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ANALYSIS OF THE EXPLOITATION FAILURE RATE IN POLISH MV NETWORKS

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The paper presents an analysis of exploitation failure rate in MV overhead and cable lines and in MV/LV transformers in the Polish power system. The reliability analysis is carried out by means of a non-parametric method using kernel estimators. The data are obtained from power networks of various operating conditions, and because of that the weight of data is taken into consideration. Additionally, the paper offers an innovative way of presenting the reliability data in a graphic form.

Keywords: reliability, failure rate in MV networks, kernel estimators.

W artykule przedstawiono wyniki analizy danych eksploatacyjnych awaryjności linii napowietrznych i kablowych średniego napięcia jak również transformatorów SN/nN krajowego systemu elektroenergetycznego. Do analizy zagadnienia zastosowano nieparametryczną metodę analizy danych niezawodnościowych sieci elektroenergetycznych z wykorzystaniem estymatorów jądrowych. Poszczególne dane pochodzą z sieci elektroenergetycznych mających różny charakter pracy, dlatego uwzględniono w analizie wagę poszczególnych danych. W artykule zaproponowano także nowy sposób prezentacji graficznej analizowanych danych niezawodnościowych.

Słowa kluczowe: niezawodność, awaryjność sieci średniego napięcia, estymatory jądrowe.

1. Introduction

The issues related to the reliability of power systems are currently in the centre of interest all over the world. Unconstrained and continuous access to electricity is generally taken for granted, and people are becoming increasingly dependent on it. What a consumer expects is both the correct operation of electrical appliances, i.e. high-quality energy [14]) and uninterrupted access, i.e. reliable supply of energy [14]. Consumers expectations can be unfulfilled due to the technical condition of the distribution network, which motivates research on methods of assessing the reliability of energy supply systems.

It is possible to analyse the power system reliability in many respects. A great number of empirical and theoretical studies have been devoted to this topic, beginning with the works of Roy Billinton and Ronald N. Allan, who jointly [2] and together with a team of co-researchers published extensively on the topic. Polish researchers have also published a number of works, of which those by Józef Paska [13, 14] or Andrzej Chojnański [5] are the most widely known.

Reliability analysis can be applied to the production, transmission or distribution of electric energy. The reliability of the distribution system concerns the three voltage levels, i.e. high, medium and low voltage, with separate reliability parameters determined for each of them, as well as the system of connections and functional dependences between the three levels [1-10, 13, 14, 16]. Real power network systems are often quite extensive, and their reliability is typically assessed by means of commonly applied indices such as SAIDI, SAIFI or MAIFI [1, 2, 9, 10]. The indices are calculated on the basis of events registered in a network operated by a distribution company. Besides, distribution companies use information on the reliability of particular groups of power devices. It has to be noted that it is difficult to collect extensive reliability data for the sake of research. The data collected by distribution companies are treated as confidential and are not available to the public, they only get published in a fragmentary

or generalized form [19]. An exception to this rule and an example of good practice is the report [1], presenting analyses of the reliability of European power systems. The publication of such reports increases availability of information and enhances transparency of reliability and energy quality policies applied in each of the countries included in the report.

Besides, a correct assessment of reliability is crucial for a distribution company to develop a strategy of exploitation and investment in the network so that the reliability targets imposed by the Energy Regulatory Office (ERO) can be met. It is very important for companies, which are obliged to report their reliability indices to ERO and if real reliability indices turn out not to meet the yearly targets, a company can be fined.

It has to be noted too, that the reliability data are collected from power networks of different operation characteristics, covering different areas and serving different numbers of customers. It is therefore necessary to take into consideration the weight of particular data in the analysis. Besides, it should also be emphasized that the classical statistical methods typically employed in the analysis of reliability data, such as Weibull, exponential or log-normal distributions, may be inadequate. These methods, referred to as parametric, rely on arbitrarily adopted typical distribution of random variables, on the basis of which parameters of the assumed distribution are subsequently obtained [5]. The reason why they are insufficient for analysing reliability problems is that the number of possible distributions is limited [12]. Non-parametric methods are free from this limitation, since no function is assumed a priori [11, 12].

It can be observed on the basis of statistical analysis of failures in power networks that the majority of breakdown occur in MV and LV lines [7-10]. The most frequent causes of failures are of random nature and include atmospheric overvoltage, SEMP, sudden changes in ambient temperature, wind, hard rime, effects of air pollution or human factors [5]. The data on failures in power networks should be reported

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in statistical sheets G-10.5, filled in by distribution companies. The sheet includes the values of indices characterizing the failures of elements in the MV and LV power networks.

This paper offers an analysis of data obtained from the sheets G-10.5 submitted by 19 distribution companies in the years 2012 – 2014. The distributions of the density functions of the reliability indices were calculated by means of nonparametric methods, which represent the variation of the phenomena under investigation in a credible way [11, 12]. Kernel estimators presented in further parts of the paper are precise in mathematical terms and are universally applicable. Combined with computational capacity of computers, kernel estimators can be successfully used in analyses of highly complex problems.

The main objective of the paper is to present an estimation of the function of the random variable distribution density on the basis of a sample. All the statistical computations have been carried out in the programming environment of the language R [15]. For each analysis of reliability data a program has been developed for calculating reliability of a group of devices under scrutiny and graphs supporting the process of statistical inferencing have been plotted.

2. Numerical data analysis

One of the fundamental problems that has to be solved in analyses of complex technological processes is obtaining functions characterizing an object under scrutiny on the basis of experimental data available. The reliability analysis of individual devices and of the whole power system requires determining adequate statistical measures on the basis of historical data. The typical cases of this problem include obtaining the mean value, the standard deviation, the regression function, or the estimation of the probability density function (pdf) for a random variable. When estimating the pdf, the type of the random variable distribution is usually arbitrarily assumed and its parameters are subsequently determined. Because of that, these methods are referred to as parametric.

2.1.1 Kernel estimators

As far as non-parametric estimation methods are concerned, the one based on kernel estimators is among the most widely applied. Non-parametric methods were put forward at the turn of the 1950s and 60s and their basic idea is derived from the pdf estimation problem, as discussed in a number of publications, e.g. [11, 12, 21-23].

A typical problem to which kernel estimators can be applied is obtaining the density function of the probabilistic distribution of the random variable on the basis of a sample (i.e. Kernel Density Estimation – KDE). To implement kernel estimators for analysing random variables, it is necessary to have access to a computer with software for statistical analyses, such as the program R.

Definition of a classical kernel estimator - KDE.

Let X be an n -dimensional random variable, with the distribution density f . Its kernel estimator $\hat{f} : \mathbb{R}^n \rightarrow [0, \infty)$ is determined on the basis of an m element random sample x_1, x_2, \dots, x_m , obtained from the variable X . The basic form of the kernel is defined by the formula:

$$\hat{f}(x) = \frac{1}{m h^n} \sum_{i=1}^m K\left(\frac{x - x_i}{h}\right) \quad (1)$$

where the function called a kernel, $K : \mathbb{R}^n \rightarrow [0, \infty)$ which is measurable, symmetrical with respect to zero and having a weak global maximum

at this point, satisfies the condition $\int_{\mathbb{R}^n} K(x) dx = 1$. The positive

coefficient h is known as bandwidth [11, 20-23].

From the statistical point of view, the form of the kernel is not significant. It is possible to choose the kernel form on an arbitrary basis. Due to the fact that many natural phenomena, including those affecting reliability data, are subject to normal distribution, it appears justified to apply the normal kernel as defined by the function:

$$K(x) = \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-x^2}{2}\right)} \quad (2)$$

The normal kernel has a disadvantage though, namely the unlimited carrier of the random variable $x \in (-\infty, \infty)$. Since the estimated values of the reliability parameters can be only positive, the form of the kernel estimator has to be modified in such a way that the random variable X can take only non-negative values. This can be done by stipulating a constraint on the kernel carrier, the role of which is to mirror the part of kernel located beyond the permitted interval, i.e. in the case of reliability random variables, it is those below zero [11, 21, 23].

Another modification of the classical KDE involves introducing weights. In this way, a weighted kernel density estimator (KDEw) is obtained.

Definition of a weighted kernel density estimator KDEw.

Let X be an n -dimensional random variable, with the distribution density f . Its kernel estimator $\hat{f} : \mathbb{R}^n \rightarrow [0, \infty)$ is obtained on the basis of an m -element random sample x_1, x_2, \dots, x_m , to which non-negative values w_1, w_2, \dots, w_m are attributed. The values satisfy the condition $\sum_{i=1}^m w_i = 1$ and are further referred to as weights of the particular elements of the variable X , as defined by the formula:

$$\hat{f}(x) = \frac{1}{h^n} \sum_{i=1}^m w_i K\left(\frac{x - x_i}{h}\right) \quad (3)$$

where the function $K : \mathbb{R}^n \rightarrow [0, \infty)$ is the kernel. It is measurable, symmetrical with respect to zero, has a weak global maximum at this point and satisfies the condition $\int_{\mathbb{R}^n} K(x) dx = 1$. The positive coefficient

h is the bandwidth, as described in [11, 23].

In the special case when all the weights w_1, w_2, \dots, w_m are equal and their sum is 1, the form of the estimator KDEw specified in (3) becomes equivalent to that in (1), i.e. the KDE.

In this paper I shall argue that the use of KDEw provides a more adequate model for the analysis of the reliability of a power system than KDE. The results of such an analysis should be however approached with caution if the weights take extremely disparate values. In such a case, before accepting the outcome of the statistical calculations, it is necessary to verify them on the basis of theoretical and practical familiarity with the phenomena under scrutiny to avoid drawing erroneous conclusions.

The quality of kernel estimators is crucially affected by the bandwidth h , which ensures the smoothness of the density function. If the parameter value is too small, there are too many local extremes of the estimator $\hat{f}(x)$, if its value is too high, the function $\hat{f}(x)$ is excessively smoothed and ceases to represent the real properties

of the random variable. A number of useful algorithms are offered in the literature for calculating the value of the parameter h , optimized on the basis of a Mean Squared Error criterion. Methods for optimizing the estimation of the parameter h are also implemented in the popular statistical analysis software, such as SAS, R, MATLAB or STATYSTYKA.

In the R program environment [15], there are a number of functions available for calculating a pdf of a random variable by means of kernel estimators. In this study, the function 'density' was used, which comes in a number of calculation options. One of them includes the parameter 'weights', defining the weights for the particular values of a random variable. The method of obtaining the parameter h selected by means of the parameter 'bw'. The method employed here for obtaining the value of the bandwidth h has the Sheather & Jones method. It is worth mentioning that the function 'density' and the software R were exploited in my previous studies too [7-10].

When the data are multidimensional, two natural generalization of the above-mentioned formulas are employed: a radial kernel [11, 23]:

$$K(x) = C K\left(\sqrt{x^T x}\right) \quad (4)$$

and product kernel:

$$K(x) = K\left([x_1, x_2, \dots, x_n]^T\right) = K(x_1) \cdot K(x_2) \cdot \dots \cdot K(x_n) \quad (5)$$

where K is a one-dimensional kernel, and C is a positive constant of such a value that the condition $\int_{\mathbb{R}^n} K(x) dx = 1$ is satisfied. Radial kernel

is more effective than product kernel, but from the perspective of applications, the difference is insignificant, as pointed out in publications [11, 23].

3. Analysis of exploitation failures in MV networks

The failure analysis of MV networks presented in this study was carried out on the basis of data obtained from 19 Polish distribution companies, covering 57 % of the area of Poland, supplying about 65 % of total energy consumption to over 10.5 million consumers (as of the end of 2015). The total length of the power lines used for energy transmission is over 465 thousand km. It can be assumed that this is a statistically significant sample for carrying out reliability analysis concerning Polish power networks.

Each Polish distribution company is obliged to submit a statistical report on the condition of electrical devices, known as G-10.5. This report includes the company's confidential data on the network condition and the functioning of particular power devices in the preceding calendar year. The report is submitted to Agencja Rynku Energii Spółka Akcyjna (ARE S.A.) for the sake of conducting statistical analysis and systematic research on power industry. ARE S.A. processes the data jointly but the results are confidential and are therefore not made available to the public. In a comprehensive report called

"Statystyka Energetyki Polskiej" [19], only mean values of indices obtained for the whole Polish power industry are published. It is not possible to obtain data on the power system reliability for particular distribution companies.

In the G10.5 report, the failure rate in MV power networks is described in terms of:

- the number of failures in MV overhead and cable lines and in MV/LV transformers;
- a failure index per 100 km of overhead and cable lines, respectively, and per 100 MV/LV transformers;
- average interruption time expressed in hours in electricity supply caused by failures in MV cable and overhead lines and in MV/LV transformers.

The total number of failures registered in the MV network devices in the period 2012 – 2014 was 53 089. A summary of failures occurring in various types of devices in MV networks is presented in Table 1.

In 2014 the distribution companies taken into account in the present analysis jointly served 128 655 km of overhead MV lines, 39 165 km of MV cable lines and 146 627 MV/LV transformers.

Table 2 presents the length of MV cable lines (l_{SN_LK}), the length of MV overhead lines (l_{SN_LN}) and the number of MV/LV transformers (l_{Tr}) (each distribution company, together with weights attributed to them ($waga$)). The value of the variable $waga$ obtained from the following formula:

$$waga_i = \frac{x_i}{\sum_{i=1}^{19} x_i} \quad (6)$$

where x_i is the value corresponding to the length of lines or the number of transformers in a company divided by the sum obtained for all the 19 companies.

The calculated values of the variable $waga$ are used for determining KDEw, discussed below.

Below a statistical analysis will be carried out of the data with respect to failure indices (w) and with respect to mean interruption duration due to failure (t) for three groups of devices: MV cable lines, MV overhead lines and MV/LV transformers.

In each of the groups the data set included 57 observations (i.e. 3 values from the period 2012–2014 in the 19 distribution companies).

The results obtained are presented at three types of graphs: a histogram, a boxplot and a pdf. The pdf is obtained by means of kernel estimators with a normal kernel, the carrier being constrained to positive values. The bandwidth coefficient h as obtained by means of the most widely accepted method developed by Sheather & Jones [17].

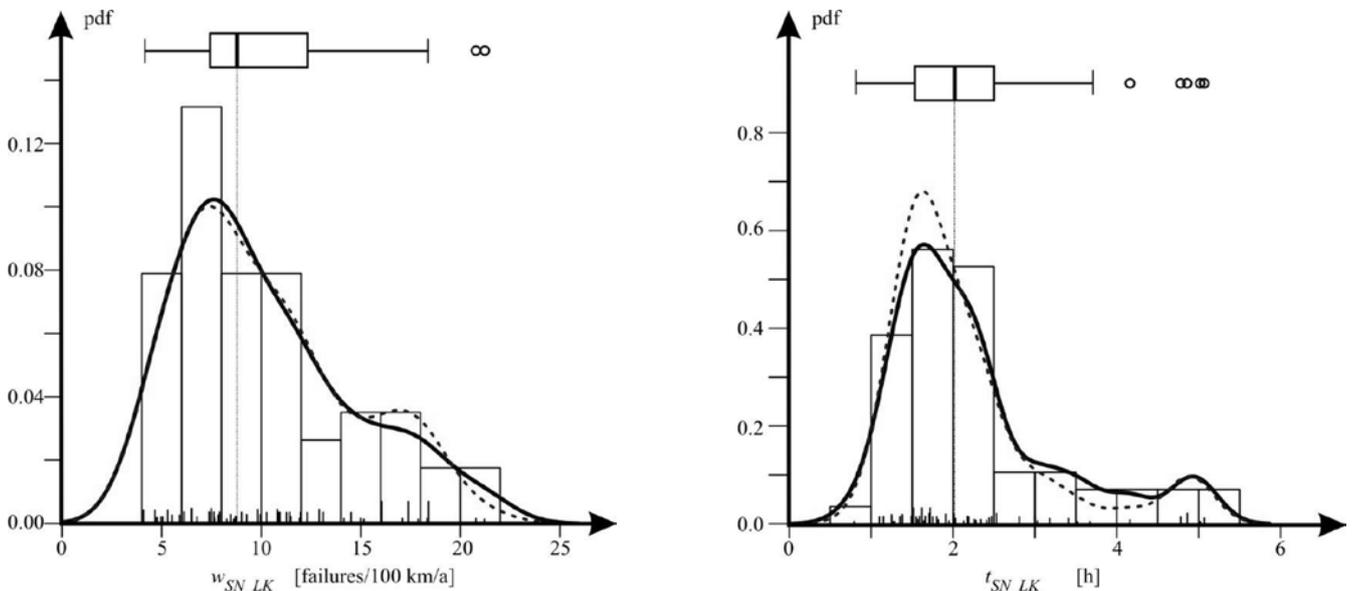
The calculations of the pdf by means of kernel estimators were carried out in two variants – the classic KDE variant without weights and the KDEw variant in which the significance of data is weighed. In non-weighted KDE each datum has the same significance in the analysis, whereas in the weighed variant the data with greater weights have greater significance in the calculations. The variables that are

Table 1. Number of failures registered in the MV network that occurred in overhead lines, cable lines and MV/LV transformers in the period 2012 – 2014

Number of failures in the MV network in the years 2012 - 2014			[%]
Total number of failures in:	MV overhead lines	38 869	73.21
	MV cable lines	11 865	22.35
	MV/LV transformers	2 355	4.44
Total		53 089	100.00

Table 2. Length of MV cable lines SN (l_{SN_LK}), length of MV overhead lines (l_{SN_LN}), number of MV/LV transformers (l_{Tr}) in each distribution company marked by a code A-T and weights attributed to the data

Company code	MV overhead lines		MV cable lines		MV/LV transformers	
	l_{SN_LK} [km]	$waga_{SN_LK}$	[m]	$waga_{SN_LN}$	l_{Tr} [Units]	$waga_{Tr}$
A	2 302	0.0588	16487	0.1281	14606	0.0996
B	2 926	0.0747	9687	0.0753	9810	0.0669
C	2 367	0.0604	1509	0.0117	3631	0.0248
D	1 700	0.0434	12715	0.0988	11744	0.0801
E	2 457	0.0627	11426	0.0888	11366	0.0775
F	2 305	0.0589	13080	0.1017	13038	0.0889
G	2 437	0.0622	14952	0.1162	17870	0.1219
H	1 074	0.0274	11416	0.0887	8916	0.0608
I	2 368	0.0605	1873	0.0146	3640	0.0248
J	1 384	0.0353	3233	0.0251	4630	0.0316
K	1 291	0.0330	3926	0.0305	4017	0.0274
L	5 036	0.1286	3011	0.0234	9350	0.0638
M	717	0.0184	2823	0.0219	2759	0.0188
N	3 341	0.0853	3441	0.0267	9938	0.0678
O	914	0.0233	2933	0.0228	2785	0.0190
P	1 815	0.0463	5080	0.0395	5189	0.0354
R	731	0.0187	3979	0.0309	4050	0.0276
S	901	0.0230	2982	0.0232	3190	0.0218
T	3 099	0.0791	4102	0.0319	6098	0.0416
Total	39 165	1.0000	128 655	1.0000	146 627	1.0000



	minimum	1 Qu	median	mean	3 Qu	maximum
w_{SN_LK} [failures/100 km/a]	4.08	7.38	8.74	10.16	12.30	21.20
t_{SN_LK} [h]	0.79	1.57	2.01	2.26	2.47	5.07

Fig. 1. Histogram, boxplot and a pdf obtained by means of KDE (continuous line) and KDEw with the weight $waga_{SN_LK}$ (interrupted line) for the failure index w_{SN_LK} and for the interruption duration t_{SN_LK} in MV cable lines

used as weights in further calculations are the length of MV overhead lines, the length of MV cable lines and the number of MV/LV transformers. In the cases under scrutiny, the weights were of similar values, i.e. the greatest differences can be of one order of magnitude. This indicates that the weighed features in the power networks are in fact comparable.

The three types of graphs represent the same information in different ways. The most popular form is a histogram representing the data distribution in a basic visual way. The boxplot represents a graphic interpretation of the distribution of the statistical features of a variable. It provides information on the median, the 1st quartile (1 Qu) Q_1 , the 3rd quartile (3 Qu) Q_3 , IQR (interquartiles) as well as outliers, exceeding 1,5 IQR, marked by circles. The KDE graphs are plotted by means of a continuous line and the KDEw graphs by means of an interrupted line, representing modal values directly visible at the pdf curve, skewness, outliers and multimodality, if it occurs.

Besides, the plots offer additional graphical information on the value and weight of the particular data d_i . Along the base of the scale on the y-axis there are markers “|”, located at places corresponding to the values of the data d_i . The height of these markers corresponds to the weight of data: the higher the marker “|”, the greater the weight of a datum and the greater its contribution to the KDEw.

The descriptions attached to some figures include tables with

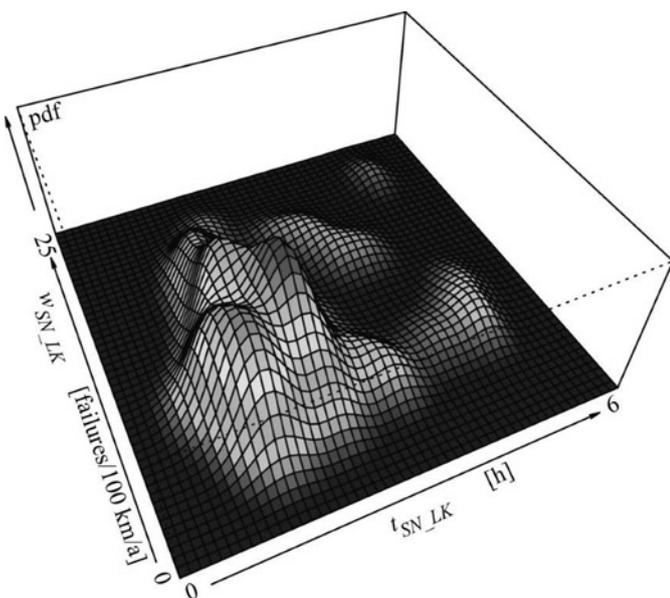


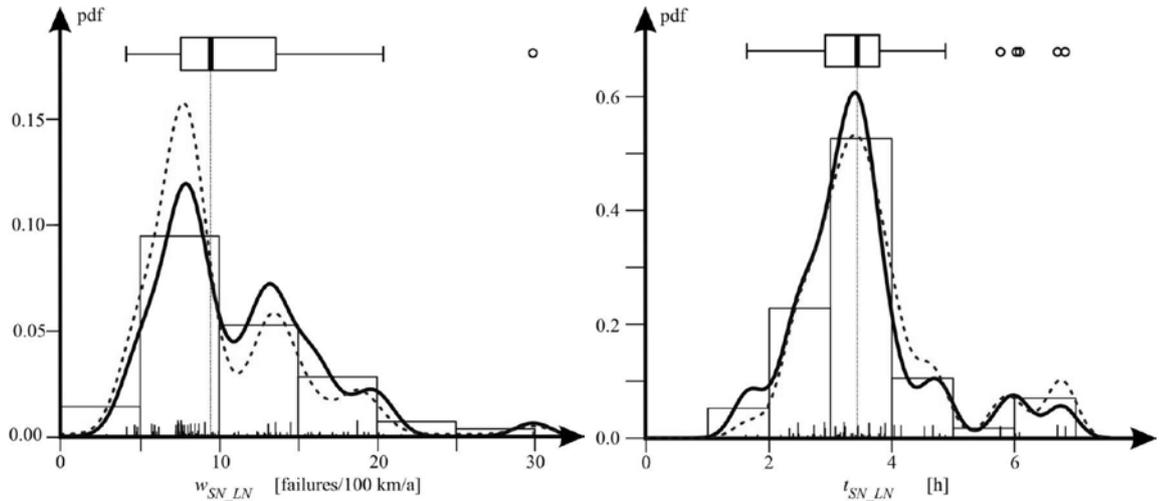
Fig. 2. Two-dimensional pdf obtained by means of KDE for the failure index w_{SN_LK} and the failure duration t_{SN_LK} in MV cable lines

selected statistical measures of the distributions, concerning the minimal and maximal values, Q_1 , Q_3 , the median and the mean value of the variables.

Figures 1 and 2 present the failure analysis of MV cable lines.

The left-hand side graph in Fig. 1 depicts the pdf vs. failure index w_{SN_LK} plot (bandwidth $h = 1.578$), whereas the right-hand side graph represents the pdf vs. the interruption time t_{SN_LK} ($h = 0.2652$) in MV cable lines.

Additionally, a two-dimensional estimation of the pdf was carried out for the two variables under analysis, i.e. the failure index and the interruption duration in MV cable lines, using the function kde2d from the library MASS of the program R. The calculations were performed with the use of a normal kernel, constraining the carrier to



	minimum	1 Qu	median	mean	3 Qu	maximum
w_{SN_LN} [failures/100 km/a]	4.14	7.59	9.50	11.02	13.61	29.89
t_{SN_LN} [h]	1.62	2.90	3.41	3.51	3.79	6.83

Fig. 3. Histogram, boxplot and a pdf obtained by means of KDE (continuous line) and KDEw with the weight $waga_{SN_LN}$ interrupted line) for the failure index w_{SN_LN} and for the interruption duration t_{SN_LN} in MV overhead lines

the positive values, and obtaining the h by means of the Sheather & Jones method. The two-dimensional graph does not take weights into account (the KDE method). Using weights would not have a significant impact on the distribution, except their values at selected points. The two-dimensional pdf representing the failure rate and duration in MV cable lines is presented in Fig. 2.

In the cable networks of the distribution companies under study there were typically about 7 failures per 100 km of cable lines, lasting on average about 1.7 hours per year. Some of the distribution companies however had a higher failure rate in cable lines than others. These companies can be easily identified on the basis of the analysis, for example for the sake of conducting benchmarking research or planning investment.

Figures 3 and 4 present the failure analyses of the MV overhead lines. The failure index w_{SN_LN} as obtained at the bandwidth $h = 1.115$, and its value for the interruption duration t_{SN_LN} MV overhead lines was 0.2402. The pdf plots for the failure index w_{SN_LN} calculated by means of KDE and KDEw differ only slightly and it can be stated that in the companies exploiting MV overhead lines of greater total length the values of w_{SN_LN} were lower.

Fig. 4 presents a two-dimensional pdf representing the failure rate and duration in MV overhead lines.

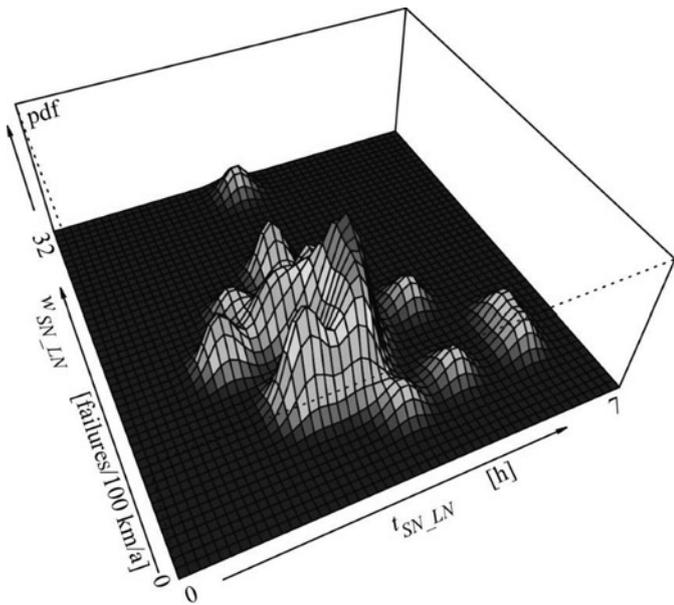


Fig. 4. Two-dimensional pdf obtained by means of KDE for the failure index w_{SN_LN} and the failure duration t_{SN_LN} in MV overhead lines

Failure rates in MV overhead lines differs significantly among the companies under study. The modal value is about 8 failures per 100 km of lines with the duration of about 3.5 hours per year. A few companies however diverged from the others with respect to the failure rate, with company T experiencing almost 30 failures per 100 km lines in 2012, and with companies P and K in 2012, and company H in 2013–2014 having the interruption duration over 6 h.

Transformers are commonly believed to be highly reliable elements of the power system. Figures 5 and 6 present exploitation reliability analysis for MV/LV transformers.

The left-hand side graph in Fig. 5 represents the failure index for MV/LV transformers (bandwidth $h = 0.0924$), whereas the right-hand side graph depicts interruption duration $h = 0.8714$). Despite high differences in the number of transformers among the distribution companies (minimal/maximal share – 1.8%/12.2%) the KDE and KDEw plots are similar. This indicates that the failure rates of MV/LV transformers in particular distribution companies are similar. Only one significant value visible in the plot is an outlier in the distribution of interruption duration (company P in 2013).

The two-dimensional pdf for the MV/LV transformer failure rate presented in Fig. 6 confirms that the population under scrutiny is largely homogeneous.

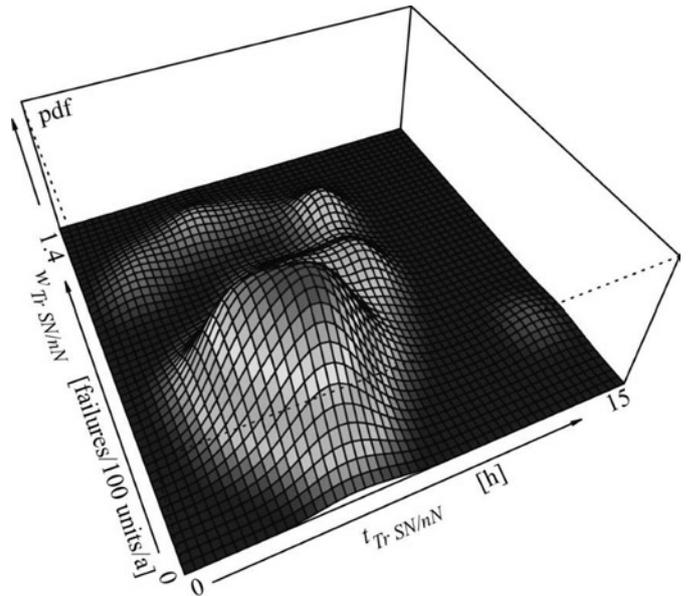
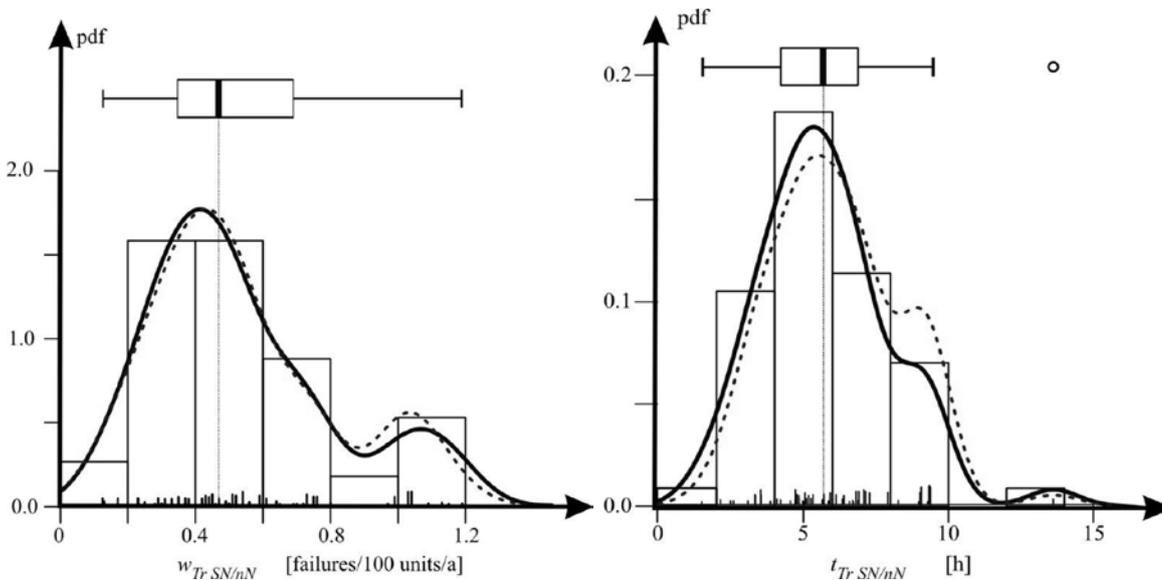


Fig. 6. Two-dimensional pdf obtained by means of KDE for the failure index $w_{Tr_{SN/nN}}$ and for the interruption duration $t_{Tr_{SN/nN}}$ in MV/LV transformers



	minimum	1 Qu	median	mean	3 Qu	maximum
$w_{Tr_{SN/nN}}$ [failures/100 units/a]	0.13	0.35	0.47	0.54	0.69	1.19
$t_{Tr_{SN/nN}}$ [h]	1.49	4.21	5.65	5.77	6.89	13.62

Fig. 5. Histogram, boxplot and a pdf obtained by means of KDE (continuous line) and KDEw with the weight $waga_{SN_LK}$ (interrupted line) for the failure index $w_{Tr_{SN/nN}}$ and for the interruption duration $t_{Tr_{SN/nN}}$ in MV/LV transformers

The plots depicting the failure rate of MV/LV transformers (Figures 5 and 6) indicate that the median of the distribution examined is 5.65 h and the modal value is about 5 h per year. The modal value of the failure index is about 4 failures per 1000 MV/LV transformers.

4. Conclusions

Thorough analysis of the power network failure rate on the basis of exploitation data collected over several years is a key issue for the evaluation of the network conditions. Results of such analyses are important for distribution companies, which can locate weak points in networks, optimize inspection timetables for particular devices and work towards increasing reliability.

The statistical analyses carried out in the study confirm that non-parametric methods are suitable for network reliability evaluation, especially that in practice the number of data is small. The results of non-parametric analysis, including those based on kernel estimators, are both credible and lucid.

Utilizing a few different types of graphic representations of the power network reliability data obtained by non-parametric methods

is a novelty aimed at a simultaneous visualization of a number of statistical measures of a variable distribution under analysis. The graphs shown in this paper can be further extended to include other elements, such as distributions obtained by means of parametric methods.

The analysis of failures in MV networks indicates high dispersion in the values of failure indices and interruption durations among the distribution companies included in the study. The modal value for the MV overhead lines was about 8 failures per 100 km of lines with the failure duration of about 3.5 hours per year. In the case of cable lines the modal value was 7 failures per 100 km of lines with an average failure duration of about 1.7 hours per year. The MV/LV transformers prove to be more reliable, since the modal value of the failure index distribution is 0.4 failures per year per 100 MV/LV transformers, with an average failure duration of 5 h.

References

1. 6th CEER Benchmarking Report on the Quality of Electricity and Gas Supply. Brussels: CEER, 2016.
2. Billinton R, Allan R. Reliability Evaluation of Power Systems. Second edition. New York: Plenum Press, 1996, <https://doi.org/10.1007/978-1-4899-1860-4>.
3. Briš R, Byczanski P, Goňo R, Rusek S. Discrete maintenance optimization of complex multi-component systems. Reliability Engineering & System Safety 2017; 168: 80-89, <https://doi.org/10.1016/j.res.2017.04.008>.
4. Brown R, Ochoa J. Distribution system reliability: default data and model validation. IEEE Transactions on Power Systems 1998; 13: 704-709, <https://doi.org/10.1109/59.667403>.
5. Chojnacki A. Analiza niezawodności eksploatacyjnej elektroenergetycznych sieci dystrybucyjnych. Kielce: Wyd. Politechniki Świętokrzyskiej, 2013.
6. Ilie I, Hernando-Gil I, Djokic S. Reliability equivalents of LV and MV distribution networks. IEEE International Energy Conference and Exhibition ENERGYCON 2012; 343-348, <https://doi.org/10.1109/EnergyCon.2012.6347779>.
7. Kornatka M. Assessment of working conditions of 110 kV line. 16th International Scientific Conference on Electric Power Engineering 2015; 296-300.
8. Kornatka M, Gawlak A. Comparative Analysis of Operating Conditions in Polish Medium-voltage and 110 kV Networks. Proceedings of the 8th International Scientific Symposium on Electrical Power Engineering 2015; 57-60.
9. Kornatka M. The weighted kernel density estimation methods for analysing reliability of electricity supply. 17th International Scientific Conference on Electric Power Engineering 2016; 2-5, <https://doi.org/10.1109/EPE.2016.7521729>.
10. Kolcun M, Kornatka M, Gawlak A, Čonka Z. Benchmarking the reliability of medium-voltage lines. Journal of Electrical Engineering 2017; 68(3): 212-215, <https://doi.org/10.1515/jee-2017-0031>.
11. Kulczycki P. Estymatory jądrowe w analizie systemowej. Warszawa: WNT, 2005.
12. Müller H, Wang J. Density and failure rate estimation with application to reliability. Contribution to the Encyclopedia of Statistics in Quality and Reliability. Department of Statistics University of California; 2007.
13. Paska J. Chosen aspects of electric power system reliability optimization. Eksploatacja i Niezawodność - Maintenance and Reliability 2013; 15 (2): 202-208.
14. Paska J. Niezawodność systemów elektroenergetycznych. Warszawa: Oficyna Wyd. Politechniki Warszawskiej, 2005.
15. R Core Team Copyright C. Środowisko i program R, 1999-2017. Available: <https://cran.r-project.org>.
16. Saleh J, Marais K. Highlights from the early (and pre-) history of reliability engineering. Reliability Engineering & System Safety 2006; 91: 249-256, <https://doi.org/10.1016/j.res.2005.01.003>.
17. Sheather S, Jones M. A reliable data-based bandwidth selection method for kernel density estimation. Journal of the Royal Statistical Society. Series B 1991; 53(3): 683-690.
18. Silverman B. Density Estimation for Statistics and Data Analysis. Chapman and Hall, 1986, <https://doi.org/10.1007/978-1-4899-3324-9>.
19. Statystyka elektroenergetyki polskiej, Warszawa: Agencja Rynku Energii S.A. 2012- 2016.
20. Turlach B. Bandwidth selection in kernel density estimation: A review. Working Paper. Université catholique de Louvain 1994.
21. Venables W, Ripley B. Modern Applied Statistics with S. New York: Springer, 2002, <https://doi.org/10.1007/978-0-387-21706-2>.
22. Wand M, Jones M. Kernel Smoothing. Vol. 60 of Monographs on Statistics and Applied Probability London: Chapman and Hall, 1995, <https://doi.org/10.1007/978-1-4899-4493-1>.
23. Wasserman L. All of Nonparametric Statistics. New York: Springer, 2007.

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