Article citation info:
ZIEJA M, WAŻNY M, STĘPIEŃ S. Outline of a method for estimating the durability of components or device assemblies while maintaining the required reliability level. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2018; 20 (2): 260–266, http://dx.doi.org/10.17531/ein.2018.2.11.

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OUTLINE OF A METHOD FOR ESTIMATING THE DURABILITY OF COMPONENTS OR DEVICE ASSEMBLIES WHILE MAINTAINING THE REQUIRED RELIABILITY LEVEL

ZARYS METODY SZACOWANIA TRWAŁOŚCI ELEMENTÓW LUB ZESPOŁÓW URZĄDZEŃ Z ZACHOWANIEM WYMAGANEGO POZIOMU NIEZAWODNOŚCI*

The paper includes a probabilistic method for evaluating the durability of components and device assemblies which operate under the impact of destructive processes. As a result of these processes, wear that causes deterioration of their cooperation conditions occurs. It is assumed that a component operates reliably when the wear does not exceed the acceptable (limit) values. In mathematical terms, this method is based on a differential equation, after the transformation of which, it is possible to obtain the Fokker-Planck type partial differential equation. The specific solution of this equation allows for obtaining the density function of the probability wear in the normal distribution form. The paper presents two methods for determining the durability. The first one involves the application of the wear density function, and the second one consists in determining the probability density function of the time of reaching the acceptable state, and its use in order to determine the component or assembly durability. The paper presents a numerical example on the aircraft technology operation process.

Keywords: reliability, durability, density function acceptable state, ageing, wear.

Praca zawiera probabilistyczną metodę oceny trwałości elementów lub zespołów urządzeń pracujących w warunkach oddziaływania procesów destrukcyjnych. W wyniku działania tychże procesów następuje zużywanie powodujące pogorszenie warunków ich współpracy. Przyjmuje się, że element pracuje niezawodnie, gdy zużycie nie przekracza wartości dopuszczalnych (granicznych). Metoda od strony matematycznej bazuje na równaniu różnicowym z którego po przekształceniu otrzymuje się równanie różniczkowe cząstkowe typu Fokkera-Plancka. Z rozwiązania szczególnego tego równania otrzymuje się funkcję gęstości prawdopodobieństwa zużywania w postaci rozkładu normalnego. W pracy przedstawione są dwa sposoby wyznaczania trwałości. Pierwszy polega na wykorzystaniu funkcji gęstości zużywania a drugi na wyznaczeniu funkcji gęstości prawdopodobieństwa czasu osiągania stanu dopuszczalnego i zastosowanie jej do wyznaczenia trwałości elementu lub zespołu. W pracy przedstawiono przykład liczbowy dotyczący procesu eksploatacji techniki lotniczej.

Słowa kluczowe: niezawodność, trwałość, funkcja gęstości stan dopuszczalny, starzenie, zużywanie

1. Introduction

In the available literature, it is possible to find a number of papers, which demonstrate the problem of the impact of the external environment, ageing and wear processes on the technical system functioning [4, 9, 13, 16, 17, 21]. Due to technical advancement and a high degree of integration of the devices used on the board of military aircraft, the development of optimal operation models is a complex task. The methods for evaluating the reliability and durability of aviation equipment based on a change in diagnostic parameters are extremely useful within this area [6, 7, 8, 12, 15, 20].

This paper includes a probabilistic method for evaluating the durability of components and the assemblies of the device that operates under the impact of ageing processes (corrosive, wear and other) in the aircraft devices [15, 18, 19]. The technical condition of some aviation equipment can be assessed with the use of diagnostic parameters. This assessment requires knowledge of limit (acceptable) values, for which it is considered that the device or assembly is in the state of usability.

In the offered durability assessment model, the following assumptions are adopted:

– the device's technical condition is defined by one diagnostic parameter "z" in the form of the parameter deviation from the nominal value:

$$z = \left| X - X^{norm} \right|, \tag{1}$$

where:

X – current value of the diagnostic parameter,
X^{norm} – nominal value of the diagnostic parameter;

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

– change in the deviation value of the diagnostic parameter takes place in the entire operation period (operation and standstill);
– "z" parameter is non-decreasing, because it is determined by the absolute value of the difference of the present and nominal values;
– increase speed of the diagnostic parameter deviation in case of random changes can be described by the following relationship:

$$\frac{dz}{dt} = c, \tag{2}$$

where:

c – random variable which characterises the component's susceptibility to ageing changes depending on its features and operating conditions,
t – calendar time.

2. Method for estimating the durability of the device component with the use of the density function of the diagnostic parameter deviation

2.1. Determination of the deviation density function taking into account the relationship (1)

The dynamics of changes in "z" deviation value in random terms will be characterised by the following differential equation:

$$U_{z,t+\Delta t} = (1-P)U_{z,t} + PU_{z-\Delta z,t}, \tag{3}$$

where:

$U_{z,t}$ – probability of the fact that in the moment of t, the diagnostic parameter value adopts z value;
P – probability of the event that the random wear occurs and that in the time interval of Δt, the deviation value will be increased by Δz value;
Δz – deviation increase.

In case, when $P=1$ equation (3) in the function notation will adopt the following form:

$$u(z,t+\Delta t) = u(z-\Delta z,t). \tag{4}$$

The equation (4) has the following form: probability of the fact that in the moment of $t+\Delta t$, the deviation value will be z is equal to the probability of the fact that in the t moment, the deviation value was equal to z-Δz. It means that along with the probability equal to unity, in the time interval of Δt, the deviation will be increased by Δz value.

The equation (4) is transformed into the partial differential equation. Therefore, the following approximations are adopted [1,2]:

$$u(z,t+\Delta t) = u(z,t) + \frac{\partial u(z,t)}{\partial t}\Delta t, \tag{5}$$

$$u(z-\Delta z,t) = u(z,t) - \frac{\partial u(z,t)}{\partial z}\Delta z + \frac{1}{2}\frac{\partial^2 u(z,t)}{\partial z^2}(\Delta z)^2. \tag{6}$$

By using (5) and (6), the equation (4) adopts the following form:

$$\frac{\partial u(z,t)}{\partial t} = -b\frac{\partial u(z,t)}{\partial z} + \frac{1}{2}a\frac{\partial^2 u(z,t)}{\partial z^2}, \tag{7}$$

where:

$b=E[c]$ – average increase in the diagnostic parameter's deviation value per time unit;
$a=E[c^2]$ – average increase square of the diagnostic parameter's deviation per time unit.

We are searching for the solution of a particular equation (7), the one, which at $t \to 0$ is coergent to the so-called Dirac function, i.e. $u(z,t) \to 0$ for $z \neq 0$ and $u(0,t) \to +\infty$, but in a way that the integral of u function is equal to "1" for all $t>0$.

The equation solution (7) adopts the following form for the above specified condition [3, 11, 14]:

$$u(z,t) = \frac{1}{\sqrt{2\pi A(t)}} e^{-\frac{(z-B(t))^2}{2A(t)}}, \tag{8}$$

where:

$$B(t) = \int_0^t b dt = bt = \overline{c}t, \qquad A(t) = \int_0^t a dt = at = \overline{c}^2 t.$$

The value of 0 in lower limits of the integrals means the adopted initial moment of time, according to which the dynamics of changes in the diagnostic parameter's value is considered – it can be, e.g. the moment of putting a given device into operation.

The density function (8) of the diagnostic parameter's deviation increase can be used for assessing the reliability of the considered device component.

2.2. Determination of reliability and durability of the component or device assembly

By having a specific density function, it is possible to record the relationship on reliability and durability due to the time of the parameter's deviation increase to the limit value. The formula adopts the following form:

$$R(t) = \int_{-\infty}^{z_d} u(z,t)dz, \tag{9}$$

where:

$u(z,t)$ – density function specified by the relationship (8);
z_d – acceptable value of the diagnostic parameter's deviation due to safety;
t – calendar time of the device operation.

Figure 1 presents a diagram of the density function course and a way of determining the reliability and durability.

The relationship (9) taking into account (8), adopts the following form:

$$R(t) = \int_{-\infty}^{z_d} \frac{1}{\sqrt{2\pi at}} e^{-\frac{(z-bt)^2}{2at}} dz. \tag{10}$$

By assuming the minimum, required value of R^* reliability, it is possible to determine t^*time, after which the reliability will decrease below the required level. The time t^* can be treated as the durability of a given component for the required, acceptable reliability value.

In this case, it is possible to obtain:

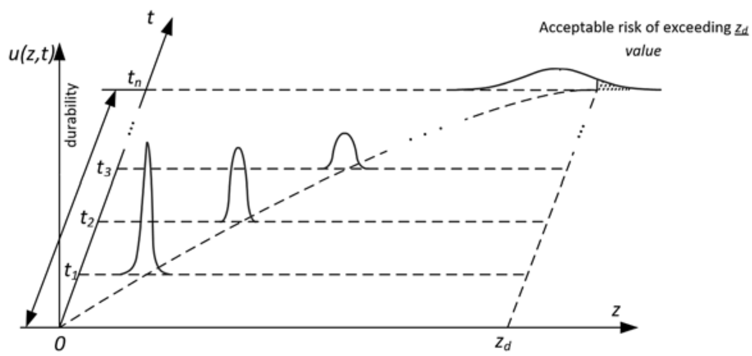

Fig. 1. Diagram of changes in the density function form

$$R^* = \int_{-\infty}^{z_d} \frac{1}{\sqrt{2\pi a t^*}} e^{-\frac{\left(z-bt^*\right)^2}{2at^*}} dz . \quad (11)$$

3. Method for estimating the durability with the use of the density function of the time exceeding the acceptable (limit) state

3.1. Determination of the time distribution of exceeding the acceptable (limit) state

The probability of exceeding the acceptable (limit) value by the diagnostic parameter with the use of the density function of changes in the diagnostic parameter's deviation (8) can be presented in the following form:

$$Q(t;z_d) = \int_{z_d}^{\infty} \frac{1}{\sqrt{2\pi at}} e^{-\frac{(z-bt)^2}{2at}} dz . \quad (12)$$

The density function of the time distribution of the first transition beyond the acceptable value z_d adopts the following form:

$$f(t) = \frac{\partial}{\partial t} Q(t;z_d) = \frac{\partial}{\partial t} \int_{z_d}^{\infty} \frac{1}{\sqrt{2\pi at}} e^{-\frac{(z-bt)^2}{2at}} dz . \quad (13)$$

Thus,

$$f(t) = \int_{z_d}^{\infty} \left\{ \frac{\partial}{\partial t} \left[\frac{1}{\sqrt{2\pi at}} e^{-\frac{(z-bt)^2}{2at}} \right] \right\} dz . \quad (14)$$

By assuming (8) definition, it is possible to obtain:

$$f(t) = \int_{z_d}^{\infty} \left\{ \frac{\partial}{\partial t} u(z,t) \right\} dz . \quad (15)$$

Furthermore, a derivative after the function time (8), adopts the following form:

$$\frac{\partial}{\partial t}\left[u(z,t)\right] = u(z,t)\left(\frac{z^2 - b^2t^2 - at}{2at^2}\right). \quad (16)$$

The relationship (16) was substituted to (14):

$$f(t) = \int_{z_d}^{\infty} \left[u(z,t)\left(\frac{z^2 - b^2t^2 - at}{2at^2}\right)\right] dz . \quad (17)$$

The primary function for the integrand in the relationship (17) is searched for. It is expected that the function in the form of:

$$w(z,t) = u(z,t)\theta(z,t),$$

is a primary function for the integrand of the relationship (17), where $\theta(z,t)$ is a sought unknown function.

That is:

$$\frac{\partial}{\partial z}\left[u(z,t)\theta(z,t)\right] = u(z,t)\left(\frac{z^2 - b^2t^2 - at}{2at^2}\right).$$

After transformations, the following equation is obtained:

$$\frac{\partial \theta(z,t)}{\partial z} - \frac{z-bt}{at}\theta(z,t) = \frac{z^2 - b^2t^2 - at}{2at^2} . \quad (18)$$

Homogeneous equation:

$$\frac{\partial \theta(z,t)}{\partial z} - \frac{z-bt}{at}\theta(z,t) = 0 .$$

Solution of the homogeneous equation:

$$\theta_0(z,t) = Ce^{\frac{z^2-2btz}{2at}},$$

where: C – arbitrary constant

The expected specific solution of the homogeneous equation has the following form:

$$\theta_s(z,t) = -\frac{z+bt}{2t} .$$

It was checked that the equation (18) fulfils the above solution. The general solution of the homogeneous equation:

$$\theta(z,t) = Ce^{\frac{z^2-2btz}{2at}} - \frac{z+bt}{2t} .$$

That is the sought primary function of the integral (17) has the following form:

$$w(z,t)=u(z,t)\left[Ce^{\frac{z^2-2btz}{2at}}-\frac{z+bt}{2t}\right].$$

Thus, by calculating the integral (17) in the specified limits, it is possible to obtain:

$$f(t)=u(z,t)\left[Ce^{\frac{z^2-2btz}{2at}}-\frac{z+bt}{2t}\right]_{z_d}^{\infty}=Cu(z,t)e^{\frac{z^2-2btz}{2at}}\Bigg|_{z_d}^{\infty}-u(z,t)\frac{z+bt}{2t}\Bigg|_{z_d}^{\infty}=$$

$$=C\frac{1}{\sqrt{2\pi at}}e^{-\frac{b^2t}{2a}}\Bigg|_{z_d}^{\infty}-u(z,t)\frac{z+bt}{2t}\Bigg|_{z_d}^{\infty}=0-0+u(z_d,t)\frac{z_d+bt}{2t}.$$

$$f(t)=\frac{z_d+bt}{2t}u(z_d,t). \tag{19}$$

The relationship (19) determines the density function of the time of the first transition of the acceptable (limit) state by the diagnostic parameter's deviation. It should be checked, whether the function (19) is a density function of time of reaching the acceptable (limit) state. The function has the following form:

$$f(t)=\frac{z_d+bt}{2t}\frac{1}{\sqrt{2\pi at}}e^{-\frac{(z_d-bt)^2}{2at}}. \tag{20}$$

The function (20) should meet the condition:

$$\int_0^{\infty}f(t)dt=1. \tag{21}$$

In order to demonstrate the validity (21), the following justification is presented:

$$\int_0^{\infty}\frac{z_d+bt}{2t}\frac{1}{\sqrt{2\pi at}}e^{-\frac{(z_d-bt)^2}{2at}}dt=1. \tag{22}$$

In order to calculate the integral that occurs in the formula (22), the following substitution is used:

$$w=\frac{z_d-bt}{\sqrt{at}}\Rightarrow dt=-\frac{2t\sqrt{at}}{z_d+bt}dw. \tag{23}$$

Transformation of the limits of integration:

$$t=0\Rightarrow w=\infty,$$

$$t=\infty\Rightarrow w=\lim_{t\to\infty}\frac{z_d-bt}{\sqrt{at}}=\lim_{t\to\infty}\frac{-2b\sqrt{at}}{a}=-\infty. \tag{24}$$

After substituting to the output integral, it is possible to obtain:

$$\int_0^{\infty}\frac{z_d+bt}{2t}\frac{1}{\sqrt{2\pi at}}e^{-\frac{(z_d-bt)^2}{2at}}dt=-\int_{\infty}^{-\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{w^2}{2}}dw=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{w^2}{2}}dw \tag{25}$$

The above integral is an integral of $N(0,1)$ normal distribution in the limits from $-\infty$ to $+\infty$ and is equal to unity. On this basis, it can be concluded that:

$$\int_0^{\infty}\frac{z_d+bt}{2t}\frac{1}{\sqrt{2\pi at}}e^{-\frac{(z_d-bt)^2}{2at}}dt=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{w^2}{2}}dw. \tag{26}$$

3.2. Evaluation of the durability of selected components of the aircraft construction with the use of the time distribution of obtaining the acceptable state

The formula for the aircraft's structural component reliability adopts the following form:

$$R(t)=1-\int_0^t f(\tau)d\tau, \tag{27}$$

where the density function $f(t)$ is determined by the following formula (19).

However, the unreliability of the aircraft's structural component can be determined on the basis of the following relationship:

$$Q(t)=\int_0^t\frac{z_d+b\tau}{2\tau}\frac{1}{\sqrt{2\pi a\tau}}e^{-\frac{(z_d-b\tau)^2}{2a\tau}}d\tau. \tag{28}$$

The integral occurring in the relationships (27) and (28) must be transformed to the more convenient form:

$$\int_0^t\frac{z_d+b\tau}{2\tau}\frac{1}{\sqrt{2\pi a\tau}}e^{-\frac{(z_d-b\tau)^2}{2a\tau}}d\tau=\left|\begin{matrix} w=\frac{z_d-b\tau}{\sqrt{a\tau}} & \tau=0\Rightarrow w=\infty \\ d\tau=-\frac{2\tau\sqrt{a\tau}}{z_d+b\tau} & \tau=t\Rightarrow w=\frac{z_d-bt}{\sqrt{at}}\end{matrix}\right|=-\int_{\infty}^{\frac{z_d-bt}{\sqrt{at}}}\frac{1}{\sqrt{2\pi}}e^{-\frac{w^2}{2}}dw.$$

After changing the limits of integration, it is possible to obtain:

$$\int_0^t\frac{z_d+b\tau}{2\tau}\frac{1}{\sqrt{2\pi a\tau}}e^{-\frac{(z_d-b\tau)^2}{2a\tau}}d\tau=\int_{\frac{z_d-bt}{\sqrt{at}}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{w^2}{2}}dw. \tag{29}$$

The reliability of a given component will adopt the following form:

$$R(t)=1-\int_{\frac{z_d-bt}{\sqrt{at}}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{w^2}{2}}dw, \tag{30}$$

or

$$R(t)=\int_{-\infty}^{\frac{z_d-bt}{\sqrt{at}}}\frac{1}{\sqrt{2\pi}}e^{-\frac{w^2}{2}}dw. \tag{31}$$

The integral occurring in the formula (31) is a value of $N(0,1)$ normal distribution function for the argument occurring in the upper limit of integration. Again, by assuming the required minimum value of R^* reliability, it is possible to determine t^* durability.

$$R^* = \int_{-\infty}^{\frac{z_d - bt^*}{\sqrt{at^*}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{w^2}{2}} dw . \quad (32)$$

The use of (11) or (32) formula in the calculation requires estimation of the values of a and b coefficients. This estimation is carried out on the basis of the data obtained from the aircraft operation process.

4. Numerical example

In order to determine the durability of the considered component, it is important to determine (estimate) the values of a and b constants. Therefore, it is assumed that the observation of the tested device in the operation process results in the provision of data on the increase of the diagnostic parameter's deviation value in the form of:

$$\left[(z_0,t_0),(z_1,t_1),(z_2,t_2),\ldots,(z_n,t_n)\right]. \quad (33)$$

The best method for determining "b" and "a" values for the held data is a method that uses a likelihood function. Its form in the general case can be presented as the relationship:

$$L = \prod_{k=0}^{n-1} g\left(t_k, z_k, \theta_1, \theta_2, \ldots, \theta_m\right), \quad (34)$$

where:

$g\left(t_k, z_k, \theta_1, \theta_2, \ldots, \theta_m\right)$ – density function of the total probability of z variable;

$\left(\theta_1, \theta_2, \ldots, \theta_m\right)$. – density function parameters;

z_k – measured wear values of z parameter respectively in the moments of time $(t_1, t_2, \ldots, t_k)$.

Finding $\left(\theta_1^*, \theta_2^*, \ldots, \theta_m^*\right)$ estimates of unknown parameters $\theta_1, \theta_2, \ldots, \theta_m$ with the use of a maximum likelihood method consists in solving the equations in the form of:

$$\frac{\partial lnL}{\partial \theta_j} = 0, \quad (35)$$

where:

j=1,2,…,m;

m – number of parameters characterising the wear process of a given technical object.

In this case, b^* and a^* estimates of unknown b and a parameters with the use of the maximum likelihood method consists in solving the system of equations:

$$\begin{cases} \dfrac{\partial lnL}{\partial b} = 0 \\ \dfrac{\partial lnL}{\partial a} = 0 \end{cases}. \quad (29)$$

By solving the system of equations (29), b^* and a^* are found.

$$b^* = \frac{z_n}{t_n}, \quad (36)$$

$$a^* = \frac{1}{n}\sum_{k=0}^{n-1} \frac{\left[\left(z_{k+1} - z_k\right) - b^*\left(t_{k+1} - t_k\right)\right]^2}{\left(t_{k+1} - t_k\right)}. \quad (37)$$

The component, which was chosen for a numerical example is 12-SAM-28 aircraft battery. Figure 2 shows a change in the time of the averaged battery capacity for held data.

In accordance with the relationship (1), the absolute value of the capacity difference and its nominal value were adopted as "z" diagnostic parameter. The change in time of "z" parameter was presented in Figure 3.

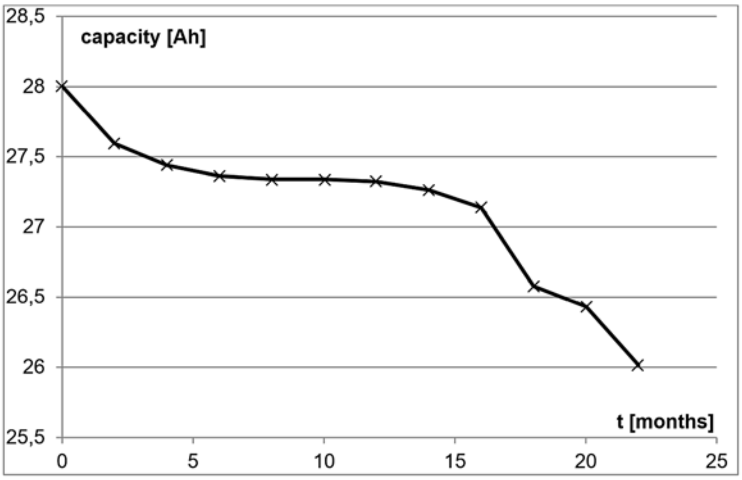

Fig. 2. The course of changes in the averaged capacity of 12-SAM-28 battery

Thus, holding the data describing the values of the diagnostic parameter in the form of $\left[(z_0,t_0),(z_1,t_1),(z_2,t_2),\ldots,(z_n,t_n)\right]$, based on (36) and (37) formulas, the values of the density function coefficients were determined:

$$b^*=0{,}09, \quad a^*=0{,}015. \quad (38)$$

The parameter z_d was determined with the use of technical documentation used for the implementation of maintenance works, in which the information on the acceptable value of the capacity of batteries was provided.

Therefore, by holding the values of parameters b_ε^*, a_ε^*, z_d, they were substituted to (11) or (32) equations by determining the relationship of t^* time on R^* probability – Figure 4. In both cases (relationship (11) or (32)) the same course was obtained.

By assuming the minimum value of R^*=0.99 reliability, the time, to which the diagnostic parameter deviation will not exceed the limit state, in accordance with the assumed probability, was determined:

$$T=63 \text{ [months]}. \quad (39)$$

The obtained value (39) can be used in the technical maintenance depending on the adopted strategy of maintenance. On the basis of the above methodology, it is possible to determine further periods, in which the control of the device diagnostic parameter should be carried out [5, 10].

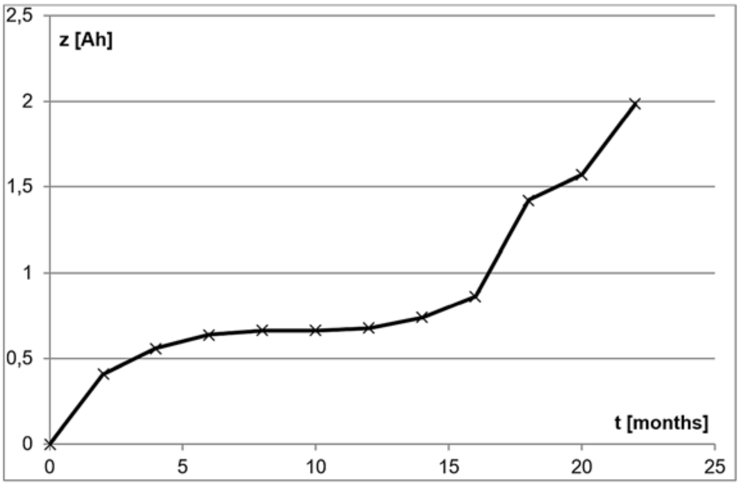

Fig. 3. Change in time of "z" parameter for 12-SAM-28 battery

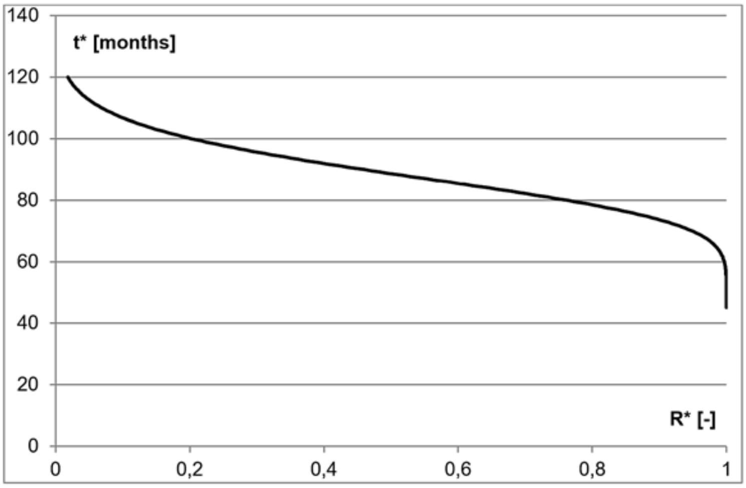

Fig. 4. Relationship of projected t^* durability on R^* reliability

5. Final remarks

In this paper, an overview of the method for estimating the durability of components or assemblies, when the increase speed of changes was of random nature, was presented. However, the method of this change was described by the following simple relationship:

$$\frac{dz}{dt} = c\,,$$

where c was a random variable determining the possibility of the parameter's deviation increase.

It is possible to generalise this method, when the speed of the deviation increase will be described by the following relationships:

$$\frac{dz}{dt} = cz\,, \tag{40}$$

$$\frac{dz}{dt} = ct^{\alpha-1}\,. \tag{41}$$

In the first case, the increase speed of changes will be of random nature similar to the exponential one. In the second case, the increase nature of changes will be similar to the intensity of damage in the Weibull distribution.

In summary, it can be concluded that the presented method seems to be correct and right, and allows to analyse the device technical condition due to the nature of changes in the values of diagnostic parameters. The presented calculation example allowed to carry out the verification of the developed model, and emphasised the developed method's application advantages. This method may be useful in further works on the improvement of both the operational process and the method of using the aircraft with the use of its on-board systems, allowing for determining the time of the device's staying in the state of usability.

Furthermore, the presented method, owing to its universal nature, can be successfully used in order to specify the residual life of any technical object, the technical condition of which is determined on the basis of the analysis of the diagnostic parameters' values.

In this paper, the presented method can be further improved and extended to other cases of increase in random changes of the exponential type. It seems that it can be used for assessing the reliability of mechanical components, in case of considering the propagation of fatigue cracks in the components subjected to the random load, and in case of using the Paris formula in order to specify the crack velocity.

References

1. DeLurgio SA. Forecasting principles and applications. University of Missouri-Kansas City: Irwin/McGraw-Hill, 1998.
2. Franck TD. Nonlinear Fokker-Planck Equations. Fundamentals and Applications. Berlin Heildenberg: Springer-Verlag, 2005.
3. Grasman J, Herwaarden OA. Asymptotic Methods for the Fokker-Planck Equation and the Exit Problem in Applications. Berlin Heildenberg: Springer-Verlag, 1999, https://doi.org/10.1007/978-3-662-03857-4.
4. Idziaszek Z, Grzesik N. Object characteristics deterioration effect on task realizability – outline method of estimation and prognosis. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2014; 16 (3): 433–440.
5. Kinnison H, Siddiqui T. Aviation Maintenance Management. The McGraw-Hill Companies, Inc. 2013.
6. Knopik L, Migawa K. Multi-state model of maintenance policy. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2018; 20 (1): 125–130, https://doi.org/10.17531/ein.2018.1.16.
7. Knopik L, Migawa K. Wdzięczny A. Profit optimization in maintenance system, Polish Martime Research, 2016, 1(89): 193-98.
8. Kołowrocki K, Soszyńska Budny J. Reliability and Safety of Complex Technical Systems and Processes. Springer 2011, https://doi.org/10.1007/978-0-85729-694-8.
9. McPherson JW. Reliability physics and engineering. New York: Springer, 2010, https://doi.org/10.1007/978-1-4419-6348-2.
10. Narayan V. Effective Maintenance Management. New York: Industrial Press Inc., 2012.
11. Pham H. Handbook of Engineering Statistics. London: Springer-Verlag 2006, https://doi.org/10.1007/978-1-84628-288-1.
12. Rasuo B., Duknic G. Optimization of the aircraft general overhaul process. Aircraft engineering and aerospace technology 2013; 85 (5):

343-354, https://doi.org/10.1108/AEAT-02-2012-0017.
13. Restel F. The Markov reliability and safety model of the railway transportation system. Safety and reliability: methodology and applications: proceedings of the European Safety and Reliability Conference, ESREL 2014, 14-18 September, 2015, Wrocław, Poland. CRC Press/ Balkema: 303-311.
14. Risken H. The Fokker-Planck Equation. Methods of Solution and Applications. Berlin Heildenberg: Springer Verlag, 1984, https://doi.org/10.1007/978-3-642-96807-5.
15. Tan CM, Singh P. Time evolution degradation physics in high power white LEDs under high temperature-humidity conditions. IEEE Transactions on Device and Materials Reliability 2014; 14(2): 742-750, https://doi.org/10.1109/TDMR.2014.2318725.
16. Ułanowicz L. Modelling of a process, which causes adhesive seizing (tacking) in precise pairs of hydraulic control devices. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2016; 18 (4): 492-500, https://doi.org/10.17531/ein.2016.4.3.
17. Valis D, Koucky M, Zak L. On approaches for non-direct determination of system deterioration. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2012; 1:33-41.
18. Wang P, Tang Y, Baeb SJ, He Y. Bayesian analysis of two-phase degradation data based on change-point Wiener process. Reliability Engineering & System Safety 2018; 170: 244-256, https://doi.org/10.1016/j.ress.2017.09.027.
19. Wang YS, Zhang CH, Zhang SF, Chen X, Tan YY. Optimal design of constant stress accelerated degradation test plan with multiple stresses and multiple degradation measures. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability 2015; 229(1): 83-93, https://doi.org/10.1177/1748006X14552312.
20. Woch M. Reliability analysis of the PZL-130 Orlik TC-II aircraft structural component under real operating conditions. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2017; 19 (2): 287–295, https://doi.org/10.17531/ein.2017.2.17.
21. Zurek J, Tomaszek H. Zieja M. Analysis of structural component's lifetime distribution considered from the aspect of the wearing with the characteristic function applied. Safety, reliability and risk analysis: Beyond the horizon. Amsterdam: Balkema 2014, 2597-2602.

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