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POISSON-DISTRIBUTED FAILURES IN THE PREDICTING OF THE COST OF CORRECTIVE MAINTENANCE

POISSONOWSKIE STRUMIENIE USZKODZEŃ W PROGNOZOWANIU KOSZTÓW OBSŁUG KOREKCYJNYCH FLOTY POJAZDÓW*

Maintaining high efficiency of using the fleet of public mass transport vehicles puts many challenges ahead of the operator. Among them, when planning periodic operational activities, the operator should take into account the assessment of possible unexpected vehicle failures and the costs of their removal under the so-called corrective maintenance. Due to the random nature of vehicle breakdowns, knowledge about stochastic processes is necessary to maintain their efficient and safe operation. The research problem formulated in the title meets these needs. Therefore, the costs of corrective maintenance of vehicles are modelled, i.e. the costs that are not included in the scheduled maintenance costs and are not related to preventive maintenance. The costs of corrective maintenance and the costs of replacement of damaged parts are unexpectedly created at random moments of operating means of transport, usually between scheduled maintenance. Due to the variety of failure processes of individual parts of the vehicle, the methods and applications of stochastic modelling for simple failures modelled by the Poisson process are presented in this paper. The basis for the application of the presented methods is to identify those parts of the vehicle that are damaged in accordance with this process. The assessment of parameters of failure processes of individual vehicle parts is made on the basis of the operational database of a homogeneous fleet of vehicles operated for 5 years. The operational database is dynamically updated with new events. On the basis of actual data on corrective maintenance of a distinguished group of damaged parts of vehicles, the possibilities and limitations of practical applications of the Poisson process to assess the risk of incurring costs in the further process of fleet operation were shown.

Keywords: *Poisson-distributed failures, corrective maintenance costs, vehicle fleet.*

Utrzymanie wysokiej efektywności użytkowania floty pojazdów publicznego transportu masowego stawia przed operatorem wiele wyzwań. Wśród nich planując okresowe działania eksploatacyjne operator powinien uwzględniać oceny możliwości pojawiania się niespodziewanych awarii pojazdów oraz kosztów ich usuwania w ramach tak zwanych obsług korekcyjnych. Ze względu na losowy charakter awarii pojazdów dla utrzymania efektywnej i bezpiecznej ich eksploatacji niezbędna okazuje się wiedza dotycząca procesów stochastycznych. Sformułowany w tytule problem badawczy wychodzi naprzeciw tym potrzebom. Modelowane są więc koszty obsług korekcyjnych pojazdów, tj. koszty, które nie są uwzględniane w ich przeglądach planowych i nie obciążają prewencyjnych napraw. Koszty robót korekcyjnych i koszty wymiany uszkodzonych części powstają niespodziewanie w losowych chwilach użytkowania środków transportu, zwykle między okresowymi przeglądami. W niniejszej pracy, ze względu na różnorodność procesów uszkodzania poszczególnych części pojazdu, przedstawiono metody i zastosowania stochastycznego modelowania dla prostych strumieni uszkodzeń, których modelem jest proces Poissona. Podstawą zastosowania przedstawionych metod jest zidentyfikowanie tych części pojazdu, które uszkodzają się zgodnie z tym procesem. Oceny parametrów procesów uszkodzeń poszczególnych części pojazdów dokonywane są na podstawie bazy danych eksploatacyjnych jednorodnej floty pojazdów eksploatowanych przez 5 lat. Wraz z upływem czasu baza danych eksploatacyjnych jest dynamicznie uzupełniana o nowe zdarzenia. Na podstawie danych rzeczywistych o obsłudze korekcyjnych wyróżnionej grupy uszkodzających się części pojazdów pokazano możliwości i ograniczenia praktycznych zastosowań procesu Poissona do oceny ryzyka poniesienia kosztów w dalszym procesie eksploatacji floty.

Słowa kluczowe: *poissonowski strumień uszkodzeń, koszty obsług korekcyjnych, flota pojazdów.*

1. Introduction

The design and construction of complex technical objects that meet high requirements in terms of energy consumption, cost-consumption, ecology, safety, availability and functionality require obtaining extensive, but also detailed knowledge concerning, among others, forecasting the frequency of failures occurring during their use [2, 5, 18, 21, 22, 26]. Due to the possible consequences, designers, manufacturers and end users of equipment and technical objects try to minimise the

possibility of various types of failures appearing during their operation. In order to minimise the costs of removing failures and ensuring the safety of using a technical object, the designer needs to know why and which adverse events may occur during the operation phase of the object. This knowledge is necessary for optimisation of the life cycle cost (LCC) of the object and is made available by the constant flow of information from the various phases of the existence of a technical object understood in the Agile Systems categories [14, 21, 27]. As a result, it reduces the vulnerability to such events and makes the

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object immune to their consequences by designing redundant systems and organising preventive actions that reduce unit operating costs [18, 24]. Despite the fact that all the requirements regarding i.a. operational availability of the technical object are taken into consideration during the designing phase, random failures are unavoidable during its operation. These failures are removed as part of unscheduled corrective maintenance (CM), which reduces availability and generates additional, unplanned operating costs.

The problem of failures is particularly important for the operator planning long-term use of the fleet of urban transport vehicles [17], [25]. For such an operator, the main cost factor in the life cycle costs, apart from the cost of purchase, are costs incurred in the vehicle operation phase [3, 4, 21]. A significant share in these costs is the cost of corrective maintenance, i.e. unplanned maintenance costs that must be incurred in order to restore the operational availability of the vehicle after incidental failures or accidents. Corrective maintenance must be carried out immediately, as the loss of operational availability of each vehicle affects the rostering of the vehicle fleet and generates costs of replacing damaged vehicles. These arguments justify the need to conduct reliability tests regarding the process of losing availability of fleet vehicles and the costs of their corrective maintenance. The present work is devoted to the development of research on these needs.

Reliability models that take into account the occurrence of random failures are usually based on stochastic processes. In particular, in the predictive problems of system reliability analysis, they constitute strong tools [1, 10, 11, 16, 19]. The classic problem is still valid: how system reliability can be improved using mathematical modelling. In this area, many interesting articles and books have already been written [7, 12, 13, 19, 20, 26]. Applications of mathematics, in particular probability theory and statistics, for problems of operation and maintenance of technical objects cover such issues as: planning of active and passive experiments, inventory management, analysis of data about failures [6, 8], preventive maintenance policy optimisation, maintenance costs analysis, management of operation and maintenance processes. In the paper [20] as part of the Performance Based Logistics (PBL) approach, the authors proposed a new stochastic model to determine the annual intensity of repairs of critical aircraft components. This model can be used to plan the base inventory level and the capacity of maintenance plants.

In [13] the authors formulated the conditions under which preventive replacements increase the availability of machine operation systems and the income per unit of their operation time. The aircraft maintenance process was analysed in [23] in accordance with the lean method. Optimisation of logistics processes is considered the most promising way to reduce maintenance time and cost of spare parts.

For complex technical objects such as vehicles it is not possible to use only one probability distribution for times between failures that cause the object to lose its operational availability. The reason for this is that the failure of complex objects is caused by hundreds of various components, which are characterised by various failure mechanisms. The estimation of parameters of vehicle failure processes caused by failures of various parts was made by the authors of this paper based on a long-term operational database for a certain fleet of urban transport vehicles. The initial reliability analysis of historical failure processes allowed to distinguish several types of them. The stochastic model of the simplest of them is the Poisson failure process. In this study, the authors present the results of research on the applicability of this stochastic process to modelling the processes of losing operational availability of vehicles and to assessment of the risk of incurring the corrective maintenance costs. The research concerns the most frequently damaged parts of those that generate high costs of corrective maintenance. Poisson processes as one of the simplest ones are a good basis for developing further research on the prediction of the costs of corrective maintenance of other failure process types. It is worth

noting that the operational database was used in [3] by the authors to study the significance of the differences in the average costs of corrective maintenance in the distinguished periods of fleet use.

This study consists of six parts. Section 2 presents the research problem and the research method. The Poisson method of modelling the vehicle corrective maintenance process together with the theoretical properties of this process was developed in section 3. Section 4 is devoted to the use of this method in the reliability and cost analysis for the event recorder system. Section 5 deals with the methods of assessing the risk of incurring corrective maintenance costs for a distinguished group of parts in the further period of vehicle operation. In section 6 a case study was carried out for an identified group of parts resulting in the loss of operational availability of vehicles. The entire study ends with a summary and conclusions.

2. Research problem and research method

The subject of the research is the cost of corrective maintenance of urban rail vehicles used for passenger transport. Vehicles of the tested fleet are homogeneous and the conditions of their operation are comparable. The vehicles covered by the research are repairable technical objects with high complexity and long life cycle. Maintaining a high level of vehicle safety and availability is provided by scheduled technical inspections during which preventive and predictive maintenance is carried out [22].

Despite compliance with scheduled maintenance, vehicles during operation at random times are subject to an unexpected failures, as a result of which they usually lose their ability to perform transport functions. Vehicle failures are caused by failures of certain parts of the vehicle. During 5 years, the vehicles of the tested fleet have been damaged several thousand times due to failures of several hundred different parts. In order to restore the operational availability of the vehicle, damaged parts are replaced by new ones as part of the corrective maintenance. Thus, the costs of corrective maintenance are modelled, i.e. costs that are not included in scheduled and preventive maintenance costs.

Corrective maintenance can be classified according to different criteria. For the operator of a fleet of urban transport vehicles, the cost criterion is important, determining who bears the costs of repairing damaged parts of the vehicle, i.e. the operator, a supplier, a part manufacturer, a guilty party, an insurer. The research is based on the fleet of a new type of vehicles covered by a warranty agreement between the operator and the supplier, which includes also further post-warranty maintenance. The problem formulated in the title of this paper consists in developing a method for assessing the risk of costs incurred by the supplier during selected corrective maintenance procedures. The stochastic Poisson processes [1, 12, 19], taking into account a repair costs table, were used to solve this research problem.

The basis for the applied stochastic modelling of cost processes are the theorems and properties of a homogeneous Poisson process. The definitions and variable naming convention used were taken from the papers [1, 6, 10, 12, 19, 22]. An interesting example of using the Poisson process to model the number of accidents in the Baltic Sea, in order to increase the safety of navigation is presented in the paper [11].

3. The Poisson model of the vehicle corrective maintenance process

The number of failures to a vehicle during a specified period of its operation or in a specified range of its mileage changes randomly for subsequent periods of operation or subsequent intervals of the vehicle's mileage. The theory of stochastic processes enables modelling a random evolution of the number of failures and corrective maintenance costs in time or in mileage of a vehicle. The Poisson process

and its extension [1, 12, 19] play a key role in building the maintenance counting models. These processes enable the construction of models of vehicle failure counting for both specific types of failures and for the assessment of the risk of incurring the costs of restoring the operational availability by the supplier.

If the conditional probability of a vehicle failure due to failure of its i -th part is constant for mileage from the interval $(l, l+\Delta l]$, regardless of the mileage of the vehicle with this part, provided that until the mileage of l this part was in working order, the random process of loss of operational availability of the vehicle caused by subsequent failures of its i -th part is a homogeneous Poisson process. Let G means the set of those parts of the vehicle that are being damaged in accordance with the described Poisson process. We assume that after the vehicle failure the damaged parts are replaced by new ones during corrective maintenance. In addition, we assume that the total cost c_i of materials and labour of the maintenance related to the replacement of the i -th part is specified in the supplier's cost table.

As a measure of the risk of incurring the cost of repairing the vehicle due to failure of the i -th part of the group G by the mileage of $(l, l+\Delta l)$ we accept the product $c_i \lambda_i \Delta l$, where λ_i it is the failure intensity of the i -th part during the specified period of vehicle use. In order to assess the level of risk of incurring costs, it is necessary to estimate the conditional probability of a vehicle failure due to the failure of the i -th part and the total costs of corrective maintenance related to this failure. We make a simplifying assumption that the cost of corrective maintenance related to the replacement of a certain part is constant in the considered periods of vehicle operation. With the assumptions made, the vehicle mileages between subsequent maintenance related to the replacement of the i -th part from the group G have the same exponential distribution with the parameter λ_i , i.e. a cumulative distribution function (CDF) of the vehicle mileage with i -th part is given by the formula $F_i(l) = 1 - \exp(-\lambda_i l)$, $l > 0$. Parameter λ_i is the failure intensity of the i -th part for a fixed vehicle mileage unit, for which we take 1 km. The arrivals of vehicle corrective maintenance caused by failure of the i -th part is a homogeneous Poisson process HPP (λ_i) with parameter λ_i .

Let $L_{i;j}$, $j = 1, 2, \dots$ denote random mileage of the vehicle at which the j -th corrective maintenance caused by replacement of the i -th part of the group G takes place and let the process $\{N_i(l), l \geq 0\}$ count vehicle maintenance to mileage l (in km) due to replacements of the i -th part. For a set mileage $l \geq 0$ random number $N_i(l)$ of corrective maintenance of the vehicle due to failure of the i -th part of the group G has a Poisson distribution, i.e.

$$\Pr(N_i(l) = j) = \frac{(\lambda_i l)^j}{j!} e^{-\lambda_i l}, \quad (1)$$

what we denote as $N_i(l) \sim \text{Poisson}(\lambda_i l)$, $i \in G$.

Random variables $L_{i;j}$, $j = 1, 2, \dots$ determine the vehicle's mileage for subsequent corrective maintenance due to replacement of the i -th part. Assuming that $L_{i;0} = 0$, we can write a difference:

$$X_{i;j} \stackrel{\text{def}}{=} L_{i;j} - L_{i;j-1}, \quad j = 1, 2, \dots \quad (2)$$

Difference $X_{i;j}$ for $j = 1$ means the mileage of the vehicle for the first corrective maintenance, and for $j \geq 2$ means vehicle mileage between successive corrective maintenance, still due to replacement of the i -th part. With all the assumptions made and within the naming convention used, all random variables $X_{i;1}, X_{i;2}, \dots$ have the same exponential distribution with the parameter λ_i what we symbolically

denote as $X_{i;j} \sim \text{EXP}(\lambda_i)$ for $j = 1, 2, \dots$. A consequence of the adopted assumptions regarding the corrective maintenance processes related to the replacement of parts from the group G is the possibility to apply the following properties adapted from the general theory of stochastic processes.

Property 1. Random number $N_i(l_1, l_2)$ of vehicle corrective maintenance due to replacements of the i -th part ($i \in G$), with a planned further mileage of it (l_1, l_2) , has a Poisson distribution with a parameter $(l_2 - l_1)\lambda_i$, i.e.

$$N_i(l_1, l_2) \sim \text{Poisson}((l_2 - l_1)\lambda_i). \quad (3)$$

Property 2. Random cost $C_i(l_1, l_2 | c_i)$ of vehicle corrective maintenance due to replacements of the i -th part during the mileage (l_1, l_2) assuming that the cost related to one replacement of this part c_i is given by the formula:

$$C_i(l_1, l_2 | c_i) = c_i N_i(l_1, l_2). \quad (4)$$

The following property follows directly from property 2.

Property 3. Expected vehicle corrective maintenance cost $\mathbb{E}C_i$ and cost variance $\mathbb{D}^2 C_i$ for maintenance caused by replacements of the i -th part, with further mileage (l_1, l_2) are defined by the formulas:

$$\mathbb{E}C_i(l_1, l_2 | c_i) = c_i \lambda_i (l_2 - l_1), \quad (5)$$

$$\mathbb{D}^2 C_i(l_1, l_2 | c_i) = c_i^2 \lambda_i (l_2 - l_1). \quad (6)$$

So the expected cost $\mathbb{E}C_i(l_1, l_2 | c_i)$ is a measure of the risk of incurring the vehicle corrective maintenance costs due to replacements of the i -th part from the group G with a planned further mileage (l_1, l_2) . Additionally, the variance can be applied to estimating the confidence interval for the risk of incurring these costs.

In the case of a fleet composed of n homogeneous vehicles operated under the same conditions, expected vehicle corrective maintenance cost $\mathbb{E}C_i(l_1, l_2; n | c_i)$ due to replacements of the i -th part of the group G with planned further mileage of vehicles (l_1, l_2) is expressed by the formula (7):

$$\mathbb{E}C_i(l_1, l_2; n | c_i) = n c_i \lambda_i (l_2 - l_1). \quad (7)$$

An alternative measure of the risk of incurring the costs of corrective maintenance due to replacements of the i -th part of the group G for the fleet of n vehicles is the most probable cost of corrective maintenance C_i^{Mo} formulated in property 4 assuming that $n\lambda_i(l_2 - l_1)$ is not an integer.

Property 4. Most probable cost (modal cost) $C_i^{Mo}(l_1, l_2; n | c_i)$ of corrective maintenance for n vehicles of a homogeneous fleet due to exchanges of the i -th part, with further mileage of the vehicles (l_1, l_2) is given by the formula:

$$C_i^{Mo}(l_1, l_2; n | c_i) = c_i \lfloor n \lambda_i (l_2 - l_1) \rfloor, \quad (8)$$

where $\lfloor \cdot \rfloor$ means the floor function.

Another very useful property of the maintenance process is determining the distribution of the vehicle mileage of the tested fleet to a given number of maintenance m .

Property 5. The mileage of the vehicle $L_{i;m}$ to its m -th corrective maintenance due to replacement of the i -th part is a random variable with the Erlang distribution, expressed as:

$$L_{i;m} \sim \text{ERL}(m; \lambda_i), \quad (9)$$

i.e. the probability density function of a random vehicle's mileage $L_{i;m}$ can be expressed as:

$$f_{i;m}(l) = \frac{\lambda_i^m l^{m-1} e^{-\lambda_i l}}{(m-1)!}, l > 0 \quad (10)$$

Hence for a random vehicle mileage $L_{i;m}$ it is possible to designate a cumulative distribution function $F_{i;m}(l)$ and its basic characteristics, i.e. the expected value $\mathbb{E}L_{i;m}$, mode $\text{Mo}(L_{i;m})$ and variance $\mathbb{D}^2 L_{i;m}$ of the vehicle mileage to the m -th corrective maintenance due to replacement of the i -th part:

$$F_{i;m}(l) = 1 - \frac{\sum_{k=0}^{m-1} e^{-\lambda_i l} (\lambda_i l)^k}{k!} \quad \text{dla } l > 0 \quad (11)$$

$$\mathbb{E}L_{i;m} = \frac{m}{\lambda_i} \quad (12)$$

$$\text{Mo}(L_{i;m}) = \frac{m-1}{\lambda_i} \quad \text{dla } m \geq 2 \quad (13)$$

$$\mathbb{D}^2 L_{i;m} = \frac{m}{\lambda_i^2} \quad (14)$$

Using the properties (12) and (14) and the cost table of corrective maintenance, it is possible to determine the expected cost and cost variance.

4. Assessment of the risk of incurring costs due to replacement of the event recorder system

The operational database applies to the fleet of $n = 45$ new vehicles used for 5 years. During this time, out of several thousand parts of which the vehicle is made, more than 500 have been damaged. The fleet supplier has granted a warranty for this time and is considering the possibility of extending it for further years. At the time under study, vehicles reached mileages of around 300,000 km. From the database, the mileages of the parts between failures have been designated as vehicle mileage with a given part.

The analysis of reliability based on the operational database shows that the mileage of the vehicles between the loss of their operational availability caused by replacements of individual parts within corrective maintenance belong to probability distribution families of the types: gamma, Weibull, exponential, normal, lognormal.

The basic problem that had to be solved was to identify these parts which force corrective maintenance of vehicles and, at the same

time, meet the assumption of the Poisson-distributed failures. For this purpose, the hypotheses on the exponentiality of the distribution of mileage between failures for selected parts were verified. Weibull ++ software was used for testing, focusing on this stage of the research primarily on parts from the 15-th vehicle construction group. This group includes electronic and electrical devices, the failure of which is immediately detected and their replacement is relatively fast within the scope of corrective maintenance, and the aging processes of the selected parts are marginal. In vehicles of the tested fleet, this part group includes: an event recorder system, a recorder module, a main monitoring module, a drive controller and a pressure aggregate from 11-th construction group. These parts have been subjected to a reliability analysis.

To assess the risk of incurring corrective maintenance costs of the vehicle fleet, an event recorder system was chosen first. This system is intended for the registration and monitoring of electric meter systems and for recording events regarding emerging hazards and failures of urban transport vehicles. The basic element of the event recorder system is the parameter recorder shown in Fig. 1. The cognitive goal is to assess the risk of incurring the corrective maintenance costs of all fleet vehicles due to the failure of this recorder.



Fig. 1. Parameter recorder.

The recorder saves on the memory card **analogue signals**, such as mileage counter, driving speed, traction current, traction network voltage, control circuit voltage and **logic signals**, such as condition of control devices, feedback signals for braking systems activation, wagon door status, switch changer state, slip signal and other signals important for safety reasons. During the tested period of operation, corrective maintenance at the supplier's cost due to exchanges of this device was registered 11 times. The total costs of the maintenance amounted to approximately 237,000 PLN, and the average cost of one maintenance amounted to approximately 21,500 PLN.

In the reliability analysis, in addition to the mileages of the 11 event recorder systems subjected to corrective maintenance, 45 censored observations concerning the mileage of recorders undamaged on the day ending the research were also included.

On the basis of the vehicle maintenance database provided by the vehicle supplier, failure data for a selected group of parts were developed. For the needs of the conducted reliability tests, data on the mileage of the damaged i -th part for the whole fleet was compiled in the form of pairs $(l_k; \delta_k), k = 1, 2, \dots, n_i$, where l_k is the mileage of the k -th instance of the i -th part (expressed in kilometres), n_i is the number of data related to the i -th part and

$$\delta_k = \begin{cases} 1, & \text{if } l_k \text{ is the observed mileage,} \\ 0, & \text{if } l_k \text{ is the censored mileage.} \end{cases} \quad (15)$$

For the event recorder system, from the operational data for the entire fleet the following pairs were obtained:

(268,707; 0),	(166,033; 1),	(106,638; 0),	(360,404; 0),	(114,686; 1),	(128,846; 0),
(284,834; 0),	(369,436; 0),	(314,522; 0),	(186,073; 1),	(153,834; 0),	(311,679; 0),
(342,241; 0),	(328,494; 0),	(354,100; 0),	(292,038; 0),	(339,492; 0),	(292,266; 0),
(319,088; 0),	(352,654; 1),	(257,803; 1),	(79,301; 0),	(310,513; 0),	(305,087; 0),
(89,378; 1),	(245,637; 0),	(308,114; 0),	(328,878; 0),	(314,589; 0),	(91,939; 1),
(176,804; 0),	(319,733; 0),	(328,480; 0),	(69,514; 1),	(258,654; 0),	(287,122; 0),
(189,518; 1),	(107,834; 0),	(271,868; 0),	(244,005; 0),	(284,525; 0),	(207,679; 0),
(314,146; 0),	(271,384; 0),	(316,331; 0),	(286,225; 0),	(58,298; 1),	(231,227; 0),
(254,974; 0),	(290,198; 0),	(299,174; 0),	(80,172; 1),	(184,615; 0),	(310,800; 0),
(339,499; 0),	(315,371; 0).				

The research covered 56 instances of the event recorder system, of which 45 were in good working order at the time the tests were completed and their observations of the mileage were cut off. The test carried out at the 5% significance level did not give grounds for rejecting the hypothesis on the exponentiality of the vehicles' mileage between their failures caused by failures of the event recorder systems. Based on the operational data, the failure intensity was estimated $\hat{\lambda} \approx 0,0000009463[1 / \text{km}]$. Assuming that during the next year of operation of the fleet, the failure intensity of the used recorder systems will not change significantly, it is possible to determine the risk of incurring the replacement costs. The annual mileage of vehicles in the fleet under investigation is about 60,000 km. Hence the risk of incurring costs measured by the expected cost of corrective maintenance caused by the replacement of this system with the mileage of vehicles from $l_1 = 300000$ [km] to $l_2 = 360000$ [km] is about 1,224 [PLN], what with the fleet of 45 vehicles makes the amount of about 55,081 [PLN]. However, the most likely total corrective maintenance cost for the fleet of 45 vehicles due to replacement of this system determined from the formula (8) is 43,000 [PLN]. The expected mileage of the vehicle for the third maintenance related to the replacement of this system is 3,170,242 km.

5. Costs of corrective maintenance of the selected group of damaged parts of the vehicle

Very practical properties of the Poisson process in assessing the cost of corrective maintenance of the vehicle due to the independent failures to parts from the selected group G is both the property of the superposition of Poisson's processes as well as the property of the Poisson process decomposition [1], [12].

Property 6 (superposition of Poisson's processes). If the numbers of corrective maintenance of the vehicle due to the replacement of parts from the set G for the mileage of $l > 0$ are independent and have Poisson distributions, i.e. $N_i(l) \sim \text{Poisson}(\lambda_i l)$, $i \in G$, than the total number of corrective vehicle maintenance $N_G(l_1, l_2)$ due to the replacement of parts from the set G with a planned further mileage of (l_1, l_2) has a Poisson distribution with a parameter $(l_2 - l_1) \sum_{i \in G} \lambda_i$, i.e.

$$\Pr(N_G(l_1, l_2) = n) = \frac{((l_2 - l_1) \sum_{i \in G} \lambda_i)^n}{n!} \exp\left(- (l_2 - l_1) \sum_{i \in G} \lambda_i\right). \quad (16)$$

Using property 4, it is now possible to designate the total random cost $C_G(l_1, l_2 | c_i, i \in G)$ of vehicle corrective maintenance due to the replacement of parts from the group G for the mileage (l_1, l_2)

$$C_G(l_1, l_2 | c_i, i \in G) = \sum_{i \in G} C_i(l_1, l_2 | c_i). \quad (17)$$

Hence the expected total cost $\mathbb{E}C_G(l_1, l_2 | c_i, i \in G)$ and the variance of the total cost $\mathbb{D}^2 C_G(l_1, l_2 | c_i, i \in G)$ are as follows:

$$\mathbb{E}C_G(l_1, l_2 | c_i, i \in G) = \sum_{i \in G} c_i \lambda_i (l_2 - l_1), \quad (18)$$

$$\mathbb{D}^2 C_G(l_1, l_2 | c_i) = \sum_{i \in G} c_i^2 \lambda_i (l_2 - l_1). \quad (19)$$

Property 7 (decomposition of the Poisson process). If the total number $N_G(l)$ of corrective vehicle maintenance for mileage $l > 0$ due to independent replacements of parts from the set G is the Poisson process with the failure intensity λ_G and the probability of vehicle corrective maintenance due to a failure of the i -th part at this time equals to p_i , wherein $N_G(l) = \sum_{i \in G} N_i(l)$ and $\sum_{i \in G} p_i = 1$, than:

$$\Pr(N_i(l_1, l_2) = n_i, i \in G) = \prod_{i \in G} \frac{(p_i \lambda (l_2 - l_1))^{n_i}}{n_i!} \exp(-p_i \lambda (l_2 - l_1)) \quad (20)$$

Equation (20) allows to determine the probability distribution of the number of replacements of individual parts from the set G with the planned further mileage of the vehicle (l_1, l_2) .

By using property 7, it is also possible to determine the expected cost of corrective maintenance caused by independent failures to a part from the set G . The risk of incurring the corrective maintenance costs for the fleet of n vehicles (based on fleet operational data to mileage l_0) related to the replacement of parts from the set G with a planned further mileage (l_1, l_2) [km] ($0 < l_0 \leq l_1 < l_2 < \infty$) – assuming the criterion of expected costs – is expressed by the following formula:

$$\mathbb{E}C_G(l_1, l_2, n | c_i, i \in G) = \sum_{i \in G} c_i p_i \lambda n (l_2 - l_1) \quad (21)$$

If the criterion of the most likely cost is taken to assess the risk of incurring costs, then:

$$C_G^{Mo}(l_1, l_2, n | c_i, i \in G) = \sum_{i \in G} c_i n \lambda_i (l_2 - l_1) \quad (22)$$

or:

$$C_G^{Mo}(l_1, l_2, n | c_i, i \in G) = \sum_{i \in G} c_i n p_i \lambda (l_2 - l_1) \quad (23)$$

The presented methods of assessing the risk of incurring the costs of corrective maintenance require meeting quite strong assumptions

regarding the distribution of failures. The reliability tests carried out show that only a small group of damaged parts of the vehicle meet these assumptions. However, the presented stochastic cost forecasting methods are a good basis for developing stochastic methods of cost forecasting for vehicle parts that meet weaker assumptions.

6. Case study for the selected group of parts

From statistical surveys of corrective maintenance of the most expensive parts from the 15-th construction group, the failure rate of the drive controller turned out to be the closest to the Poisson model. Additionally, the failure rate of the recorder module and the main monitoring module did not show a significant difference with the Poisson model, characterised by a constant failure intensity. In contrast, the failure process of the assembly set was characterised by a significantly decreasing intensity, so it did not meet the accepted requirement. Among the costly corrective maintenance from outside of the 15-th construction group, the failure rate of a pressure aggregate was similar to the Poisson model. The identified parts, which are characterised by failure rate that does not differ significantly from the Poisson model, were assigned to the group G . The theoretical basis of risk assessment methods for the corrective maintenance cost, which were described in the previous section, were applied to the identified group of parts G . Group of parts G is composed of a pressure aggregate, an event recorder system, a recorder module, a main monitoring module and a drive controller. There is only one instance of each of these parts inside one vehicle. Statistics on the number of corrective maintenance and its costs related to the replacement of these parts for the fleet of 45 vehicles are presented in Table 1.

The results of the point and interval estimation performed for the parameters of the two-parameter Weibull distribution for selected vehicle parts are presented in Table 2. The assumption was made that the distributions of mileage between failures belong to families of two-parameter Weibull distributions with the parameters determined by the probability density function:

$$f(l|\beta,\eta) = \frac{\beta}{\eta} \left(\frac{l}{\eta}\right)^{\beta-1} \exp\left(-\left(\frac{l}{\eta}\right)^\beta\right) \mathbb{I}_{(0,\infty)}(l), \quad (24)$$

where $\eta > 0$ is a scale parameter and $\beta > 0$ is a shape parameter.

For the estimation of Weibull distribution parameters with right-censored data, the maximum likelihood estimation (MLE) method was used [9]. Table 2 presents the results of point and interval estimation for Weibull distribution parameters. A 95% confidence level was assumed.

On the basis of operational data it is not possible to reject the hypotheses about the

Table 1. Data regarding costs of corrective maintenance due to replacement of selected parts of the vehicle (own work)

No.	Part name	Construction group	Number of maintenance	Total maintenance costs [PLN]
1	Pressure aggregate	11	27	610,200
2	Event recorder system	15	11	237,138
3	Recorder module	15	12	184,920
4	Main monitoring module	15	54	287,712
5	Drive controller	15	23	208,357

Table 2. Results of estimation of the parameters of the part's mileage assuming a two-parameter Weibull distribution

No.	Part name	Lower end of the interval for β	$\hat{\beta}$	Upper end of the interval for β	Lower end of the interval for η	$\hat{\eta}$	Upper end of the interval for η
1	Pressure aggregate	0.6954	1.0932	1.7187	293,870	635,599	1,145,688
2	Event recorder system	0.54656	0.9117	1.52082	450,409	1,209,913	3,250,134
3	Recorder module	0.71235	1.0414	1.52245	469,975	731,649	1,139,018
4	Main monitoring module	0.77217	0.96850	1.21475	190,126	253,444	337,850
5	Drive controller	0.62877	1.02535	1.67208	297,741	664,105	1,481,273

exponentiality of mileage to failure of a part from the group G on the assumed significance level of 0.05. Hence, further inference is based on the assumption of the Poisson distribution of failures. For the selected parts, the results of estimation of the expected mileage to failure, 95% confidence intervals for the expected mileage and the intensity of their failures are presented in Table 3.

Based on the estimated failure intensities and the unit costs of corrective maintenance related to replacements of the selected group of parts G , the risk of incurring corrective maintenance costs due to the replacement of these parts for the assumed further annual mileage of vehicles from the tested fleet from $l_1 = 300,000$ [km] to $l_2 = 360,000$ [km] was determined.

The estimated total cost of corrective maintenance related to the replacement of parts from the group G for one vehicle is 7,100 [PLN], which, with a fleet of 45 vehicles, amounts to almost 320,000 [PLN]. Alternatively, the forecast based on the most probable corrective maintenance cost for these parts is 274,590 [PLN] for the vehicle fleet.

Table 3. The results of the estimation of the parameters of the part mileage, assuming the exponentiality

No.	Part name	Confidence Bounds Lower	Mean Time (km)	Confidence Bounds Upper	Failure Rate [1/km]
1	Pressure aggregate	376,975	554,163	814,633	0.0000018045
2	Event recorder system	613,590	1,056,719	1,819,870	0.0000009463
3	Recorder module	508,933	765,372	1,151,023	0.0000013066
4	Main monitoring module	186,564	241,867	313,564	0.0000041345
5	Drive controller	397,799	602,362	912,119	0.0000016601

Table 4. Estimated costs of corrective maintenance of parts from the group *G* during the next year of operation

No.	Part name	Estimated costs		The most probable costs for the fleet
		for a vehicle	for the fleet	
1	Pressure aggregate	2,447	110,111	90,400
2	Event recorder system	1,224	55,081	43,116
3	Recorder module	1,208	54,364	46,230
4	Main monitoring module	1,322	59,477	58,608
5	Drive controller	902	40,605	36,236

7. Conclusions

Stochastic modelling of corrective maintenance costs presented in the paper, which takes into account reliability characteristics of repairable objects, such as vehicles, is an excellent method of supporting decision-making processes in their maintenance and allows for more rational use of the public transport fleet.

As a result of the conducted research, the main idea of stochastic cost forecasting of selected corrective maintenance of urban transport means was presented, which is of key importance in supporting effective management of vehicle fleet operations.

The developed methods have been implemented to assess the costs of corrective maintenance of the fleet of urban transport vehicles. Parameters of the vehicle corrective maintenance process models were estimated on the basis of the operational database containing

information from the period of 5 years of operation of a homogeneous fleet of trams. To predict the costs of corrective maintenance, parts of the vehicle that meet the assumption regarding the Poisson-distributed failures were identified.

The issue of corrective maintenance costs is up-to-date due to the currently developed practical possibilities of the process perception of the activities of transport companies and is indispensable in the study of the life cycle costs of means of transport. In their final form, the presented methods will be used for supporting an IT system managing the operation and maintenance of fleet of urban transport vehicles.

The authors see further development of the conducted research in such a weakening the assumptions of the failure process models, so that the application possibilities in the field of corrective maintenance cost prediction can be broadened. Knowledge about corrective maintenance is essential in optimising preventive maintenance and reducing unscheduled vehicle downtime costs. In conclusion, it is worth noting that in recent years, intensive research has been carried out on the optimisation of strategies and methods for maintenance of technical objects [8, 15, 16, 26, 27].

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References

- Andrzejczak K.: Stochastic modelling of the repairable system, *Journal of KONBiN* 3(35) 2015, 5-14.
- Andrzejczak K.: Methods of the forecasting in the modeling of means of transport maintenance (in Polish). *Rozprawy nr 496*. Wydawnictwo Politechniki Poznańskiej, Poznań 2013.
- Andrzejczak K., Selech J.: Quantile analysis of the operating costs of public transport fleet, *Transport Problems*, vol. 12 (3), 103-111, 2017.
- Andrzejczak K, Selech J., Investigating the trends of average costs of corrective maintenance of public transport vehicles, *Journal of KONBiN* 41(2017), 207-226. DOI 10.1515/jok-2017-0011.
- Andrzejczak K., Młyńczak M., Selech J. Assessment model of operational effectiveness related to newly operated public means of transport. *Proceedings of the 27th European Safety and Reliability Conference (ESREL 2017)*, Portorož, Slovenia, Editors: Marko Čepin, Radim Briš, pages 3455-3461.
- Ascher H., Feingold H.: *Repairable Systems Reliability: Modeling, Inference, Misconceptions and their Causes*. Marcel Decker, New York, 1984.
- Bobrowski D.: *Modele i metody matematyczne teorii niezawodności w przykładach i zadaniach*. 1985, Warszawa, WNT.
- Chen Y., Cowling P., Polack F., Remde S., Mourdjis P., Dynamic optimisation of preventative and corrective maintenance schedules for a large scale urban drainage system. *European Journal of Operational Research*, 257 (2), 494-510. <https://doi.org/10.1016/j.ejor.2016.07.027>
- Ferreira L.A., Silva J.L. Parameter estimation for Weibull distribution with right censored data using EM algorithm. *Eksplatacja i Niezawodność – Maintenance and Reliability* 2017; 19(2): 310–315, <http://dx.doi.org/10.17531/ein.2017.2.20> .
- Grabski F.: Stochastyczny model bezpieczeństwa obiektu w procesie eksploatacji. *Problemy Eksploatacji*, 1/2011 (80), 89-102.
- Grabski F.: Nonhomogenous Poisson proces application to modeling accidents number at Baltic Sea waters and ports, *Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars*, Vol. 8, Number 1, 2017, 39-46.
- Jokiel-Rokita A., Magiera R.: (2010). Parameter estimation in non-homogeneous Poisson process models for software reliability. Technical report, Wrocław University of Technology, Institute of Mathematics and Computer Science.
- Knopik L, Migawa K. Multi-state model of maintenance policy. *Eksplatacja i Niezawodność – Maintenance and Reliability* 2018; 20 (1): 125–130, <http://dx.doi.org/10.17531/ein.2018.1.16>.
- Kusz A., Marciniak A., Skwarcz J. Implementation of computation process in a bayesian network on the example of unit operating costs determination. *Eksplatacja i Niezawodność – Maintenance and Reliability* 2015; 17 (2): 266–272, <http://dx.doi.org/10.17531/ein.2015.2.14>.
- Legát V, Mošna F, Aleš Z, Jurča V. Preventive maintenance models – higher operational reliability. *Eksplatacja i Niezawodność – Maintenance and Reliability* 2017; 19 (1): 134–141, <http://dx.doi.org/10.17531/ein.2017.1.19>.
- Lee H, Cha JH. New stochastic models for preventive maintenance and maintenance optimization. *European Journal of Operational Research* 2016; 255 (1): 80-90, <https://doi.org/10.1016/j.ejor.2016.04.020>.
- Macián V, Tormos B, Riechi J. Time replacement optimization model: comparative analysis of urban transport fleets using Monte Carlo Simulation. *Eksplatacja i Niezawodność – Maintenance and Reliability* 2017; 19 (2): 151–157, <http://dx.doi.org/10.17531/ein.2017.2.1>
- Młyńczak M. 2012. *Metodyka badań eksploatacyjnych obiektów technicznych*, Wrocław, Oficyna Wydawnicza Politechniki Wrocławskiej, s. 204.

19. Nakagawa T.: Stochastic Processes with Applications to Reliability Theory. Springer, 2011.
20. Nataša Kontrec, Stefan Panić, Milena Petrović, Hranislav Milošević. A stochastic model for estimation of repair rate for system operating under performance based logistics. *Eksploatacja i Niezawodność – Maintenance and Reliability* 2018; 20 (1): 68–72, <http://dx.doi.org/10.17531/ein.2018.1.9>.
21. Nowakowski T, Młyńczak M, Werbińska-Wojciechowska S, Dziaduch I, Tubis A. Life Cycle Costs of passenger transportation system. Case study of Wrocław city agglomeration 5. *JPSRA* 2014; 5: 109-120.
22. Omdahl T.P., ed.: Reliability, Availability and Maintainability (RAM) Dictionary, ASQC Quality Press, Milwaukee, Wisconsin, 1988.
23. Pogačnik B, Duhovnik J, Tavčar J. Aircraft fault forecasting at maintenance service on the basis of historic data and aircraft parameters. *Eksploatacja i Niezawodność – Maintenance and Reliability* 2017; 19 (4): 624–633, <http://dx.doi.org/10.17531/ein.2017.4.17>.
24. Raposo H, Farinha JT, Ferreira L, Galar D. An integrated econometric model for bus replacement and determination of reserve fleet size based on predictive maintenance. *Eksploatacja i Niezawodność – Maintenance and Reliability* 2017; 19 (3): 358–368, <http://dx.doi.org/10.17531/ein.2017.3.6>.
25. Rymarz J, Niewczas A, Krzyżak A. Comparison of operational availability of public city buses by analysis of variance. *Eksploatacja i Niezawodność–Maintenance and Reliability* 2016; 18 (3):373–378, <http://dx.doi.org/10.17531/ein.2016.3.8>.
26. Sharma A, Yadava G S, Deshmukh S G. A literature review and future perspectives on maintenance optimization, *Journal of Quality in Maintenance Engineering* 2011; 17 (1): 5 – 25, <https://doi.org/10.1108/1355251111116222>.
27. Stenström C, Norrbin P, Parida A, Kumar U. Preventive and corrective maintenance - cost comparison and cost-benefit analysis. *Structure and Infrastructure Engineering* 2016; 12 (5): 603-617, <https://doi.org/10.1080/15732479.2015.1032983>

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