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A COMPREHENSIVE AND PRACTICAL RELIABILITY ALLOCATION METHOD CONSIDERING FAILURE EFFECTS AND RELIABILITY COSTS

KOMPLEKSOWA I PRAKTYCZNA METODA ALOKACJI NIEZAWODNOŚCI UWZGLĘDNIAJĄCA SKUTKI USZKODZEŃ I KOSZTY NIEZAWODNOŚCI

In view of the drawbacks in existing allocation methods which are incomplete considerations and poor practicality, a comprehensive fuzzy allocation method considering failure effects and reliability costs is proposed. Fuzzy linguistics and triangular fuzzy numbers are used to evaluate the uncertainty and subjective factors in allocation process. The traditional risk priority numbers (RPNs) are modified to overcome the shortages which are the same factor weights and equal difference of failure effects in original methods. State of the arts, components intricacy and working conditions are used to construct the reliability costs model, which solves the difficulties of costs statistics and avoids the sophisticated calculations which exist in current allocation methods. The relationship between reliability costs and potential risk of subsystem is studied and the value range of it is given in this paper. A case example is given to illustrative the scientificity and practicability of proposed allocation method.

Keywords: *reliability allocation, reliability cost, failure modes and effect analysis (FMEA), relative reliability, fuzzy methods.*

Ze względu na niedostatki istniejących metod alokacji, które nie dają pełnego obrazu problematyki i mają słabe zastosowanie w praktyce, w artykule zaproponowano kompleksową metodę alokacji opartą na logice rozmytej, uwzględniającą skutki uszkodzeń i koszty niezawodności. W pracy wykorzystano lingwistykę rozmytą i trójkątne liczby rozmyte do oceny niepewności i czynników subiektywnych w procesie alokacji. Zmodyfikowano tradycyjny wskaźnik liczby priorytetowej ryzyka (RPN), co pozwoliło na poprawę mankamentów charakteryzujących oryginalną metodę, t.j. takie same współczynniki wagowe i równoważność skutków uszkodzeń o różnym stopniu ciężkości. Na podstawie wiedzy o stanie techniki, złożoności komponentów i warunkach pracy, skonstruowano model kosztów niezawodności, który rozwiązuje trudności dotyczące sporządzania statystyki kosztów i pozwala uniknąć skomplikowanych obliczeń stosowanych w obecnych metodach alokacji. Zbadano związek między kosztami niezawodności a potencjalnym ryzykiem podsystemu, oraz podano jego zakres wartości. Prezentowane studium przypadku demonstruje możliwe zastosowania i efektywność proponowanej metody.

Słowa kluczowe: *alokacja niezawodności; koszty niezawodności; analiza przyczyn i skutków uszkodzenia (FMEA); względna niezawodność; metody rozmyte.*

1. Introduction

Reliability allocation is a vital step of reliability design. A scientific allocation method can make the system owns the highest reliability while expending the minimum costs. The current allocation methods including traditional methods, risk priority number based (RPN-based) methods and cost-based methods etc. The above-mentioned allocation methods have following shortages ubiquitously:

(1) The traditional allocation methods do not consider the failure effect on system, which makes the results incredible.
(2) The same weight of factors and equivalence relationship between different severities cause the results of RPN-based methods deviating from reality.
(3) It is difficult to obtain the concrete cost statistics, and the calculation process of cost function is too complex, which makes the cost-based allocation methods impractical.

In addition, the current allocation methods just allocate from single aspect, either considering the failure effects or manufacturing costs, which are lack of a comprehensive consideration. Thus, it is difficult to optimize the allocation results.

Therefore, this paper proposes a comprehensive allocation method considering both failure effects and reliability costs. The risk priority numbers in current RPN-based methods are modified to represent the failure effects. Based on the inspiration of generalized cost function, the reliability costs are described by the current and the highest reliability of system. State of the art, working conditions and subsystem intricacy are considered synthetically to construct a semi-quantitative cost function. Value range of the relationship between the costs and potential risk of subsystem is given to avoid the smaller weight factor is too small to take into consideration, which ensures the scientificity of the results.

2. Literature review

Reliability allocation is to distribute the reliability target of system to its component subsystems actually through a specific method. It must satisfy the reliability requirements of system but also a variety of constrains. The most basically requirement is to solve the following inequality:

$$\begin{cases} R_s(R_1, R_2, \cdots, R_i, \cdots, R_n) \geq R_s^* \\ \vec{g}_s(R_1, R_2, \cdots, R_i, \cdots, R_n) \leq \vec{g}_s^* \end{cases}. \tag{1}$$

Where R_s^* is the reliability target of system, $\vec{g}_s^*$ is the constrain of system which includes cost, volume and mass factors etc.., R_i is the reliability target of subsystem i.

Assuming that a serial system is composed by k subsystems, $\lambda^*(t)$ is the target failure rate of system. $\lambda^*_i(t)$ is the allocated failure rate of subsystem i which can be expressed as:

$$\lambda^*_i(t) = \omega_i \cdot \lambda^*(t), \quad t \geq 0,\ i=1,2,\ldots,k. \tag{2}$$

where ω_i is the allocation weight of subsystem i, which can be obtained by the following equation:

$$\omega_i = \frac{n_i}{\sum_{i=1}^{k} n_i}, \quad i=1,2,\ldots,k. \tag{3}$$

where n_i is the estimated value of subsystem i, it could be component numbers in subsystems or failure rate or others [9]. The various allocation methods in the end are the different selection of n_i.

2.1. Traditional allocation methods

Traditional reliability allocation is a method that considers single or multiple factors, judge subsystems by objective or subjective information and finally calculates allocation weight of subsystems through a certain of combination operations. It is aimed at guiding new designs by the current reliability level of systems, that is, the higher reliability the existing subsystem, the lower failure rate the corresponding new subsystem allocated.

State of the art, intricacy, operating time and working conditions of system are closely related to its reliability level, therefore, these factors are always regarded as the consideration factor while lacking of reliability data. Many scholars utilized various operations to allocate subsystems weight by the consideration of above four factors.
or summation [14]:

$$n_i = \sum_{j=1}^{4} A_{ij}, \ i=1,2,\ldots,k. \tag{4}$$

where A_{ij} is the estimated value of factor j for subsystem i, which value rang is the natural number from 1 to 10.
or multiplication [7]:

$$n_i = \prod_{j=1}^{4} A_{ij}, \ i=1,2,\ldots,k. \tag{5}$$

or mixed operations [1]:

$$n_i = A_{i1}(A_{i2} + A_{i3} + A_{i4}), \ i=1,2,\ldots,k. \tag{6}$$

where A_{i1} is the state of the art of subsystem i.

Karmiol [13] allocated the reliability index by Eq.(4) while considering the state of the art, intricacy, criticality and operating time as evaluating factors.

To solve the problem of the same weight between judging factors, the evaluation results are modified by factor weights or expert weights in some papers[19,27]. O'Hagan[20] presents a calculating method of relative weight a_j by maximal entropy, the estimated value n_i of subsystem i is given as:

$$n_i = \sum_{j=1}^{n} a_j A_{ij}, \ i=1,2,\ldots,k. \tag{7}$$

Wang et al.[23] take the failure frequency, failure severity, subsystems maintainability and complexity etc. seven factors into account, evaluate the allocated value of subsystems by Eq. (7) after seven factors were compared each other by both quantitative and qualitative information. Where a_j is the relative weight of factor j to others. A_{ij} is the relative value of subsystem i to subsystem j.

Though the traditional allocation methods can works in a certain extent in system allocation, these methods do not take the failure effects into consideration, nor take the manufacturing costs of system into consideration.

2.2. RPN-based allocation methods

It is inevitable for any systems to have no failure during it runtime. Various failures bring different influences to system, even if the same failure mode occurs in different subsystems. Whatever the failure happens, it would cause a loss to system more or less. Therefore, it must take the potential failures and failure effects into consideration while the reliability of system is allocated.

Recently, some scholars [11, 26] proposed the RPN-based allocation methods. RPN is the scale of failure criticality, measuring the severity(S), occurrence (O) and detection (D) though an ordinal scales from 1 to 10 in the failure modes and effects analysis (FMEA) of system. The RPN of failure mode j in subsystem i as given below:

$$RPN_{ij} = S_{ij} \times O_{ij} \times D_{ij}, \tag{8}$$

When the detection is considered in the severity of failures[5,11], the Eq. (8) can be rewritten as below:

$$RPN_{ij} = S_{ij} \times O_{ij}, \tag{9}$$

Assuming that there is N failure modes in the system, Itabashi-Campbell [11] proposed the estimated value of subsystem i can be given by Eq.(10) or Eq.(11) according to the different intentions of allocators.

$$n_i = B_i, \tag{10}$$

$$n_i = 1 - \frac{B_i}{\sum_{i=1}^{k} B_i}, \tag{11}$$

where:

$$B_i = \frac{1}{N}\sum_{j=1}^{N} S_{ij} \times O_{ij} \ , \tag{12}$$

Many researchers [18,24] point out it is unreasonable to give the same weight to risk factors, for instance, the failure mode S_1=2, O_1=8 and S_2=8, O_2=2 has the same RPN in this manner though it was not the case in reality.

To overcome the defects of this method, a new allocation approach was presented by Kim et al. [15]. The original severity is modified by an exponential function, assuming that S_{ij} is the original severity of failure mode *j* in subsystem *i*, the new severity is given as :

$$\tilde{S}_{ij} = \exp(\alpha S_{ij}) \ , \tag{13}$$

where *a* is the severity coefficient, which is depended on the designer intention. *a* must get a higher value while the designer take failure effects more seriously and vice versa.

The evaluation criterion of subsystem *i* is given by:

$$n_i = \frac{1}{m_i \tilde{S}_i F_i} \ , \tag{14}$$

where:

$$\tilde{S}_i = \max(\tilde{S}_{i1}, \tilde{S}_{i2}, ..., \tilde{S}_{iN_i}) \ , \tag{15}$$

$$j_i = \arg\max_j \tilde{S}_{ij} \ , \tag{16}$$

m_i is the number of failure mode which having the same severity with $\tilde{S}_i$. F_i is the frequency ratio of failure mode j_i in subsystem *i*.

Though this method solves the shortcoming of equal weighed in general RPN-based methods, it is still unreasonable. The evaluation values in the paper are specific numbers which is far away from the actual due to the subjectivity and uncertainty in judgment processes [2,24]. Furthermore, whatever the RPN-based allocation methods only consider the failure effects on system, and ignore the necessary manufacturing costs of system with a specific reliability during it produced.

2.3. Cost-based allocation methods

Generally speaking, everybody wants the system with higher reliability, but the higher reliability of system, the more manufacturing costs needed, and sometimes even lose more than gained. Therefore, the manufacturing cost is the essential factor that must be taken into accounts in any systems development.

The current cost-based allocation methods mainly focused on the optimal planning of allocation, there are two main ways to consider the costs. The one is regarding costs as a specific constant which obtained from statistics or assumption, the other is considering the cost as an increasing function with the reliability of system[3,8,10,12, 21].

Todinov [22] regards the costs and losses of system as the consideration factors in allocation. Assuming Q_i is the manufacture costs of subsystem *i,* the losses caused by failures of subsystems is the constant *L*, the total costs C_i of subsystems as given below:

$$C_i = Q_i + L \ , \tag{17}$$

Wang et al. [23] weight the costs through cost sensitivity, which is obtained from the experts experience by a scale of 0 to 1 value to represents the relationship between the costs and reliability of subsystem *i.* :

$$C_i = \frac{\Delta C_i}{\Delta R_i} \ , \tag{18}$$

where ΔC_i is the increased costs of subsystem *i*. ΔR_i is the improved reliability of subsystem *i*.

In the actual project, however, the cost of systems is hard to collect with the changing of technological and price level. In addition, it is unreasonable to treat the costs of various failures as a constant while there is a big difference effects between all kinds of failure modes.

Dale et al. [4] proposed the six basic properties of cost function in 1986, regarding costs as the increasing function with reliabilities. And then many scholars set up cost function model on this basis. Based on the six properties, Li et al. [17] establish the cost function of diesel engine as:

$$c(R_i) = f_i \ln \frac{R_{i,\max} - R_{i,\min}}{R_{i,\max} - R_i} \ , \tag{19}$$

where R_i is the allocated reliability of subsystem *i*. f_i is the cost coefficient of subsystem *i,* where $0<f_i<1$. $R_{i,\max}$ and $R_{i,\min}$ is the maximum reliability under the current technologies and the current reliability of subsystem *i* respectively. $c(R_i)$ is the improvement costs of subsystem *i* from the reliability $R_{i,\min}$ to R_i.

According to the three properties mentioned in [16] that a cost function must be a positive definite function and non-decreasing and increasing rapidly as reliability close to 1, Elegbede [6] presents the total costs, which is expressed by:

$$C_s = \sum_{i=1}^{s}\sum_{j=1}^{k_i} k_i \cdot h_i\left(\frac{\log(1-R_i)}{k_i}\right) \ , \tag{20}$$

where k_i is the number of components in subsystem *i*. R_i is the reliability of subsystem *i*. *s* is the number of subsystems and $h_i(\cdot)$ is the function with the three properties.

Though cost function could describe the relationship between the costs and reliability of subsystems in a certain extent, it is poor practicability in the practical application due to its complicated computing processes.

Recently, Yadav et al. [25] notice the efforts of reliability improvement and describe it as a function which related to failure rate. The modified evaluation criteria of subsystem *i* based on the method presented by Kim is given by:

$$n_i = \frac{m_i \tilde{s}_i}{\delta_i e_i} \ , \tag{21}$$

where δ_i is the difficulty coefficient of subsystem *i* for improvement. e_i is the effort coefficient where $e_i = \ln \lambda_i \, / \sum_{i=1}^{k} \ln \lambda_i$.

Though the approach presented by Yadav noticed the influence between the reliability of subsystems and improvement efforts, it ignored the real determinant factor of efforts is the current technology level of subsystems rather than failure rate. After the severities and efforts are modified, each subsystem is multiplied with different degree of difficulty coefficients according to the subjective consciousness of the allocators, which is equivalent to modify the efforts twice. It is no doubt to increase the subjectivity in the allocation process, resulting in lower credibility of the distribution results.

The deficiencies stated above urgently require a more thoughtful and credible allocation method.

3. Proposed allocation method

Aimed at these defects mentioned above, we present the solution in this section. Fuzzy linguistic is used to describe the uncertainty subjective information in the allocation process. A more practical reliability allocation method which integrates failure modes and the necessary manufacture cost of system in a specific reliability is proposed. The steps and basis of proposed approach are shown in detail as following.

Step 1 Influential factors determination

The effect on system caused by the failures of components is defined as the potential risk (*PR*) of subsystems. Any system is made up of several subsystems, and there are several potential failure modes in each subsystem. The potential risk is determined by both severity and occurrence of failure mode in subsystems. Therefore, the *S* and *O* must be considered in the allocation process. Secondly, the improvement of reliability in any systems must increase the manufacture costs. Everyone expects that the system owns a higher reliability, but it is always not the case due to the constraint of costs even if the existing technological level could achieve. Due to these reasons, the corresponding costs (*C*) of system in a specific reliability must be considered.

The precise costs data of system is hard to collect, and it is not feasibility for various products even the data has gotten. Allocation methods based on cost function are too complicated in computing process to be applied in practical application. From the previous researches (as mentioned before), the necessary costs of reliability improvement is constrained by both the current reliability level and the highest reliability level under the circumstance of subsystem. And the reliability of systems is closely associated with the state of the art (*SA*), subsystem intricacy (*SI*), operating time (*OT*) and environmental conditions (*EC*). For these reasons, we set the four factors as the related factors of the corresponding manufacturing costs of system in a specific reliability. To simplify the allocation process, the operating time is ignored for the reason that it is same in a system even under different technological levels. Finally, the influence set *K* is expressed as

$K=\{PR, C\}=\{(O,S), (SA, SI, EC)\}$={(occurrence, severity), (state of the art, intricacy, environmental condition)}

Step 2 Experts rating

Experts are asked to rate the influence factor set K on the basis of objective information and subjective judgment. Since mainly of the collected data are incomplete or imprecise, also the opinions of design makers are essentially vague, information description using single numbers often leads to errors in judgment. Fuzzy linguistic and triangular fuzzy numbers are used to rate factors in this paper, shown in Table 1 and Figure 1. Specifically, for the failure modes, higher occurrence and severity, higher score. For the rating of subsystems, assume that the best state of the art and environmental condition and the lowest intricacy of subsystems under the existing circumstance get the full marks (10), the closer to the limitations, the higher scores the subsystem rated.

To obtain the clear decision-numbers, the fuzzy rating results must be defuzzified. The current defuzzification methods mainly include the mean of maxima (MOM), center of area (COA) and α-cut etc. [18]. Different methods lead to various results. COA method is applied while considering the demand of simple practicality in allocation process, the expression of COA method is given as:

Table 1 Fuzzy ratio scale and membership function of linguistic terms

Linguistic variable	Triangular fuzzy number
Very low(VL)	(0,0,1)
Low(L)	(0,1,3)
Medium low(ML)	(1,3,5)
Medium(M)	(3,5,7)
Medium high(MH)	(5,7,9)
High(VH)	(7,9,10)
Very high(VH)	(9,10,10)

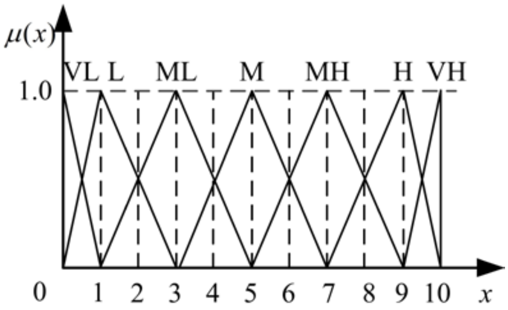

Fig. 1. Membership functions of triangular fuzzy numbers

$$x(a)=a_1+\frac{1}{3}[(a_3-a_1)+(a_2-a_1)]\,, \tag{22}$$

where *x(a)* is the defuzzified value, a_1, a_2, a_3 is the upper limit, most probable value and the lower limit value respectively.

Step 3 PR_i determination

Different severities of failure modes have different effects on system. To solve the unreasonable of factor weights in the allocation of RPN-based methods, the severity is modified as Eq.(23) based on the approach proposed by Kim et al. [15]. Not only does this method make up for the equal weight of factors, but solves the linearity of various severities which are criticized in RPN-based methods:

$$S'_{ij}=a^{S_{ij}}\ ,\ a>1. \tag{23}$$

where *a* is the risk coefficient related to the type of products, the more serious the failure effects of this product, the higher value of *a* must be selected.

The failure mode numbers, severity and occurrence of each failure modes in a subsystem codetermine the potential risk, where the single loss is depend on the severity of failure mode and the loss frequency in a certain time is determined by both the number of failure modes and occurrence. Therefore, we proposed that the potential risk of subsystems should expressed as:

$$PR_i=\sum_{j=1}^{N_i} O_{ij}S'_{ij}\ . \tag{24}$$

Step 4 C determination

Plenty of papers show that it is not the simple linear relationship between costs and the improvement of systems reliability. Costs increase with the improvement of system reliability and would be a very high value while the reliability closes to the ultimate value under the current circumstance. Based on this property, the corresponding costs of system for its reliability we proposed is given as:

$$C'_i = \log_b(1 - \frac{C_i}{C_{\max}}), \tag{25}$$

$$C_i = SA_i \times SI_i \times EC_i, \tag{26}$$

where C'_i is the final cost rating of subsystem i. C_i is the defuzzification rating of subsystem i, the higher C_i indicates that the subsystem owns the higher reliability level and the lower potential for reliability improvement. $C_{\max}$ is the ultimate value of subsystem i in the current technological level. b is the cost coefficient where $b \in (0,1)$.

As shown in Figure 2, the cost described by Eq. (25) has following two features. Firstly, the higher reliability of system, the more costs it needed for the improvement of equal reliability ΔC, that is $\Delta C'_1 > \Delta C'_2$. Secondly, at the same level of reliability, different types of products have different effort coefficients, and the cost of raising the same reliability is different, that is $C'_{i2} > C'_{i3}$.

To avoid ignoring the smaller value of factor due to the larger value of others in allocation process, the potential risk and manufacturing costs of subsystems should kept in the same magnitudes, that is to say, PR_i and C'_i should satisfy the limitation of $10^{-1} \le PR_i/C'_i \le 10$ while b is limited as:

$$\frac{10\ln(1 - C_{i\min}/C_{\max})}{\exp(PR_{\min})} \le b \le \frac{\ln(1 - C_{i\max}/C_{\max})}{10\exp(PR_{\max})}, \tag{27}$$

Step 5 System allocation methods

The reliability of system is allocated to its components and is satisfied by the reliability combination of subsystems in the end. The basic target of reliability allocation is minimizing the possibility damages of system by a reasonable method which requires that the potential risks and the necessary reliability costs of subsystems must be weighted. The larger value of PR, the more serious the possible failure damage of a subsystem is. The smaller value of C', the higher the potential for reliability improvement of a subsystem is. The lower failure rate must be assigned to the subsystem which has higher potential risk and lower manufacturing costs for the sake of optimal results:

$$n_i = \sum_{i=1}^{k} PR_i - PR_i + C'_i, \tag{28}$$

Plugging this into Eq.(3) , the final allocation weight is expressed by:

$$\omega_i = \frac{\sum_{i=1}^{k} PR_i - PR_i + C'_i}{\sum_{i=1}^{k}(\sum_{i=1}^{k} PR_i - PR_i + C'_i)}, \; i=1,2,\ldots,k, \tag{29}$$

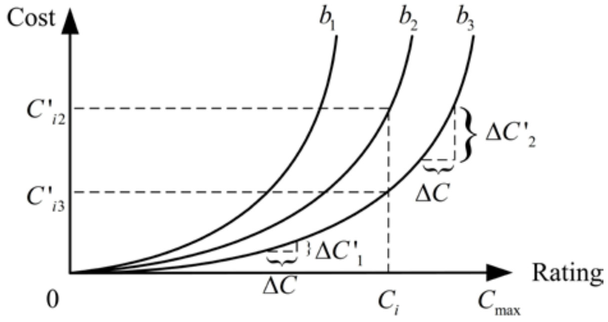

Fig. 2. Transformed cost rating

4. Illustrative example

To illustrate the effectiveness of proposed method further, the reliability allocation of spindle system of numerical control machine is employed in this section.

The spindle system is made up of spindle, bearing group, cooling system, broaching mechanism and rotation driving. Assume that the target failure rate of spindle system $\lambda^*=0.002$. Now three experts (E1, E2 and E3) are asked to rate the failure modes and subsystems by the linguistic variables as shown in Table 1. The rating results are shown in Table 2. The defuzzified results which are translated from linguistic variables to triangular numbers are expressed in Table 3.

Table 4 shows the allocations results of three methods. The results of RPN-based allocation method are calculated by Eq. (11) due to the attention of allocator is minimizing the potential risk of system. Results of proposed method are obtained at the circumstance of $a=\sqrt{e}$, b=0.997. The traditional allocation results are modified based on Eq. (5) for the reason that the rating principle of traditional method is opposite to this paper.

As shown in Table 4 and Figure 3, subsystems are allocated diverse failure rates under different methods. Cooling system is given the highest failure rate in the proposed method which is the same result of RPN-based approach. As shown in Table 3 and Table 4, cooling system has the highest rating of C_i which means owning the highest relative reliability, and it must cost more than others to increase the same reliability. Meanwhile, it has the lowest rating of PR_i which means having the minimal effects on system when failure happens. Therefore, it is more reasonable to assign the highest failure rate to cooling system than others.

The broaching mechanism is given the lowest failure rate in proposed method while the lowest failure is given to the cooling system and spindle in traditional and RPN-based method respectively. It can be explained that, the traditional allocation method is concentrates on the current reliability of system and assigns the lower failure rate to the subsystem with higher reliability, while RPN-based method focuses on the failure effects of subsystems to system and assigns the lower failure rate to the subsystem with more serious effects for minimizing the probable losses. Table 3 and Table 4 show that, cooling system has the highest rating of C_i and the spindle owns the highest mean value of failure modes rating which means that the cooling system has the highest relative reliability and the spindle has the most serious failure effect to system. Therefore, the lowest failure rating is given to cooling system and spindle respectively. However, both traditional and RPN-based approaches are considered only unilaterally, without optimizing allocation results. Though the spindle owns the largest PR, the necessary reliability costs are massive due to its high relative reliability, and it is more unreasonable to allocate the lowest failure rate to cooling system. The broaching mechanism owns a low relative reliability in subsystems which means having a big room for reliability improvement, it more necessary to pay more attention to it while PR is the second in subsystems. Therefore, it is the optimization

Table 2. Ratings of subsystems and failure modes assessed by experts

i	Subsystems	SA_i			EC_i			SI_i			Failure modes	O_{ij}			S_{ij}		
		E1	E2	E3	E1	E2	E3	E1	E2	E3		E1	E2	E3	E1	E2	E3
1	Spindle	MH	H	H	VH	H	MH	MH	VH	H	Orientation error (FM_{11})	H	MH	H	M	M	MH
											Accuracy error (FM_{12})	ML	L	L	M	MH	M
											Abnormal sound (FM_{13})	VH	H	VH	ML	M	M
											Over-heat (FM_{14})	ML	L	L	L	VL	L
2	Bearing group	VH	H	VH	H	M	MH	H	VH	H	Excessive clearances (FM_{21})	H	M	L	H	ML	M
											Ball drops out (FM_{22})	VL	VL	VL	VH	VH	VH
3	Cooling system	H	H	H	VH	H	VH	VH	H	H	Cannot refrigerate (FM_{31})	M	L	ML	MH	ML	ML
											Leak (FM_{32})	VH	VH	H	L	L	ML
4	Broaching mechanism	H	MH	MH	H	MH	MH	H	H	MH	Loose (FM_{41})	MH	H	ML	M	L	MH
											Fracture (FM_{42})	L	VL	L	H	VH	VH
5	Rotation driving	MH	H	MH	H	H	H	H	H	MH	Jam (FM_{51})	L	M	ML	MH	MH	H

result to allocating the minimum failure rate to broaching mechanism.

Table 5 and Figure 4 show the allocated failure rate of subsystems under various cost coefficient b. In order to satisfy the demand that PR_i and C_i should kept in the same magnitudes, b is limited to the interval of [0.9867,0.9998] by using Eq.(27) . As shown in Table 5 and Figure 4, the allocated failure rates change with different value of b. When b is close to the lower limit (b=0.990), the lowest failure rate is assigned to spindle while the highest is assigned to cooling system. When b is close to the upper limit (b=0.999), the lowest failure rate assigned to broaching mechanism while the assigned failure rate of spindle rises to the third. This can be explained that b must get a higher value in Eq. (25) while the production costs are higher or the designers are more concerned about costs than the failure effects on system, in other word, b must get a lower value in Eq. (25) while the failure effects are more serious than its manufacturing costs. When b=0.990, the necessary reliability costs are considered lesser than the losses of failures by designers, failure effects of

Table 3. Defuzzified fuzzy ratings of subsystems and failure modes

i	SA_i	EC_i	SI_i	C_i	Failure modes	O_{ij}	S_{ij}	$\sum_{j=1}^{k} O_i S_i$
1	8.11	8. 67	8. 67	609.6198	FM_{11}	8.34	5.67	98.6734
					FM_{12}	1.79	5.67	
					FM_{13}	9.11	4.33	
					FM_{14}	1.79	1.00	
2	9.33	7.00	9.00	587.7900	FM_{21}	5.00	5.56	30.9911
					FM_{22}	0.33	9.67	
3	8.67	9.33	9.00	728.0199	FM_{31}	3.00	4.33	30.2079
					FM_{32}	9.11	1.89	
4	7.56	7.56	8.11	463.5157	FM_{41}	6.22	4.44	36.7268
					FM_{42}	1.00	9.11	
5	7.56	8. 67	8.11	531.5716	FM_{51}	3.00	7.67	23.01

Table 4. Comparison of the results obtained from different allocation methods

i	PR_i	$\sum_{i=1}^{k} PR_i$ -PR_i	C'_i	ω_i	λ^*_i		
					Traditional	RPN-based	Proposed
1	254.8591896	462.9606091	313.0741438	0.177486987	0.000395632	**0.000372372**	0.000354974
2	122.1218372	595.6979614	294.9641172	0.203703414	0.000399369	0.000419830	0.000407407
3	49.58213563	668.2376630	433.3574532	0.251945931	**0.000375361**	**0.000421856**	**0.000503892**
4	152.3763532	565.4434454	207.2611430	0.176725345	**0.000420645**	0.000404992	**0.000353451**
5	138.8802830	578.9395156	252.4112766	0.190138324	0.000408994	0.000380951	0.000380277
Total	717.8197986	2871.279195	1501.068134	1	0.002		

Table 5. Influence of cost coefficient b on the allocation results

i	λ^*_i			
	b=0.990	b=0.995	b=0.997	b=0.999
1	**0.000335271**	**0.000345062**	0.000354974	0.000380302
2	0.000411972	0.000409703	0.000407407	0.000401539
3	**0.000480593**	**0.000492170**	**0.000503892**	**0.000533843**
4	0.000377952	0.000365777	**0.000353451**	**0.000321954**
5	0.000394213	0.000387288	0.000380277	0.000362362
Total	0.002			

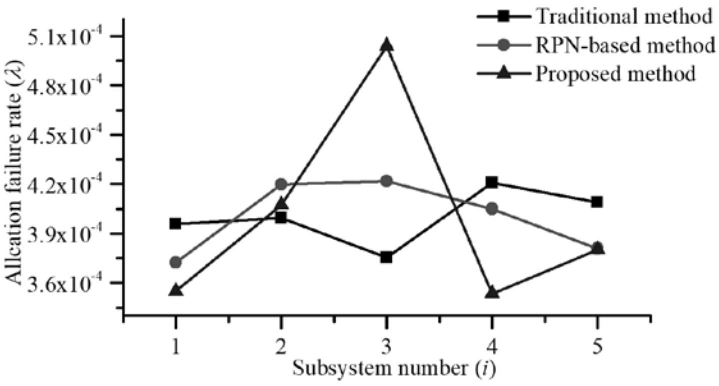

Fig. 3. Comparison of the results obtained from different allocation methods

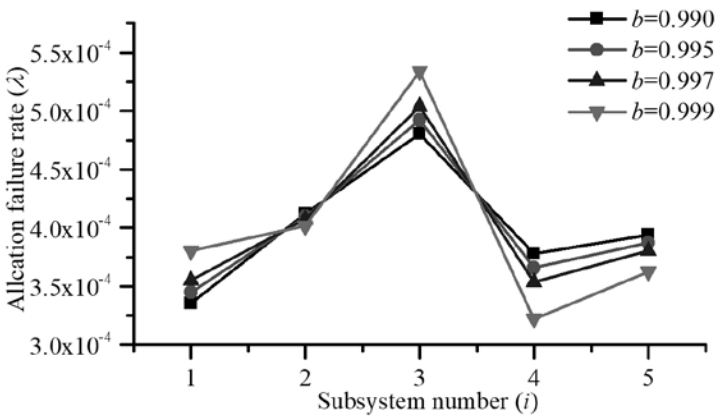

Fig. 4. Influence of cost coefficient b on the allocation results

subsystems is predominant during the allocation process so that the rank of allocation results is similar to the results of RPN-based. As for the difference between broaching mechanism and rotation driving can be explain that the values in RPN-based method are mean values, and the weight of broaching mechanism is raised after averaged. When b=0.999, the designers are more focused on design costs. The failure effects of subsystems are slighter relatively while the costs of subsystems are predominant. The ranks of allocation results are opposite to traditional method. When b exceed the limitation, the lower weight will ignored due to the far less than the higher, which would decrease the credibility of the allocation results.

5. Conclusions

This paper provided a comprehensive reliability allocation method considering failure effects and the necessary costs of system in a specific reliability. The potential risks and reliability costs of subsystems are considered as the allocation factors, the modified RPNs are used to represent the potential risks of subsystems, and the reliability costs model is created by using relative reliability of subsystems. An allocation model is constructed for the purpose of optimizing results which solves the weaknesses of incomplete considerations and poor practicability in the existing reliability allocation methods. It is more flexible while the risk and cost coefficients are considered which can be adjusted with various purposes or allocating objects. Uncertainty factors in allocation process are accounted by fuzzy method and the presented value range of risk and cost coefficients ensures the balance of weight factors, which both enhance the credibility of results.

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References

1. Bracha VJ. The methods of reliability engineering .Machine Design 1964; 7:70–6.
2. Certa A, Hopps F, Inghilleri R, et al. A Dempster-Shafer Theory-based approach to the Failure Mode, Effects and Criticality Analysis (FMECA) under epistemic uncertainty: application to the propulsion system of a fishing vessel. Reliability Engineering & System Safety 2017; 159(69):79, https://doi.org/10.1016/j.ress.2016.10.018 .
3. Chen J, Duan M, Zhang Y. Decision-making of spare subsea trees with multi-restrictive factors in deepwater development. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2016; 18 (4): 590–598, http://dx.doi.org/10.17531/ein.2016.4.14.
4. Dale C J, Winterbottom A. Optimal Allocation of Effort to Improve System Reliability. IEEE Transactions on Reliability 1986; 35(2):188-191, https://doi.org/10.1109/tr.1986.4335401 .
5. Department of the Army. TM 5-689-4. Failure modes, effects and criticality analysis (FMECA) for command, control, communications, computer, intelligence, surveillance, and reconnaissance (C4ISR) facilities 2006 Sep.
6. Elegbede A O C, Chu C, Adjallah K H, et al. Reliability allocation through cost minimization. IEEE Transactions on Reliability 2003; 52(1):106-111, https://doi.org/10.1109/tr.2002.807242 .
7. Gong Q X. Reliability Engineering Handbook. National Defense Industry Press, 2007.

8. Gölbaşı O. Risk-Based Reliability Allocation Methodology to Set a Maintenance Priority Among System Components: A Case Study in Mining[J]. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2017; 19(2):191-202, http://dx.doi.org/10.17531/ein.2017.2.6.
9. Heydorn R P. Reliability Engineering Handbook. Prentice-Hall, 2001.
10. Huang H Z, Liu Z J, Li Y, et al. A warranty cost model with intermittent and heterogeneous usage. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2008; 40(4):9-15.
11. Itabashi-Campbell R R, Yadav O P. System Reliability Allocation based on FMEA Criticality// SAE World Congress & Exhibition 2009.
12. Jaśkowski P. Methodology for enhancing reliability of predictive project schedules in construction. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2015; 17(3):470-479, https://doi.org/ 10.17531/ ein.2015.3.20 .
13. Karmiol ED. Reliability apportionment. Preliminary Report EIAM 5, Task II. General electric. Schenectady, NY 1965.
14. Kececioglu D. Reliability engineering handbook (vol. 1). Prentice Hall, 1992.
15. Kim K O, Yang Y, Zuo M J. A new reliability allocation weight for reducing the occurrence of severe failure effects. Reliability Engineering & System Safety 2013; 117(117):81–88, https://doi.org/10.1016/ j.ress.2013.04.002 .
16. Kuo W, Prasad V R, Tillman F A, et al. Optimal Reliability Design: Fundamentals and Applications. Microelectronics Journal 2001; 32(10):911-911.
17. Li J, Guo J Z, Zhou H Q, et al. Research on the Method of Reliability Allocation of Diesel Engine Base on the Cost Function. Applied Mechanics & Materials 2012; 271-272(2446):1115-1120, https://doi.org/10.4028/www.scientific.net/amm.271-272.1115 .
18. Liu H C, You J X, You X Y, et al. A novel approach for failure mode and effects analysis using combination weighting and fuzzy VIKOR method. Applied Soft Computing 2015; 28(C):579-588, https://doi.org/10.1016/j.asoc.2014.11.036 .
19. Liu Y, Yu W, Li Y, et al. Reliability allocation based on interval analysis and grey system theory. China Mechanical Engineering 2015; 26(11):1521-1526, https://doi.org/10.1016/j.infsof.2016.09.010 .
20. O'Hagan M. Aggregating Template or Rule Antecedents In Real-time Expert Systems With Fuzzy Set Logic// Asilomar Conference on. IEEE Xplore 1988; 681-689, https://doi.org/10.1109/ acssc.1988.754637 .
21. Qiu X, Ali S, Yue T, et al. Reliability-Redundancy-Location Allocation with Maximum Reliability and Minimum Cost Using Search Techniques. Information & Software Technology 2016; 82:36-54, https://doi.org/10.1016/j.infsof.2016.09.010 .
22. Todinov M T. Risk-based reliability allocation and topological optimization based on minimizing the total cost. International Journal of Reliability & Safety 2007; 1(4):489-512(24), https://doi.org/10.1504/ijrs.2007.016261 .
23. Wang Y, Yam R C M, Zuo M J, et al. A comprehensive reliability allocation method for design of CNC lathes. Reliability Engineering and System Safety 2001; 72(3):247-252, https://doi.org/10.1016/ s0951-8320(01)00018-7 .
24. Xiao N, Huang H Z, Li Y, et al. Multiple failure modes analysis and weighted risk priority number evaluation in FMEA. Engineering Failure Analysis 2011; 18(4):1162-1170, https://doi.org/10.1016/ j.engfailanal.2011.02.004 .
25. Yadav O P, Zhuang X. A practical reliability allocation method considering modified criticality factors. Reliability Engineering & System Safety 2014; 129:57-65, https://doi.org/10.1016/j.ress.2014.04.003 .
26. Yadav O P. System reliability allocation methodology based on three- dimensional analyses. International Journal of Reliability & Safety 2007; 1: 360–75, https://doi.org/10.1504/ijrs.2007.014969 .
27. Zhang G B, Jian L, Wang G Q. Fuzzy reliability allocation of CNC machine tools based on task. Computer Integrated Manufacturing Systems Cims 2012; 18(4):768-774.

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