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A NEW ASSESSMENT METHOD OF MECHANISM RELIABILITY BASED ON CHANCE MEASURE UNDER FUZZY AND RANDOM UNCERTAINTIES

NOWA METODA OCENY NIEZAWODNOŚCI MECHANIZMÓW OPARTA NA POMIARZE SZANSY WYSTĄPIENIA ZDARZENIA W WARUNKACH NIEPEWNOŚCI ROZMYTEJ I LOSOWEJ

The traditional reliability analysis methods based on probability theory and fuzzy set theory have been widely used in engineering practice. However, these methods are unable directly measure the uncertainty of mechanism reliability with uncertain variables, i.e., subjective random and fuzzy variables. In order to address this problem, a new quantification method for the mechanism reliability based on chance theory is presented to simultaneously satisfy the duality of randomness and the subadditivity of fuzziness in the reliability problem. Considering the fact that systems usually have multilevel performance and the components have multimode failures, this paper proposes a chance theory based multi-state performance reliability model. In the proposed method, the chance measure is adopted instead of probability and possibility measures to quantify the mechanism reliability for the subjective probability or fuzzy variables. The hybrid variables are utilized to represent the random and fuzzy parameters, based on which solutions are derived to analyze the chance theory based mechanism reliability with chance distributions. Since the input parameters of the model contain fuzziness and randomness simultaneously, an algorithm based on chance measure is designed. The experimental results on the case application demonstrate the validity of the proposed method.

Keywords: *chance measure, reliability assessment, uncertainty quantification, mechanism reliability.*

Tradycyjne metody analizy niezawodności oparte na teorii prawdopodobieństwa i teorii zbiorów rozmytych znajdują szerokie zastosowanie w praktyce inżynierskiej. Jednak metod tych nie można stosować do bezpośredniego pomiaru niepewności niezawodności przy niepewnych zmiennych, tj. subiektywnych zmiennych losowych i rozmytych. Aby zaradzić temu problemowi, przedstawiono nową metodę kwantyfikacji niezawodności opartą na teorii szansy, która jednocześnie spełnia aksjomaty dwoistości losowości oraz subaddytywności związanej z rozmytością w problemach niezawodności. Biorąc pod uwagę fakt, że systemy zazwyczaj charakteryzują się wielopoziomową strukturą, a uszkodzenia elementów składowych mają charakter wieloprzyczynowy, w niniejszym artykule zaproponowano model niezawodności eksploatacji systemu wielostanowego oparty na teorii szansy. W proponowanej metodzie, zamiast miar prawdopodobieństwa i możliwości, do kwantyfikacji niezawodności, w przypadku gdy dane są subiektywne zmienne losowe lub zmienne rozmyte, przyjęto miarę szansy wystąpienia zdarzenia. Do reprezentacji parametrów losowych i rozmytych wykorzystano zmienne hybrydowe, które stanowią podstawę dla wyprowadzenia rozwiązań w celu analizy niezawodności mechanizmu opartej na teorii szansy z rozkładem szans. Ponieważ parametry wejściowe modelu noszą jednocześnie znamiona rozmytości i losowości, opracowano algorytm oparty na mierze szansy. Wyniki eksperymentalne otrzymane na podstawie studium przypadku dowodzą poprawności proponowanej metody.

Słowa kluczowe: *miara szansy, ocena niezawodności, kwantyfikacja niepewności, niezawodność mechanizmu.*

1. Introduction

In practical engineering, various uncertainties are unavoidable due to the complicated environmental factors, incomplete knowledge and inevitable measurement errors [14]. Thus, the mechanism reliability analysis requires proper modeling of all sources of uncertainty. The reliability analysis based on probability theory and Boolean algebra has made many important achievements and has been widely used in engineering practice. The probabilistic reliability is considered as the most valuable issue in engineering. In probabilistic framework, the uncertainties are modeled as random variables or stochastic processes by using a large amount of sample statistical information [1,8]. The application of probabilistic reliability requires sufficient information to construct precise probability density functions of uncertain parameters, but the sample information is not always adequate in the early stage of numerical analysis and optimization design [2].

In practical engineering, besides the randomness that can be modeled by probabilistic theory with probability distribution functions, epistemic uncertainty is another issue, caused by factors such as loss of information, limited knowledge, and inevitable man-made mistakes [11] which cannot be well explained by randomness and probabilistic models. For uncertain problems in practical engineering, a random variable is always employed to represent a kind of subject probability that is conducted by experts' judgments (subjective interpretation) and the uncertainty of this variable is actually the fuzziness that comes from experts' judgments [29]. In order to overcome this shortcoming, Zadeh [36] developed fuzzy set theory in 1965. In 1975, Kaufmann [11] first used the fuzzy theory in reliability engineering. Up until now, the fuzzy theory has received widespread attention for reliability problems with subjective uncertainties [3, 9]. In order to measure a fuzzy event, Zadeh [37] proposed the concept of possibility measure, and afterward many researchers introduced it into fuzzy reliability analysis [4, 32]. Although, the possibility measure has been widely used, it does not obey the law of truth conservation and is inconsistent with the law of excluded middle and the law of contradiction. The main reason is that the possibility measure has no self-duality property. However, a self-dual measure is definitely required in both theory and practice. In order to define a self-dual measure, Liu [15, 16] presented the concept of credibility measure in 2002, and constructed an axiomatic foundation for credibility theory. In addition, Li [17] provided a sufficient and necessary condition for credibility measure. From then on, the credibility theory became a branch of mathematics for studying the behavior of fuzzy phenomena. A survey of credibility theory can be found in Liu [18].

Fuzziness and randomness are the two basic types of uncertainties, and both may appear in a structural system simultaneously. A fuzzy probabilistic model was proposed by Holický [10] in 2006, which combined the two types of uncertainties in the newly defined fuzzy probabilistic measures of structural reliability, the damage function and the fuzzy probability of failure. In most practical situations, some input parameters of system might be represented with probability distribution functions, while some with membership functions [20]. For completeness, the different knowledge conditions for each uncertainty parameter derive "hybrid" uncertain variables in structural reliability analysis [33]. Therefore, randomness and fuzziness should be jointly considered to comprehensively and correctly analyze the reliability of systems, resulting in a hybrid model with random and fuzzy variables [5].

Numerous approaches have been proposed to solve the aforementioned hybrid reliability problems in systems. Most of these approaches separate random and fuzzy parameters based on a double-sampling framework [20]. In order to avoid the deterioration of efficiency and accuracy, several works have attempted to combine the stochastic expansions with traditional optimization methods [7, 30]. They are mainly concentrated on explaining the fuzzy variables by adopting the probability theory, and calculating the reliability based on a probability measure. However, the probability measure with additivity used by these methods fails to satisfy the subadditivity axiom of fuzziness, and the possibility measure cannot satisfy the duality axiom of randomness [11]. Therefore, a reliability quantification model based on probability theory and the one based on credibility theory frequently yield infeasible solutions with large differences and paradoxical results [14]. In other words, neither probability theory nor credibility theory can deal with mechanism reliability problems under epistemic uncertainty with hybrid subjective random and fuzzy variables, because the measures of the two theories cannot satisfy the duality and subadditivity simultaneously [11]. Thus, the development of a framework of hybrid reliability models that integrates the merits of different uncertainties is necessary.

In order to achieve a reasonable solution to these reliability problems, and solve the limitations of the two measures, the chance theory and the chance measure proposed by Liu [21] are introduced in mechanism reliability, including the normality, the duality, the subadditivity, and the product axioms. Chance theory is a hybrid of probability theory and credibility theory. This theory relies on the chance measure to describe the belief degrees of events affected by epistemic uncertainty. It provides a concrete mathematical description of different types of uncertain parameters in the chance space. The "chance measure" in the range of [0, 1] is adopted to represent the chance level about the occurrence of a particular event. Different from the chance theory used in this paper, Liu [28] combines the probability theory and the uncertain theory into a chance theory that also includes the normality, the duality, the subadditivity, and the product axioms. This theory relies on the chance measure to describe the belief degrees of events affected by human uncertainty and objective randomness. Uncertainty theory is a powerful tool for interpreting human uncertainty that was founded by Liu [19] and refined by Liu [26]. The study of uncertain random reliability analysis was started by Wen-Kang [35] with the concept of reliability index. They proposed a formula to calculate the reliability of a Boolean system involving both random and uncertain variables. However, in engineering practice, it is often necessary to evaluate the reliability of the structure in combination with the limit state function, and the parameters in the function may contain various types of uncertainty information (for instance about randomness and fuzziness). The reliability evaluation index based on the Boolean system is not suitable for this situation. Comparatively speaking, both of these chance theories not only have different theoretical foundation, but the types of uncertainty information considered by them are also different. In this paper, the mixture of fuzziness and randomness is mainly considered. Therefore, the chance theory based on the probability measure and the credibility measure is selected. Probability measure is used to deal with the parameters with sufficient information, while the credibility measure is employed to deal with the fuzzy variables. Moreover, Liu [18, 23] introduced a hybrid variable in 2006 as a tool to describe the quantities with fuzziness and randomness, and then proposed a general framework of hybrid programming. Based on the chance theory and the limit state function of structures, this paper explores a new quantification model, and applies it to quantify the performance reliability of structural systems with the hybrid uncertainty problem.

The remainder of this paper is organized as follows. In Section 2.1, some useful concepts in the credibility theory and the chance theory such as credibility measure, hybrid variable, and chance distribution are described; In Section 2.2, formulas based on the chance measure and the chance distributions are derived to quantify the uncertainty reliability and the uncertainty of failure with the performance state function of systems; According to the formulations based on the chance theory and the performance reliability theory, a chance theory based performance reliability model is defined in Section 3; An algorithm based on chance measure is designed in Section 4 followed by an engineering case presented in Section 5; Finally, conclusions are presented in Section 6.

2. Chance theory based quantification for mechanism reliability

2.1. Preliminaries of chance theory

Let Θ be a nonempty set with P as the power set of Θ. For any $A \in P(\Theta)$, Liu [24] presented a credibility measure $Cr\{A\}$ to express the chance that fuzzy event A occurs.

Definition 1 (Credibility measure [18]) The set function Cr is called a credibility measure if it satisfies the normality, monotonicity, self-duality, and maximality axioms.

Theorem 1 (Liu [24]) A fuzzy variable is a (measurable) function from a credibility space (Θ, P, Cr) to the set of real numbers.

Theorem 2 (Liu [24]) Let ξ be a fuzzy variable defined on the credibility space (Θ, P, Cr). Then its membership function is derived from the credibility measure by:

$$\mu(x) = (2Cr\{\xi = x\}) \wedge 1 \tag{1}$$

Theorem 3 (Credibility Inversion Theorem [24]) Let ξ be a fuzzy variable with membership function μ. Then for any set B of real numbers, we have:

$$Cr\{\xi \in B\} = \frac{1}{2}(\sup_{x\in B} \mu(x) + 1 - \sup_{x\in B^c} \mu(x)) \tag{2}$$

Theorem 4 (Product Credibility Axiom [18]) Let Θ_k be nonempty sets on which Cr_k ($k = 1,2,...,n$) satisfy the four axioms, respectively, and $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$. Then, for each $(\theta_1, \theta_2, ..., \theta_n) \in \Theta$

$$Cr\{(\theta_1, \theta_2, ..., \theta_n)\} = Cr_1\{\theta_1\} \wedge Cr_2\{\theta_2\} \wedge \cdots \wedge Cr_n\{\theta_n\} \tag{3}$$

Theorem 5 (Credibility Subadditivity Theorem [16]) The credibility measure is subadditive. That is, for any events A, B

$$Cr\{A \cup B\} \le Cr\{A\} + Cr\{B\} \tag{4}$$

Chance measure was introduced by Liu [21] in 2009 as a tool to describe the quantities with fuzziness and randomness.

Definition 2 (Chance space [16]) Suppose that (Θ, P, Cr) is a credibility space and (Ω, A, Pr) is a probability space. Then, the product $(\Theta, P, Cr) \times (\Omega, A, Pr)$ is called a chance space.

Theorem 6 Let $(\Theta, P, Cr) \times (\Omega, A, Pr)$ be a chance space. A subset $\Lambda \subset \Theta \times \Omega$ is called an event if $\Lambda(\theta) \in A$ for each $\theta \in \Theta$, where $\Lambda(\theta) = \{\omega \in \Omega | (\theta, \omega) \in \Lambda\}$.

Definition 3 (Chance measure [21]) Let $(\Theta, P, Cr) \times (\Omega, A, Pr)$ be a chance space. Then a chance measure of event Λ is defined as:

$$Ch\{\Lambda\} = \begin{cases} \sup_{\theta\in\Theta}(Cr\{\theta\} \wedge \Pr\{\Lambda(\theta)\}), if & \sup_{\theta\in\Theta}(Cr\{\theta\} \wedge \Pr\{\Lambda(\theta)\}) < 0.5 \\ 1 - \sup_{\theta\in\Theta}(Cr\{\theta\} \wedge \Pr\{\Lambda^c(\theta)\}), if & \sup_{\theta\in\Theta}(Cr\{\theta\} \wedge \Pr\{\Lambda(\theta)\}) \ge 0.5 \end{cases} \tag{5}$$

Axioms 1 (Normality axiom). For the universal set $\Theta \times \Omega$, $Ch\{\Theta \times \Omega\} = 1$.

Axioms 2 (Duality axiom).For each event Λ, $Ch\{\Lambda\} + Ch\{\Lambda^c\} = 1$, where Λ^c is the complement set of Λ.

Axioms 3 (Subadditivity axiom).The chance measure is subadditive. That is, for any events Λ_1 and Λ_2

$$Ch\{\Lambda_1 \cup \Lambda_2\} \le Ch\{\Lambda_1\} + Ch\{\Lambda_2\} \tag{6}$$

Definition 4 (Chance distribution [21]) The chance distribution $\Phi : \Re \to [0,1]$ of a hybrid variable ξ is defined by:

$$\Phi(x) = Ch\{(\theta, \omega) \in \Theta \times \Omega | \xi(\theta, \omega) \le x\} \tag{7}$$

As two special hybrid variables, the chance distribution of a random variable ξ is just its probability distribution:

$$\Phi(x) = Ch\{\xi \le x\} = \Pr\{\xi \le x\} \tag{8}$$

and the chance distribution of a fuzzy variable ξ is just its uncertainty distribution:

$$\Phi(x) = Ch\{\xi \le x\} = Cr\{\xi \le x\} \tag{9}$$

In many cases, fuzziness and randomness simultaneously appear in a system. In order to describe these phenomena, Liu [18] introduced a hybrid variable as a measurable function from a chance space to the set of real numbers.

Definition 5 (Hybrid variable [18]) Let ξ be a measurable mapping function from a chance space $(\Theta, P, Cr) \times (\Omega, A, Pr)$ to the set of real numbers. Then, $\xi(\theta, \omega)$ is called a hybrid variable. If $\xi_1, \xi_2, ..., \xi_n$ are hybrid variables, and $f : \Re^n \to \Re$ is a measurable function, then $\xi = f(\xi_1, \xi_2, ..., \xi_n)$ is a hybrid variable defined as:

$$\xi(\theta, \omega) = f(\xi_1(\theta, \omega), \xi_2(\theta, \omega), ..., \xi_n(\theta, \omega)), \forall (\theta, \omega) \in \Theta \times \Omega \tag{10}$$

A hybrid variable $\xi(\theta, \omega)$ degenerates to a random variable when it does not vary with θ and to a fuzzy variable when it does not vary with ω. Fuzzy random variable [34] and random fuzzy [25] variable are instances of hybrid variable.

Example Let $\xi_1, \xi_2, \ldots, \xi_m$ and $\eta_1, \eta_2, \ldots, \eta_n$ be random and fuzzy variables, respectively. If f is a measurable function, then:

$$\tau = f(\xi_1, \xi_2, ..., \xi_m, \eta_1, \eta_2, ...\eta_n) \tag{11}$$

is a hybrid variable determined by:

$$\tau(\theta, \omega) = f(\xi_1(\omega), \xi_2(\omega), ..., \xi_m(\omega), \eta_1(\theta), \eta_2(\theta), ...\eta_n(\theta)) \tag{12}$$

for all $(\theta, \omega) \in (\Theta, P, Cr) \times (\Omega, A, Pr)$.

2.2. Chance theory based reliability

As generally known, reliability can typically be measured by the probability of structure functions that satisfy certain requirements. The structure functions can be expressed by the state function, which is determined by the failure criteria. Assume that $\mathbf{x} = (x_1, x_2, ..., x_n)$ are the n -dimensional input variables denoting the various factors that affect the structure functioning. Then, $G = g(x_1, x_2, ..., x_n)$ is the state function of systems, and $G = 0$ is the limit state function of variable space, which is also called the critical surface. The basic variable space can be divided into two parts, failure region and safe region, by the critical surface. Particularly, when the input variables $\mathbf{x} = (x_1, x_2, ..., x_n)$ consist of the random variables $\mathbf{x_R} = (x_{R1}, x_{R2}, ..., x_{Rn_r})$ and the fuzzy variables $\tilde{\mathbf{x}}_\mathbf{F} = (\tilde{x}_{F1}, \tilde{x}_{F2}, ..., \tilde{x}_{Fn_f})$ simultaneously, the state function of the structural system is a hybrid variable $G = g(\mathbf{x_R}, \tilde{\mathbf{x}}_\mathbf{F})$. With the state function of structural systems under epistemic uncertainty, this paper proposes a reliability definition based on the chance theory.

Definition 6 (Chance theory based reliability) Given a chance space $(\Theta, P, Cr) \times (\Omega, A, Pr)$, let the state function $G = g(\mathbf{x_R}, \tilde{\mathbf{x}}_\mathbf{F})$, where $G > 0$ and $G \le 0$ indicate safe state and failure state, respectively. Then the chance measure of occurrence of a failure event is defined as:

$$Ch_{failure} = Ch\{g(\mathbf{x_R}, \tilde{\mathbf{x}}_\mathbf{F}) \le 0\} \tag{13}$$

From the duality axiom of the chance theory, the chance theory based reliability of a structural system where $G = g(\mathbf{x_R}, \tilde{\mathbf{x}}_\mathbf{F}) > 0$ can be formulated as:

$$Ch_{reliability} = 1 - Ch_{failure} \tag{14}$$

Following the probability definition of reliability, $Ch_{reliability}$ denotes the chance theory based reliability to quantify the uncertainty of a safe event in a system with the numerical value of [0, 1], and $Ch_{failure}$ describes the chance of occurring a failure event. Due to the similarity with the probability definition of failure and reliability, the numerical value of Ch is used instead of frequency to represent the chance with which it is believed that the event will occur. The higher $Ch_{reliability}$ is the more chance the reliability event will happen.

If the chance distribution Φ is given, the failure uncertainty with Eq.(13) can be obtained by:

$$Ch_{failure} = Ch\{G \le 0\} = \Phi(0) \tag{15}$$

Meanwhile, the chance theory based reliability Eq. (14) transform into:

$$Ch_{reliability} = Ch\{G > 0\} = 1 - Ch\{G \le 0\} = 1 - \Phi(0) \tag{16}$$

3. Mechanism reliability analysis method under random and fuzzy uncertainties

3.1. Performance reliability assessment of multi-state system

In the real world, usually systems and their components perform their tasks at several levels of performance or exhibit multiple performance levels or states, and most systems gradually degrade and have a wide range of working efficiencies [13]. Thus, the state of the system may range from perfect functioning to complete failure in engineering practice. The fundamental of multi-state systems was introduced by Murchland [31] in the middle of 1970s. For a multi-state system, $\mathbf{S} = (S_0, S_2, ..., S_q)$ is the vector of function states of the system, it is assumed that there are *w* failure modes, the failure state space is $\bar{\Omega} = \{S_0, ..., S_{w-1}\}$, and $\Omega = \{S_w, ..., S_q\}$ is the state space that the system can work. The critical surface is given by the function $g_i(x_1, x_2, ..., x_n) = 0$ for failure mode $S_i (i = 0,1,..., w-1)$. It should be noted that for each function g_i, it is necessary that all $x_j (j = 1,2,...,n)$ are included in the function. Suppose that $Y = f_t(x_1, x_2, ..., x_n)$ represents a joint probability density function of performance variables at time t, then the probability that the structure will fail in mode S_i $(i = 0,1,..., w-1)$ up to time t is given by (for a larger-is-better case):

$$F_i(t) = \Pr(f_t(x_1, x_2, ..., x_n) \in S_i) = \Pr(g_i(x_1, x_2, ..., x_n) \le 0) \tag{17}$$

where S_i is the space determined by $g_i(x_1, x_2, ..., x_n) \le 0$, $i = 0,1,..., w-1$. The reliability considering only failure mode S_i is given by:

$$R_i(t) = 1 - F_i(t) \tag{18}$$

The overall mechanism reliability considering all *w* failure modes can be evaluated as:

$$R(t) = 1 - F(t) = 1 - \Pr(g_i(x_1, x_2, ..., x_n) \le 0, i = 0,1,..., w-1) \tag{19}$$

According to the above content, chance theory is combined with performance reliability theory and a definition of performance reliability is proposed as follows:

Definition 7 The chance based performance reliability is defined as the chance for the system (component) to function properly over a period of time t. In order to express this relationship mathematically, the variable T is defined to be the time to failure of the system (component), $T \ge 0$ .Then the performance reliability can be expressed as:

$$R(t) = 1 - \Phi(t) = Ch\{T \ge t\} \tag{20}$$

where $R(t) \ge 0$ and $R(0) = 1$.

From definition 7 and Eq.(13), the chance theory based performance reliability function can be represented as:

$$\begin{aligned} R\{Y\} &= Ch\{Y \in \Omega\} \\ &= Ch\{f_i(x_1, x_2, ..., x_n) \in \Omega, w \le i \le q\} \\ &= Ch\{g_i(x_1, x_2, ..., x_n) > 0, w \le i \le q\} \\ &= Ch_{reliability} \end{aligned} \tag{21}$$

where $\Omega = \{S_w, ..., S_q\}$ is the state space that the system can work.

When the performance variables $\mathbf{x} = (x_1, x_2, ..., x_n)$ consist of the random variables $\mathbf{x_R} = (x_{R1}, x_{R2}, ..., x_{Rn_r})$ and the fuzzy variables $\tilde{\mathbf{x}}_\mathbf{F} = (\tilde{x}_{F1}, \tilde{x}_{F2}, ..., \tilde{x}_{Fn_f})$ simultaneously, the randomness and fuzziness should be jointly considered. Therefore, a multi-state performance reliability model with random and fuzzy variables is proposed in the next section to comprehensively and correctly analyze the reliability of system.

3.2. Multi-state performance reliability model considering random and fuzzy variables

A system with two-state spaces is considered as an example, and the multi-state performance reliability model is established as shown in Fig.1.

According to the performance reliability model proposed in literature [6], a multi-state performance reliability model based on hybrid variables is proposed. In this section, the chance theory is applied to establish a new model for the performance reliability of the structural system in the presence of both random and fuzzy information. From the mathematical view, the conceptual system model is shown in Fig.2.

The symbols and the parameters are defined as: t is the operation time of the system; $\mathbf{Y} = (Y_1(t), ..., Y_n(t))$ is the vector of performance parameters of the system; $\mathbf{y} = [y_{ij}(t)]_{n \times q}$, $i = 1,2,...,n$, $j = 0,1,2,...,q$ denotes the criteria of the system; $\{Y(t) \in \Omega\}$ is the set of events that product's state is normal; and $R\{Y(t) \in \Omega\}$ is the performance reliability of the system.

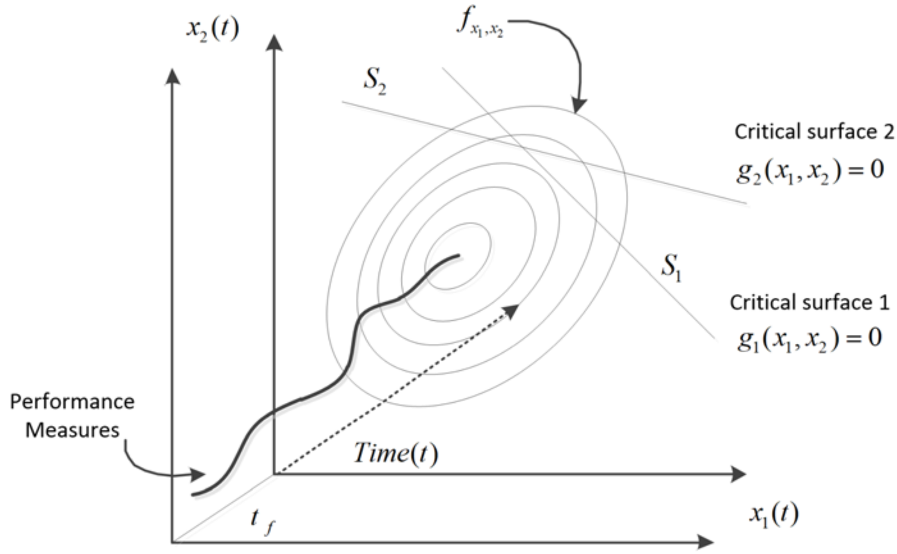

Fig. 1. The two-state performance reliability assessment with multiple input variables

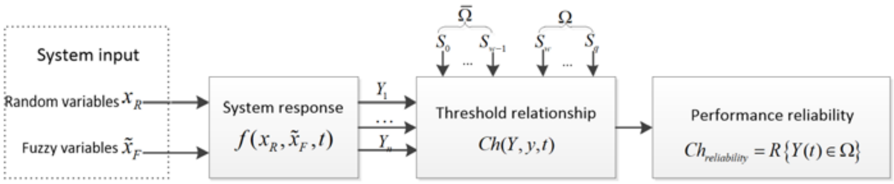

Fig. 2. Performance reliability model considering fuzzy and random factors

In Fig.2, $f(\mathbf{x_R},\tilde{\mathbf{x}}_\mathbf{F},t)$ is the state function of the system. From the definition 5 it can be seen that $f(\mathbf{x_R},\tilde{\mathbf{x}}_\mathbf{F},t)$ is a hybrid variable. The chance measure $Ch(\mathbf{Y},\mathbf{y},t)$ is a transforming operator that can transform inputs $\mathbf{Y}$ and $\mathbf{y}$ into $R\{Y(t)\in\Omega\}$. Thus, the mathematical model of the system is:

$$\begin{cases} Y(t)=f(x_R(t),\tilde{x}_F(t)) \\ Ch(Y(t),y(t))=R\{Y(t)\in\Omega\} \end{cases} \tag{22}$$

4. Simulation method of mechanism reliability model considering random and fuzzy variables

4.1. Statistical analysis model

In order to solve the general fuzzy programming models, Liu [23] proposed a fuzzy simulation algorithm based on credibility distribution and applied it to the solutions of uncertain functions and their expected functions. Later on, Chen [2] proposed a fuzzy based performance reliability algorithm, and designed a Monte Carlo method to estimate the fuzzy based model. According to these two methods, the following procedure may be used to handle the general simulation models considering random and fuzzy variables. θ_k is randomly generated from the credibility space (Θ,P,Cr), and ω_k the probability space (Ω,A,Pr), followed by writing $\nu_k=(2Cr\{\theta_k\})\wedge 1$ and producing $\tilde{x}_{Fk}=\xi(\theta_k)$, and $x_{Rk}=\omega_k$, $k=1,2,...,N$ respectively. Equivalently, $\tilde{x}_{Fk}$ and x_{Rk} are randomly generated and $\nu_k=\mu(\tilde{x}_{Fk})$ is written for $k=1,2,...,N$, where μ is the membership function of $\tilde{x}_F$.

Let the system simulate for N-times under the influence of random and fuzzy causes. The sampled values of characteristics parameters x_R and $\tilde{x}_F$ are $x_{R1r}, x_{R2r},\ldots,x_{Rn_rr}$ and $\tilde{x}_{F1r}$, $\tilde{x}_{F2r},\ldots,\tilde{x}_{Fn_fr}$ in the r-th simulation, respectively. From Eq.(23), the sampled values of performance parameters are:

$$\begin{cases} Y_{1r}=f_1(\mathbf{x_{Rr}},\tilde{\mathbf{x}}_\mathbf{Fr}) \\ \quad ...... \\ Y_{nr}=f_n(\mathbf{x_{Rr}},\tilde{\mathbf{x}}_\mathbf{Fr}) \end{cases} \tag{23}$$

According to the model proposed in Section 3.2, the incident $\mathbf{S}=[S_0,S_1,...,S_q]$ is determined by the performance parameter $\mathbf{Y}=(Y_{1r}(t),...,Y_{nr}(t))$ and criterion $\mathbf{y}=\left[y_{ij}(t)\right]_{n\times q}$, $i=1,2,...,n$, $j=0,1,2,...,q$. The relationship is denoted by matrix:

$$\mathbf{Y}\times\mathbf{y}=\mathbf{S} \tag{24}$$

Eq.(24) is a logic expression that can be explained as follow: the incident S_0 occurs when the element of vector $(Y_{1r},Y_{2r},...,Y_{nr})$ meets the element of vector $(y_{10},y_{20},...,y_{n0})$. $S_1,S_2,...,S_q$ can be explained in a similar way. Alternatively, if one of the vectors $(Y_{1r},Y_{2r},...,Y_{nr})$ meets criterion y_j, and the others meet criterion y_{j+1}, the function state of the system is still S_j.

According to the results of N-times simulations, if the results of k-times simulations belong to Ω, then the reliability obtained by statistical analysis is $\frac{k}{N}$.

4.2. Chance theory based reliability simulation method

Assume that the structural system is reliable when the system state belongs to the set $\Omega=\{S_w,S_{w+1},...,S_q\}$, and is unreliable when the system state belongs to the set $\bar{\Omega}=\{S_1,S_2,...,S_{w-1}\}$. Thus, according to Eq.(21), the critical surface is given by the function $g_i(x_1,x_2,...,x_n)=0$ for state S_i, and the performance reliability of the system is:

$$\begin{aligned} Ch_{reliability} &= Ch\{Y\in\Omega\} \\ &= Ch\left\{f_j(x_R,\tilde{x}_F)\in\Omega, j=1,2,...,n\right\} \\ &= 1-Ch(g_i(x_R,\tilde{x}_F)\le 0, j=1,2,...,n) \end{aligned} \tag{25}$$

In order to compute the uncertain function $Ch\left\{g_j(x_R,\tilde{x}_F)\le 0, j=1,2,...,n\right\}$, using the method proposed in Section 4.1, $\tilde{x}_{F1},\tilde{x}_{F2},...,\tilde{x}_{FN}$ and $x_{R1},x_{R2},...,x_{RN}$ are randomly generated from the credibility space (θ, P, Cr) and the probability space $(\Omega, A, \Pr)$, respectively. For each $\tilde{x}_{Fk}$,

$$\Pr\left\{g_j(x_R, \tilde{x}_{Fk}) \le 0, for \quad all \quad j\right\}, \Pr\left\{g_j(x_R, \tilde{x}_{Fk}) > 0, for \quad some \quad j\right\} \quad (26)$$

is estimated by stochastic simulation via the samples $x_{R1}, x_{R2}, ..., x_{RN}$.

Besides, by using the credibility inversion theorem Eq.(2), the expression in Eq.(5) can be presented as follows:

$$\max_{1\le k\le N} Cr\left\{\tilde{x}_{Fk}\right\} \wedge \Pr\left\{g_j(x_R, \tilde{x}_{Fk}) \le 0, for \quad all \quad j\right\}$$
$$= \max_{1\le k\le N}\left\{\frac{1}{2}(\sup_{\tilde{x}_{Fk}\in\Omega} \mu(\tilde{x}_{Fk}) + 1 - \sup_{\tilde{x}_{Fk}\in\bar{\Omega}} \mu(\tilde{x}_{Fk}))\right\} \wedge \Pr\left\{g_j(x_R, \tilde{x}_{Fk}) \le 0, for \quad all \quad j\right\}$$
$$and$$
$$\max_{1\le k\le N} Cr\left\{\tilde{x}_{Fk}\right\} \wedge \Pr\left\{g_j(x_R, \tilde{x}_{Fk}) > 0, for \quad some \quad j\right\}$$
$$= \max_{1\le k\le N}\left\{\frac{1}{2}(\sup_{\tilde{x}_{Fk}\in\Omega} \mu(\tilde{x}_{Fk}) + 1 - \sup_{\tilde{x}_{Fk}\in\bar{\Omega}} \mu(\tilde{x}_{Fk}))\right\} \wedge \Pr\left\{g_j(x_R, \tilde{x}_{Fk}) > 0, for \quad some \quad j\right\} \quad (27)$$

Then according to Eq.(5), after simulating for N-times, if:

$$\max_{1\le k\le N} Cr\left\{\tilde{x}_{Fk}\right\} \wedge \Pr\left\{g_j(x_R, \tilde{x}_{Fk}) \le 0, for \quad all \quad j\right\} < 0.5 \quad (28)$$

then the value of $Ch\left\{g_j(x_R, \tilde{x}_F) \le 0, j = 1,2,...,n\right\}$ is:

$$Ch\left\{g_j(x_R, \tilde{x}_F) \le 0\right\} = \max_{1\le k\le N} Cr\left\{\tilde{x}_{Fk}\right\} \wedge \Pr\left\{g_j(x_R, \tilde{x}_{Fk}) \le 0, for \quad all \quad j\right\} \quad j = 1,2,...,n \quad (29)$$

Otherwise, if:

$$\max_{1\le k\le N} Cr\left\{\tilde{x}_{Fk}\right\} \wedge \Pr\left\{g_j(x_R, \tilde{x}_{Fk}) \le 0, for \quad all \quad j\right\} \ge 0.5,$$

then:

$$Ch\left\{g_j(x_R, \tilde{x}_F) \le 0\right\} = 1 - \max_{1\le k\le N} Cr\left\{\tilde{x}_{Fk}\right\} \wedge \Pr\left\{g_j(x_R, \tilde{x}_{Fk}) > 0, for \quad some \quad j\right\} \quad j = 1,2,...,n \quad (30)$$

Finally, the chance theory based reliability $Ch_{reliability}$ of the multi-state system can be calculated by Eq.(25).

Considering fuzzy and random variables, the chance theory based simulation method is proposed to conduct reliability calculation in Eq.(25). Five steps are involved in the proposed method. The first three steps are the simulation process, while the last two steps are the analytical process. The flowchart of the proposed method is shown in Fig.3.

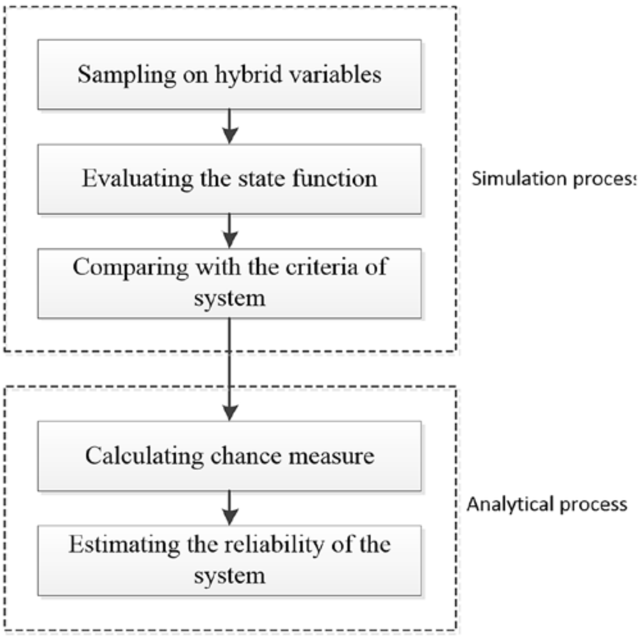

Fig. 3. The flowchart of the chance measure based simulation method

5. An illustrated example

The main failure model of the harmonic gear reducer is due to the increase of the clearance between the components caused by the wear cumulating [27]. The clearance affects the transmission error $\Delta\varphi_{hg}$ and the backlash j_φ of the harmonic gear reducer, reducing the accuracy.

Fig.4 shows a harmonic gear reducer, the wave generator H is active, circular spline is fixed Z_R, and flex spline Z_G is the output.

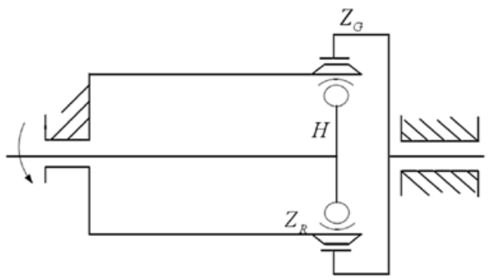

Fig. 4. Sketch map of the harmonic gear reducer

The speed of wave generator is $n = 100r/\min$, load is $T = 10N \cdot m$, the gear pressure angle is $\alpha = 28.6^\circ$, the teeth number of the flex spline and the circular spline are $z_1 = 200$ and $z_2 = 202$, respectively and the harmonic gear ratio is $i = 100$. When the transmission error and the backlash are greater than 0.15°, the harmonic gear reducer is considered to be invalid. Because of the high accuracy and lubrication conditions, the harmonic gear reducer can be considered to work in the stable wear stage. The wear rate is constant and the wear amount increases linearly with time. If the wear rate is μ, the cumulative wear is $W(t) = \mu t$. Therefore, the calculation formula of the transmission error $\Delta\varphi_{hg}$ of the harmonic gear reducer considering the amount of wear is:

$$\Delta\varphi_{hg} = \pm\frac{k_B}{\sqrt{N}}\left[0.2\times(\Delta F_{\Sigma j} + \frac{\pi d_1}{1.2Ui}(\Delta\rho_n + W(t))) + 0.3\times\sqrt{(\Delta F_1)^2 + (\Delta F_2)^2 + (E_{rb})^2 + (\frac{\pi d_1}{1.2Ui})^2[\sum_1^6(\Delta\rho_j)^2 + W(t)^2]}\right]\frac{412.8}{d_1} \quad (31)$$

The backlash of the harmonic gear reducer j_φ is calculated as:

$$j_\varphi = \frac{6.876\tan\alpha(E_M + u_r + 2(\Delta f_a + W(t)) - 2\Delta F_r)}{mz_2} \quad (32)$$

According to the failure criterion, four states of the harmonic gear reducer can be defined:

$S_0 : \Delta\varphi_{hg} < 0.15^\circ$, $j_\varphi < 0.15^\circ$;

$S_1 : \Delta\varphi_{hg} < 0.15^\circ$, $j_\varphi \geq 0.15^\circ$;

$S_2 : \Delta\varphi_{hg} \geq 0.15^\circ$, $j_\varphi < 0.15^\circ$;

$S_3 : \Delta\varphi_{hg} \geq 0.15^\circ$, $j_\varphi \geq 0.15^\circ$.

Thus $\Omega = \{S_0\}$ is the state space that the harmonic gear reducer can work, and $\bar{\Omega} = \{S_1, S_2, S_3\}$ is the state space that the harmonic gear reducer cannot work. Considering the effect of wear, when the operation time is t =8000 hours, the reliability level of the harmonic reducer can be evaluated using the proposed performance reliability simulation method.

Since the manufacturing installation dimensions of flex splines, circular splines, wave generator and bearings include both randomness and fuzziness, the error factors that affect the transmission error and the backlash can be processed as random and fuzzy variables. The random variables in the manufacturing installation error can be regarded as normal distribution and the membership function of fuzzy variables can be determined according to the actual experience and opinions of experts. The distribution parameters of random and fuzzy variables in Eq.(31) and Eq.(32) are presented in Table 1 and 2.

The variables $N, \Delta F_1, \Delta F_2, \Delta\rho_n, \Delta f_a, \Delta F_r$ are defined as fuzzy input variables with the membership functions as follows:

Thus, fuzziness and randomness appear in the structural system simultaneously. An algorithm based on chance theory is designed. The steps in the proposed algorithm are as follows:

Algorithm. Chance theory based performance reliability simulation

Step1: The total simulation number of the system is *N*.

Step2: Set the characteristic parameters of fuzzy and random variables by mechanism designer, as shown in Table 2.

Step3: Refer to the method in Section 4.1 and generate random numbers based on the characteristic parameters as samples of fuzzy and random variables.

Step4: Take the samples into the calculation formulas of error $\Delta\varphi_{hg}$ and j_φ.

Step5: Compare the calculation result with the fault criterion and determine whether the error satisfies the criteria or not. The system is considered as success when criteria is satisfied, otherwise it is failure.

Step6: Repeat steps (3) to (6) *N*-times.

Step7: Use the simulation method of statistical analysis model in Section 4.1 to estimate the number of success *k* via the samples and obtain the reliability by *k*/*N*.

Step8: Estimate the chance theory based reliability using the method of section 4.2.

Using the models and the algorithm in Section 4, the reliability simulation test for a harmonic gear reducer is performed. Firstly, the actual simulation is conducted to decide the critical index and the failure criterion of the system. Then the integrated reliability of the failure and the accuracy error factors are simulated. Using the statistical method, the integrated factors that affect the accuracy of the harmonic gear reducer are considered. After *N*=10000 cycles in simulation, the number of success is 8627. Besides, the chance theory based reliability is obtained to be 0.8491 using Eq.(25). That is, after

Table 1. Random variables

Random variables(unit)	Meaning	Distribution form	Mean value	Coefficient of variation
$E_{rb}(mm)$	The radial motion of the working axis	Normal	0.01	0.050
m	Gear modulus	Normal	0.3	0.033
$\Delta\rho_1$ $\Delta\rho_2$ $\Delta\rho_3$ $\Delta\rho_4$ $\Delta\rho_5$ $\Delta\rho_6(mm)$	Radial error of the wave generator	Normal	0.01	0.100
E_M	The M value deviation	Normal	0.038	0.132
$u_r(mm)$	Radial clearance of flexible bearings	Normal	0.003	0.0333
$\mu(mm)$	Wear rate	Normal	1.140×10^{-6}	0.136

Table 2. Fuzzy variables

Parameters(unit)	Meaning	Value	Type
N	The number of teeth actually engaged	0.4/0.5/0.6	Fuzzy triangular variable
$\Delta F_i(mm)$	Integrated error of circular spline	0.005/0.075/0.01	Fuzzy triangular variable
$\Delta F_2(mm)$	Integrated error of flex spline	0.005/0.075/0.01	Fuzzy triangular variable
$\Delta\rho_n(mm)$	Integrated radial error of the wave generator	0.01/0.02/0.03/0.04	Fuzzy trapezoidal variable
$\Delta f_a(mm)$	Radial error of the long axis	0.014/0.021/0.028	Fuzzy triangular variable
$\Delta F_r(mm)$	Coaxial error of gear ring	0.009/0.015/0.021	Fuzzy triangular variable

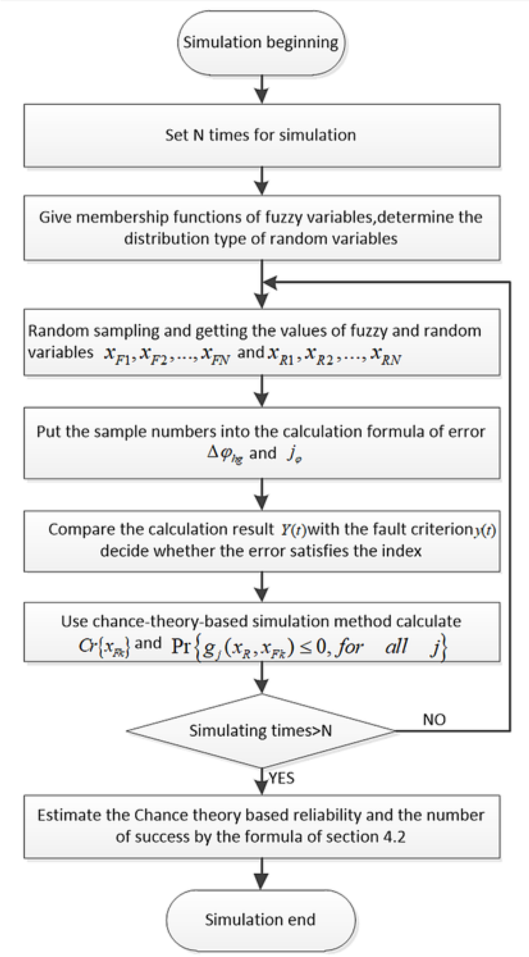

Fig. 5. The flowchart of chance theory based performance reliability simulation

working for 8000 hours, the reliability of harmonic gear reducer drops to 0.8491.

Furthermore, depending on the working time t, and using the above simulation method, a set of corresponding results about the number of success k and the chance theory based reliability can be obtained. Accordingly, the trend of the reliability of the harmonic reducer with time t can be observed. The quadratic polynomial fitting method is used to fit the results. As shown in Fig.6, curve 1 is the result of quadratic polynomial fitting of the statistical analysis, and the curve 2 is the fitting result of the chance based reliability.

Table 3. Reliability at different values of time

Time t	Success number (/10000)	$Ch_{reliability}$	Time t	Success number (/10000)	$Ch_{reliability}$
0	0.9860	0.9728	6000	0.9059	0.8638
500	0.9635	0.9569	8500	0.8507	0.8283
1000	0.9666	0.9426	10000	0.8524	0.8058
1500	0.9546	0.9263	11000	0.8261	0.7848
2000	0.9486	0.9358	12000	0.7448	0.7255
2500	0.9588	0.9453	15000	0.6221	0.6037
3000	0.9441	0.8958	16500	0.5296	0.4714
3500	0.9426	0.9239	17500	0.5273	0.4967
4000	0.9307	0.9105	18500	0.4530	0.3930
5000	0.9276	0.8966	19000	0.4604	0.3991

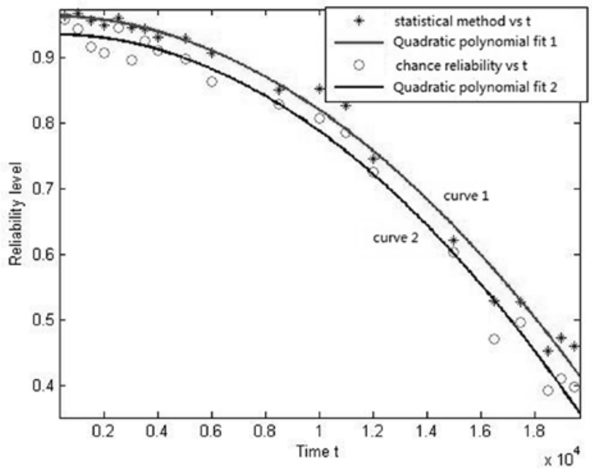

Fig. 6. Time-dependent reliability of the harmonic gear reducer

From the results shown in Table 3 and Fig.6, the chance theory based reliability is 0.9728 at time t=0, which is mainly caused by the manufacturing and installation errors of the flex splines, circular splines, and wave generators of harmonic gear reducers. However, as the working time increases, the wear between the inner wall of the flex spline and the outer wall of the flexible bearing increases with respect to the moving interface, and the reliability decreases gradually. The main reason is that the wear amount gradually accumulates and the wear standard deviation becomes larger over time, which increases the uncertainty of the result. Hence, the influence of the uncertainty on the reliability of the harmonic gear reducer becomes larger. Thus, the results obtained by the chance theory based performance model are consistent with the changes in the reliability of the actual situation affected by the uncertainty factors.

As shown in Fig.6, the trend of the two fitting curves is basically the same. It can be seen from the figure that the reliability obtained by the proposed chance theory based hybrid performance model is smaller than that by the statistical method. Thus, using the statistical method in Section 4.1 to determine the reliability of the structure with hybrid variables will negatively affect the accuracy and the belief degree of the results. That is, the reliability obtained by the proposed method is more conservative and can ensure the structure security to a larger extent. The result of the proposed method presents more realistic estimation of the structure reliability compared to the classic methods.

6. Conclusions

In this paper, the chance theory and the multi-state performance reliability model are used to deal with the epistemic uncertainty, especially with the subjective randomness and fuzziness mixed in the mechanism reliability analysis. A chance theory based quantification method integrating the design for performance reliability is proposed. A performance reliability model is established based on fuzzy and random factors related to the structure failure. The hybrid variables and the performance state function are used to represent the subjective randomness and fuzziness. The chance measure is adopted to quantify the reliability of the structures. In the framework of the chance theory, a definition of the chance theory based performance reliability is provided and a formulation is proposed to analyze the reliability in a structural system from a viewpoint of the chance measure. With the proposed formulations, not only the duality of random events can be described, but also the subadditivity of fuzzy events can be explained. A harmonic gear reducer considering wear failure is provided as a case in order to demonstrate and validate the correctness and effectiveness of the proposed algorithm. The results obtained from the case experiment have demonstrated that the trend of the chance theory based reliability is basically consistent with that of the statistical analysis results. Moreover, the comparative results have shown that the results of the proposed chance theory based simulation method are more conservative and this method can reflect the influence of the hybrid variables on the structural reliability more accurately than the classical methods. Thus, as a new method to deal with fuzzy and random hybrid uncertainties, the proposed chance theory based reliability model can effectively solve the existing problems in engineering practice.

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