "2"

	State 2		State 1	
Workstations	**Fitted Distribution**	**Anderson Darling (p-value, p-value[a])**	**Fitted Distribution**	**Anderson Darling (p-value, LRT p-value)**
WS1	**Weibull**	0.439(>0.250)	Weibull	0.265 (>0.250)
WS2	**3-parameter Weibull**	0.594 (0.129)	3-parameter Weibull	0.995(0.014)
WS3	**Weibull**	0.765 (0.043)	Weibull	0.575 (0.139)
WS4	**Weibull**	0.905 (0.021)	3-parameter Weibull	0.759 (0.050)
WS5	**3-parameter Weibull**	0.702(0.073)	3-parameter Weibull	0.846 (0.032)
WS6	**Weibull**	0.877 (0.024)	3-parameter Weibull	0.827(0.036)
WS7	**3-parameter Weibull**	0.933 (0.020)	3-parameter Weibull	0.446(0.301)

WS 1: rotor packaging, WS 2: rotor die casting, WS 3: shaft removing, WS 4: rotor shaft grouping, WS 5: grinding, WS 6: rotor turning, WS 7: rotor balancing

all the workstations we do not observe any symmetry. We then try to determine the lifetime distributions at each state for the workstations.

We obtained the Anderson Darling statistics for the lifetimes spent at each state for each workstation, and they are given in Table 3. The values in Table 3 are obtained by using Minitab 15. The smaller values for Anderson Darling statistics with a high significance value($p>0.01$) show a best fit for the related distributions for the lifetimes. Mostly the lifetime data for each state fits with Weibull and 3-parameter

Table 4. Best-fit Distribution Analysis for TTF for state "2" and state "1"

	State "2"	State "1"
Work Stations	**ML Estimates of Distribution Parameters**	**ML Estimates of Distribution Parameters**
WS1	$\hat{\beta}$: 0.61133 $\hat{\alpha}$: 5271.99858	$\hat{\beta}$: 1.09476 $\hat{\alpha}$: 46.15090
WS2	$\hat{\beta}$: 0.71141 $\hat{\alpha}$: 373.45120 $\hat{\lambda}$: 24.91098	$\hat{\beta}$: 0.87339 $\hat{\alpha}$: 21.80811 $\hat{\lambda}$: 10.08887
WS3	$\hat{\beta}$: 0.75265 $\hat{\alpha}$: 6110.05142	$\hat{\beta}$: 0.91626 $\hat{\alpha}$: 23.16936
WS4	$\hat{\beta}$: 0.76699 $\hat{\alpha}$: 1878.22315	$\hat{\beta}$: 0.71561 $\hat{\alpha}$: 4.05998 $\hat{\lambda}$: 4.00950
WS5	$\hat{\beta}$: 0.67251 $\hat{\alpha}$: 439.39004 $\hat{\lambda}$: 26.91321	$\hat{\beta}$: 0.82224 $\hat{\alpha}$: 5.27115 $\hat{\lambda}$: 5.94
WS6	$\hat{\beta}$: 0.61696 $\hat{\alpha}$: 475.10034	$\hat{\beta}$: 0.89558 $\hat{\alpha}$: 3.44008 $\hat{\lambda}$: 2.97
WS7	$\hat{\beta}$: 0.68217 $\hat{\alpha}$: 387.57255 $\hat{\lambda}$: 23.922	$\hat{\beta}$: 1.03632 $\hat{\alpha}$: 1.75971 $\hat{\lambda}$: 2.97

WS 1: rotor packaging, WS 2: rotor die casting, WS 3: shaft removing, WS 4: rotor shaft grouping, WS 5: grinding, WS 6: rotor turning, WS 7: rotor balancing, β; shape parameter, α; scale parameter, λ; threshold

Weibull distributions. The LRT-p values obtained for the 3-parameter Weibull distributions are 0.000. This supports the significance of the third parameter(threshold) in the distribution.

Then the parameters of the Weibull distributions are estimated by maximum likelihood estimation (MLE) method. The scale, location and the threshold parameter estimators are given in Table 4 for each workstation.

4. Markov degradation process

A discrete state continuous time stochastic process $\varphi(t) \in \{0,1,2,\ldots,M\}$ is called Markov chain if, for $t_1 < t_2 < \ldots < t_n$, its conditional mass function satisfies the relation:

$$P\{\varphi(t_n) = k_n | \varphi(t_{n-1}) = k_{n-1},\ldots,\varphi(t_2) = k_2, \varphi(t_1) = k_1\} \quad (1)$$
$$= P\{\varphi(t_n) = k_n | \varphi(t_{n-1}) = k_{n-1}\}$$

Let $t_{n-1} = t$ and $t_n = t + \Delta t$, then (1) becomes:

$$P\{\varphi(t+\Delta t) = j | \varphi(t) = i\} = P_{i,j}(t,\Delta t), \quad (2)$$

for $i, j \in \{0,1,2,\ldots,M\}$. (2) shows the transition probabilities of non-homogeneous continous time Markov process. They satisfy the following properties:

$$P_{i,j}(t,\Delta t) \geq 0 \text{ and } \sum_{j=0}^{M} P_{i,j}(t,\Delta t) = 1$$

for $t, \Delta t > 0$. The transition degradation rate from state i to state j is obtained by:

$$\lambda_{i,j}(t) = \lim_{\Delta t \to 0} \frac{P_{i,j}(t,\Delta t)}{\Delta t} \quad (3)$$

and transition rate from state j to state j is:

$$\lambda_{j,j}(t) = -\sum_{k=0,k\neq j}^{M} \lambda_{j,k}(t), \quad (4)$$

By using the Chapman-Kolmogorov equation given in [21] and the total probability formula, $P_{i,j}(t+\Delta t)$ can be written as follows:

$$P_{i,j}(t+\Delta t) = \sum_{k=0}^{M} P_{i,k}(t) P_{k,j}(t,\Delta t) \quad (5)$$

By the use of equation (5) and some intermediate steps, one can obtain the state equations of a machine having non-homogenous continuous time Markov degradation process as the following:

$$P_j^{'}(t) = -P_j(t) \sum_{k=0,k\neq j}^{M} \lambda_{j,k}(t) + \sum_{k=0,k\neq j}^{M} P_k(t).\lambda_{k,j}(t), \quad (6)$$

where $P_M(0) = 1$ and $P_k(0) = 0$ for $k \neq M$ and $P_j(t)$ is the probability that the machine tool is in state j at time t .Thus by solving equation (6) the state probabilities of a machine tool can easily be obtained. This method is practically used to find state distribution of machine tools [17]. However, it becomes difficult to use whennumber of states becomes large. In this case, in order to find the state distribution of machines Sheu and Zhang [22] proposed a recursive approach which is more efficient to use in the cases where the number of states is large. According to this method the probability of a machine tool to be at state j at time t is obtained by:

$$P_j(t) = P\{\varphi(t) = j\} = \sum_{k=j+1}^{M} \int_0^t P_k(\tau_{M+1-k}) \lambda_{k,j}(\tau_{M+1-k}) \times \exp\left[-\int_{\tau_{M+1-k}}^{t} \sum_{l=0}^{j-1} \lambda_{j,l}(s) ds\right] d\tau_{M+1-k} \quad (7)$$

where $j = M-1, M-2, \ldots 1, 0.$ Also the other properties $P_M(0) = 1$ and $P_k(0) = 0$ for $k = M-1, M-2, \ldots, 1, 0$ are also satisfied. We get benefit of this method and use equation (7) especially in finding the state distributions of each workstation whose degradation process follows NHCTMP.

5. Performance evaluation of a multi-state manufacturing system with independent and nonidentically distributed multi-state workstations

Each of the workstations is considered to be a multi-state component of a three-state system. The system also is assumed to have independent and non-identical three-state components. In this study NHCTMP is used to describe the age-dependent performance degradation process for the components. In the evaluation of the state probabilities of the components, minor degradation in which each element degrades to the nearest state from its current state is considered in this study for the simplicity of calculations. Let T_{j_1,j_2} be the lifetime of a multi-state component spent at state j_1 before proceeding to the next state j_2 . Because the components' degradation process follows a NHCTMP, we determined based on the data that the components' lifetimes spent at each state, T_{j_1,j_2} has a Weibull distribution types with a scale parameter α_i and a shape parameter β_i i.e. $T_{j_1,j_2} \sim Weibull(\alpha_i, \beta_i)$ or with a threshold parameter λ_i, i.e. $T_{j_1,j_2} \sim Weibull(\alpha_i, \beta_i, \lambda_i)$ and the components' probability density functions are given respectively by:

$$f_{j_1,j_2}(t) = \frac{\beta_i}{(\alpha_i)^{\beta_i}} t^{\beta_i - 1} e^{-\left(\frac{t}{\alpha_i}\right)^{\beta_i}}, \quad (8)$$

$$f_{j_1,j_2}(t) = \frac{\beta_i}{(\alpha_i)^{\beta_i}} (t-\lambda_i)^{\beta_i - 1} e^{-\left(\frac{t-\lambda_i}{\alpha_i}\right)^{\beta_i}} \quad (9)$$

Thus the transient degradation rates for the lifetimes of components following Weibull (α_i, β_i) and $Weibull(\alpha_i, \beta_i, \lambda_i)$. are given respectively by:

$$\lambda_{j_1,j_2}^{(i)}(t) = \frac{\beta_i t^{\beta_i - 1}}{(\alpha_i)^{\beta_i}}, \quad (10)$$

$$\lambda_{j_1,j_2}^{(i)}(t) = \frac{\beta_i (t-\lambda_i)^{\beta_i - 1}}{(\alpha_i)^{\beta_i}} \quad (11)$$

where $j \in \{1,2\}, 0 \le j \le j-1$, $i = 1,2,...,7$. Then for different values of α_i, β_i and λ_i, the instantaneous degradation rates of components are given in Table 5.

Table 5. Instantaneous degradation rates of workstations

i	$\lambda_{2,1}^{(i)}(t)$	$\lambda_{1,0}^{(i)}(t)$
1	$0.003273t^{-0.39}$	$0.016729t^{0.09}$
2	$0.010601(t-24.91)^{-0.29}$	$0.059103(t-10.09)^{-0.13}$
3	$0.001085t^{-0.25}$	$0.051057t^{-0.08}$
4	$0.002321t^{-0.23}$	$0.262539(t-4.01)^{-0.28}$
5	$0.011366(t-26.91)^{-0.33}$	$0.20986(t-5.94)^{-0.18}$
6	$0.013578t^{-0.38}$	$0.296033(t-2.97)^{-0.1}$
7	$0.011806(t-0.24)^{-0.32}$	$0.577697(t-2.97)^{0.04}$

WS 1: rotor packaging, WS 2: rotor die casting, WS 3: shaft removing, WS 4: rotor shaft grouping, WS 5: grinding, WS 6: rotor turning, WS 7: rotor balancing

Under the assumptions related to the components of the system mentioned above we first obtain the state probabilities of each component (workstation) under NHCTMP by the equation (7) proposed by Shue and Zhang [22]. Then the probability of the i th workstation being at state 2 (working with its full performance) and at state 1 (working with partial performance), are obtained respectively by:

$$P_2^{(i)}(t) = P\{\varphi(t)=2\} = \exp\left[-\int_0^t \lambda_{2,1}^{(i)}(s)\,ds\right] \quad (12)$$

$$P_1^{(i)}(t) = P\{\varphi(t)=1\} = \int_0^t \exp\left[-\int_0^{\tau_1} \lambda_{2,1}^{(i)}(s)\,ds\right]\lambda_{2,1}^{(i)}(\tau_1)\exp\left[-\int_{\tau_1}^t \lambda_{1,0}^{(i)}(s)\,ds\right]d\tau_1 \quad (13)$$

In the calculation of the related state probabilities for especially the components having lifetimes distributed with 3-parameter Weibull distribution, the limits of the related integrals in equation (12)-(13), is determined based on the estimated threshold parameter of the related distribution. In order to find the reliability of the system first we need to calculate the survival probabilities of the components, the probability of a workstation being at state j or above at time t, $P\{T_i^{\ge j} > t\} = P\{\varphi(t) \ge j\}, i = 1,2,\ldots,7, j = 0,1,2$. The related results are given in Table 6. According to the results:

The data in Table 6 show that the workstation with the highest reliability in the rotor line is shaft removing, whereas the two workstations with lower reliability are the rotor die casting and rotor balancing workstations.

The probability of working with full performance of the shaft removing workstation, which has the highest reliability, in 500 minutes, which corresponds to almost one shift, is 0.86, while that of working with partial performance is 0.01 (0.865-0.858). This workstation's probability of complete failure in 500 minutes is 0.14 (1-0.858). In 1500 minutes making up one-day working period (3 shifts), this workstation's probability of working with full performance is 0.706; probability of working with partial performance is 0.004; and probability of complete failure is 0.29. Also, as shown in Table 2, the shaft removing workstation's average life expectancy with full performance is 7366 minutes, whereas its average life expectancy with partial performance is 24.25 minutes.

Among the workstations with lower reliability, the rotor die casting workstation's probability of working with full performance in 500 minutes is 0.305, whereas that of the rotor balancing workstation is 0.317. In 500 minutes, the rotor die casting workstation's probability of working with partial performance is 0.03, while that of the rotor balancing workstation is 0.006. In the same time period, the rotor die casting workstation's probability of complete failure is 0.54, and that of the rotor balancing workstation is 0.666. In 1500 minutes corresponding to an almost one day working period, the rotor die casting workstation's and the rotor balancing workstation's probabilities of working with full performance are 0.07 and 0.084, respectively. In the same time period, the rotor die casting workstation's probability of partial performance is 0.006, whereas that of the rotor balancing workstation is 0.001. Their probabilities of complete failure are 0.924 and 0.915, respectively, in the same time period. As shown in Table 2, the rotor die casting workstation's average life expectancies with full performance and partial performance are 493 minutes and 33.49 minutes, respectively. Likewise, the rotor balancing workstation's av-

Table 6. The survival probabilities for the workstations for each state depending on time

	50 (min.)		100(min)		150(min)		350(min)		500(min)		1000(min)		1500(min)		3500(min.)	
	$P(T^{\ge 2} > t)$	$P(T^{\ge 1} > t)$	$P(T^{\ge 2} > t)$	$P(T^{\ge 1} > t)$	$P(T^{\ge 2} > t)$	$P(T^{\ge 1} > t)$	$P(T^{\ge 2} > t)$	$P(T^{\ge 1} > t)$	$P(T^{\ge 2} > t)$	$P(T^{\ge 1} > t)$	$P(T^{\ge 2} > t)$	$P(T^{\ge 1} > t)$	$P(T^{\ge 2} > t)$	$P(T^{\ge 1} > t)$	$P(T^{\ge 2} > t)$	$P(T^{\ge 1} > t)$
WS1	0.943	0.973	0.915	0.939	0.892	0.911	0.826	0.836	0.788	0.797	0.696	0.701	0.628	0.632	0.459	0.461
WS2	0.863	0.976	0.726	0.831	0.631	0.714	0.404	0.445	0.305	0.334	0.138	0.149	0.07	0.076	0.0076	0.0081
WS3	0.973	0.984	0.955	0.965	0.94	0.949	0.89	0.897	0.858	0.865	0.773	0.778	0.706	0.710	0.518	0.521
WS4	0.941	0.955	0.901	0.914	0.867	0.88	0.76	0.771	0.697	0.707	0.54	0.548	0.431	0.438	0.199	0.202
WS5	0.87	0.939	0.74	0.781	0.653	0.683	0.443	0.459	0.349	0.361	0.182	0.187	0.105	0.108	0.018	0.019
WS6	0.781	0.803	0.683	0.699	0.613	0.625	0.437	0.444	0.356	0.361	0.205	0.207	0.13	0.131	0.032	0.032
WS7	0.853	0.892	0.719	0.743	0.628	0.646	0.411	0.42	0.317	0.323	0.154	0.156	0.084	0.085	0.01178	0.01183

WS 1: rotor packaging, WS 2: rotor die casting, WS 3: shaft removing, WS 4: rotor shaft grouping, WS 5: grinding, WS 6: rotor turning,WS7: rotor balancing

erage life expectancies with full performance and partial performance are 528.45 minutes and 4.70 minutes, respectively.

We also present the state probability graphs for one of the workstations with the best performance; WS 3-shaft removing in Figure 2. When we examine (a), the probability of WS3 being at state 2 based on time t, the related probability starts from 1, because at the beginning the component is at its perfect state, it decreases as the time passes and converges to zero in the limit case. When we examine (b), the probability of WS3 being at state 1 based on time t, it starts from zero, because at the beginning the component has zero probability of working partially, goes up to a point and then decreases as time passes. When we examine (c), the probability of WS3 being at state 0 based on time t, it starts from zero and increase by the time, converges to the probability 1 in the limit case. Similar conclusions can also be made for the other workstations.

Then we also obtained the survival probabilities for the manufacturing system with the seven workstations connected in series within the system. Due to the series structure of the system the probabilities of the system being at state j or above depending on time is calculated by:

$$P\left\{T_{system}^{\geq j} > t\right\} = \prod_{i=1}^{7} P\{T_i^{\geq j} > t\} \quad (14)$$

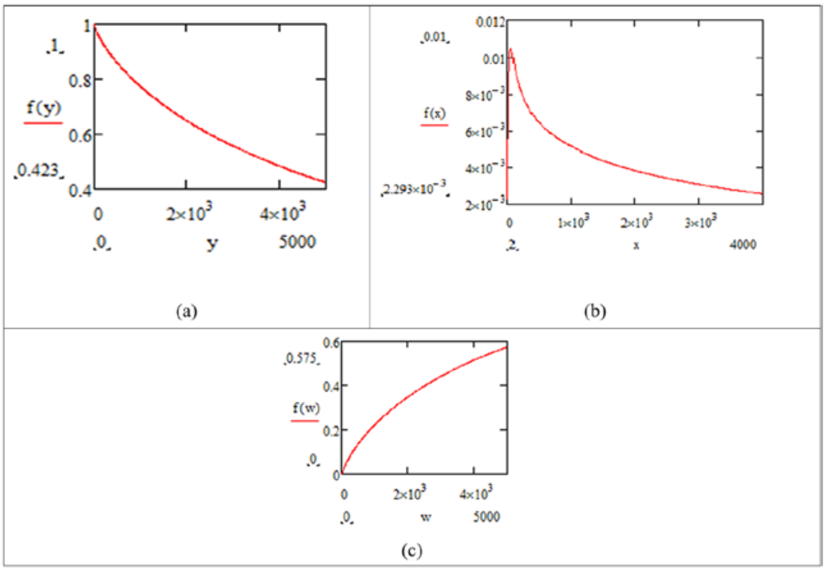

Fig. 2. State probabilities related to the WS 3

The results are given with Figure 3. The probabilities of the system being at state 2 and at state 1 or above are represented by the solid and the dashed lines, respectively. The probability of the system working with its full performance within 50 minutes is 0.432. However the probability of the system working above a partial performance within 50 minutes is 0.600. In addition, we can say that the system showed a partial performance of 17% in the first 50 minutes, 7% in the first 100 minutes, and 4% in the first 150 minutes.

Mean residual lifetime (MRL) is another performance characteristic that is used to evaluate the stochastic behaviour of a system or its components over time. The MRL function of the ith component of the system can be calculated by the following equation:

$$m_i^{\geq j}(t) = E\left\{T_i^{\geq j} - t \middle| T_i^{\geq j} \geq t\right\} = \int_0^{\infty} P(T_i^{\geq j} \geq t + x | T_i^{\geq j} \geq t)dx = \int_0^{\infty} \frac{P(T_i^{\geq j} \geq t + x)}{P\left(T_i^{\geq j} \geq t\right)}dx \quad (15)$$

MRL results are obtained by the use of equation (15) and given in Table 7. When we consider the MRL of each workstation, we observe that for each workstation MRLs are increasing depending on

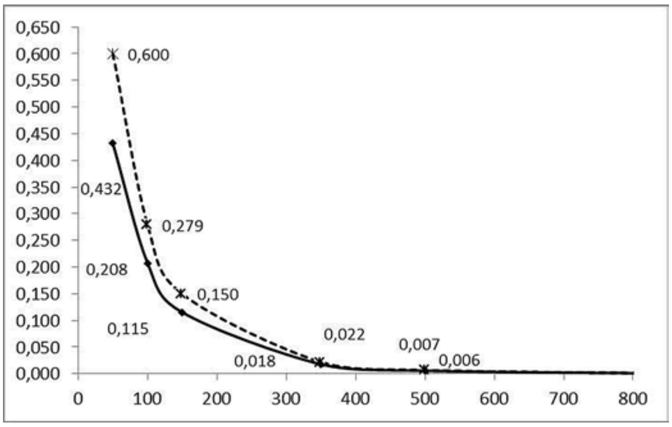

Fig. 3. Probabilities that the rotor line of electric motors manufacturing system is at state 2 and at state 1 or above at time t

time. This result is due to the fact that the machines found in each workstation are repairable. We can also observe this from the shape parameters, β_i, which are smaller than 1 for each state distributions estimated based on the data. A corrective and preventive maintenance plans are both considered within the manufacturing plant. Whenever a machine is failed, the operators fix the machine as soon as possible

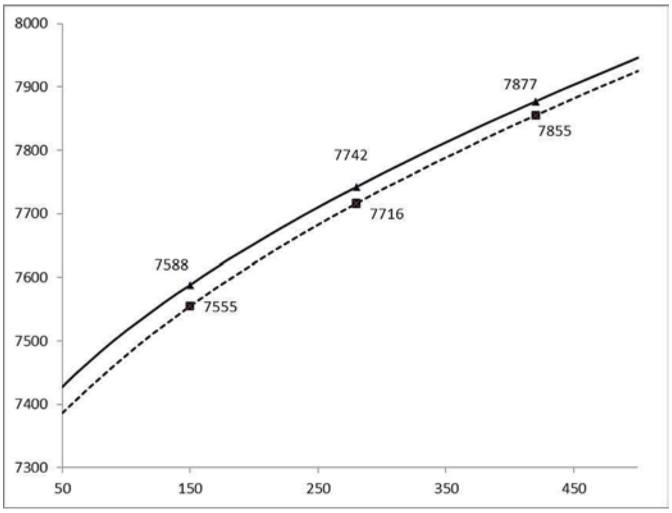

Fig. 4. MRLs of WS3(shaft removing) at state 2 and at state 1 or above at time t

Table 7. MRL for the workstations for each state

	50 (min.)		150(min)		500(min)	
	$m_i^{\geq 2}(t)$	$m_i^{\geq 1}(t)$	$m_i^{\geq 2}(t)$	$m_i^{\geq 1}(t)$	$m_i^{\geq 2}(t)$	$m_i^{\geq 1}(t)$
WS1	8181	7961	8546	8396	9300	9232
WS2	489	475	561	541	684	675
WS3	7427	7386	7588	7555	7946	7925
WS4	2277	2272	2366	2363	2554	2550
WS5	617	591	716	705	885	879
WS6	823	811	936	929	1156	1152
WS7	542	528	593	584	777	773

WS 1: rotor packaging, WS 2: rotor die casting, WS 3: shaft removing, WS 4: rotor shaft grouping, WS 5: grinding, WS 6: rotor turning, WS 7: rotor balancing

and makes it work with its perfect performance again. The MRLs at state 2 are greater than the MRLs at state 1 or above. This is also an obvious result comes with the increasing MRLs over time. Also because WS3 has a great performance among the other workstations its MRLs are greater than the others. The related MRLs of WS 3 regarding each state are also given in Figure 4. The solid and the dashed lines in Figure 4 represent the mean residual lifetime of WS3 at state 2 and at state 1 or above, respectively.

6. Conclusion

The studies regarding the reliability analysis of multi-state structures are important, because in practical applications we encounter many engineering systems defined with a multi-state structure. However, there is not enough research about a multi-state manufacturing system and its reliability evaluation methods. Thus, in this paper we have studied the dynamic reliability analysis of a manufacturing system with seven workstations. The system and the workstations are considered to be multi-state having three performance states such as full performance, partial performance and failure. Defining the workstations' performances as multi-state based on the number of working machines during the manufacturing process makes this study original and a challenging problem. That can also attract interests of the others dealing with the performance improvement studies of the manufacturing processes in the field of quality and reliability engineering. Also one of the stochastic processes, Markov process, is used to solve the reliability problem of such a system. Especially the degradation process of the workstations is explained by a nonhomogeneous continuous time Markov process because this is more appropriate to use in order to reflect the age effect on the state change of the workstations. The dynamic performances of the workstations and the system are obtained and discussed. One of the striking results found within this study besides the performance evaluations of the workstations is that depending on the data and also the estimated lifetime distributions, we have observed increasing MRLs for all the workstations for each state.This result also showed the maintenance activities are perfectly performed by the manufacturing plant.

Reliability analyses provide guidance for the improvement of production lines. The analysis results both determine the bottlenecks in production lines and provide decision support in production planning and maintenance activities in the lines. The rotor die casting and rotor balancing workstations in the rotor manufacturing line are the workstations that create bottlenecks. These workstations affect the performance of the entire line since the line is serially connected. For this reason, failures should be reduced by making failure mode effects analyses in these workstations. Preventive maintenance works are also important in these workstations. Moreover, the rotor line's perfect functioning and partial working performances in 500 minutes and 1500 minutes should be used in the production planning activities of this line.

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