**SIMULATION-BASED ANALYSIS OF THE IMPACT OF VEHICLE MASS ON STOPPING DISTANCE**

Results of experimental testing of motor truck tyres in dynamic braking conditions have been presented. With the measurement results being used as an example, higher normal wheel loads have been shown to result in longer time of rise in the longitudinal tangential tyre reaction force and in lower values of both the peak and sliding tyre-road adhesion coefficient. The data presented include results of simulation of the process of emergency braking of a motor truck whose mass can vary within wide limits. It can be seen from these results that an increase in the vehicle mass may considerably lengthen the vehicle stopping distance in emergency braking conditions.

*Keywords*: motor vehicle safety, stopping distance, tyre testing.

**1. Introduction**

The tyre-road adhesion (“grip”) may be decisive for the vehicle behaviour in the conditions of extreme braking or drive along a road bend close to the limiting tyre-road adhesion [13, 14, 15, 17, 3]. In the braking process, the vehicle stopping distance may be expressed as [2, 16, 12]:

\[
s_z = v_0 \left( t_{rk} + t_{rs} + \frac{t_n}{2} \right) + \frac{v_0^2}{2a_h} \tag{1}
\]

where:

- \(a_h\) – average braking deceleration
- \(v_0\) – initial vehicle velocity
- \(t_{rk}\) – driver reaction time
- \(t_{rs}\) – braking system response time
- \(t_n\) – braking force/deceleration rise time

In the emergency braking conditions, the braking deceleration value \(a_h\) is limited by the adhesion force that can develop between the vehicle tyres and the road surface (“tyre-road adhesion force”). In the classic approach, the tyre-road adhesion force of each road wheel depends on the adhesion coefficient \(\mu_j\), which is constant (at wheel lockup), equation (1) takes the following form [2]:

\[
s_z = v_0 \left( t_{rk} + \frac{t_n}{2} \right) + \frac{v_0^2}{2\mu_j g} \tag{2}
\]

As it can be noticed, the vehicle mass is not present in this equation, which suggests that it does not influence the vehicle stopping distance at emergency braking. However, author’s experience and literature data show that the tyre-road adhesion coefficient may vary with increasing normal wheel load [9, 8, 11] and a growth in vehicle mass may lengthen the emergency braking distance [18].

Changes in the mass (and weight) of passenger cars are in general rather small while the mass of present-day motor trucks may vary significantly, as the Maximum Authorized Mass (MAM), corresponding to the Gross Vehicle Weight (GVW), may be up to three times as big as the unladen mass.

Results of author’s experimental research on vehicle tyres show that the normal wheel load value has a considerable impact on the course of changes in the values of the physical quantities that characterize the course of the braking process. This has been shown in Figure 1. The course of the whole process of dynamic wheel braking has been described in other publications [7, 8, 6]. It can be seen in example measurement results that in spite of applying a step signal \(U_h\) controlling the opening of the air brake control valve, the force \(F_{zh}\) that clamps the brake pads on the brake disc rises with a definite time delay and with a specific limited rate until it reaches its maximum value. A similar time delay and characteristic growth rate is observed for the longitudinal reaction \(s_x\) and longitudinal reaction force \(F_{sx}\) (hereinafter referred to as “longitudinal reaction”) transmitted by the tyre. A time history of this force has been shown in Figure 1b in the form of a unit force (\(\mu_k = \frac{F_{zh}}{F_z}\)) vs time curve.

At the specific design of the disc brake calliper, the maximum value of the brake pad clamping force \(F_{zh}\), which determines the wheel braking torque value \(M_p\), is limited by the value of the air pressure applied to the brake actuator.

Therefore, a change in the normal wheel load value \(F_z\) should not be expected to cause changes in the time history and maximum value of the brake pad clamping force \(F_{zh}\) and, thus, of the wheel braking torque value \(M_p\), which limits the wheel deceleration value \(a_h\).
ing torque $M_t$. However, the wheel braking dynamics actually does change, as it can be seen in Figure 2.

The measurement results show that a growth in the normal wheel load $F_z$ during dynamic braking of the wheel causes:

- lengthening of the time of drop in the angular wheel velocity until the wheel is locked up (Figures 2a, 2b);
- lengthening of the time of rise in the value of the longitudinal reaction $F_x$ transmitted by the vehicle tyre until the value of the tyre-road adhesion force for the wheel locked up is reached (Figure 2b);
- decline in the peak ($\mu_1$) and sliding ($\mu_2$) tyre-road adhesion coefficient (Figure 2c).

Based on the presented results of laboratory tests of a wheel with a pneumatic tyre, and with reference to equation (2), a statement may be made that an increase in the vehicle mass directly causing a growth in the value of the normal load on each road wheel of the vehicle may result in an elongation of the vehicle stopping distance in the emergency braking process by:

- lengthening of the time of rise in the braking force up to a value corresponding to that of the tyre-road adhesion force;
- decline in the tyre-road adhesion coefficient.

These conclusions are important from the point of view of safety of vehicle motion and reconstruction of a road event during which emergency braking of a vehicle took place [15, 20, 19]. This problem chiefly applies to motor trucks, where the load mass may exceed the unladen vehicle mass.

The lengthening of the vehicle stopping distance due to an increase in the vehicle mass may be estimated by a simulation method. In the work described herein, simulation tests were planned and carried out which were aimed at presenting the impact of a growth in the vehicle mass and, thus, in the normal loads on vehicle wheels on the elongation of the vehicle stopping distance in an emergency braking process on the grounds of results of experimental tyre tests carried out in laboratory conditions.

2. Impact of normal wheel load on the process of growth in the value of the longitudinal tangential reaction transmitted by the tyre

To enable the execution of the simulation tests planned, a simplified description of the process of growth in the longitudinal tangential reaction transmitted by the tyre during dynamic braking had to be prepared and parametrized.

The process of growth in the longitudinal reaction $F_x$ during dynamic braking of a vehicle wheel may be described in a simplified way by a linear relation, with the use of the following quantities (Figure 3):

- limiting longitudinal reaction value $F_{x,max}$, achieved and maintained during the wheel braking process;
- longitudinal reaction $F_x$ rise time $t_{rh}$.

During the longitudinal reaction rise time $t_{rh}$, the angular wheel velocity $\omega$ is decreasing, which means a simultaneous growth in the longitudinal tyre slip $s_x$ (Figure 3). For the purposes of this analysis, the limiting value $F_{x,max}$ of the longitudinal reaction $F_x$ may be deter-
mined from the tyre-road adhesion coefficient. The results of author’s experimental tests on a vehicle wheel with a tyre in braking conditions as presented in Figure 2 as well as the results of similar tests described in the literature [7, 8] show that the normal wheel load has an impact on the coefficient of adhesion of the individual wheel to the road surface. With a growth in the normal wheel load, both the peak and sliding tyre-road adhesion coefficient (\(\mu_1\) and \(\mu_2\), respectively) are declining. For the emergency braking, where vehicle wheels are locked up, an assumption may be made that the real value of the tyre-road adhesion coefficient is distributed around the sliding coefficient value \(\mu_2\) (Figure 3).

It can also be seen in Figure 3 that the limiting longitudinal reaction value \(F_{x,\text{max}}\) was achieved in a time much shorter than the wheel lock-up time \(t_b\). From the point of view of this analysis, it is important that in both cases under consideration, the longitudinal tangential reaction \(F_x\) reached the hypothetical limiting value \(F_{x,\text{max}}\) corresponding to the sliding tyre-road adhesion force when the angular wheel velocity declined from the initial angular velocity \(\omega_0\) to a value of about \(\omega = 4/5 \omega_0\). Similar relative drops in the angular velocity occur when the wheel is braked from other initial velocity levels.

The impact of the normal wheel load on the tyre-road adhesion in the wheel lock-up condition, determined with taking into account the wheel rolling velocity, has been shown in Figure 4. The sliding tyre-road adhesion coefficient values \(\mu_2\), determined in a wide range of changes in the normal wheel load, may be directly used for estimating the vehicle stopping distance from equation (2).

Based on the conclusions drawn from an analysis of the measurement results, the following simplifying assumptions were adopted for the purposes of carrying out the simulation tests planned:

- in the dynamic braking process, the maximum braking torque \(M_h\) is applied to the wheel in a stepwise manner and its value is determined by the capacity of the wheel brake control mechanism,
- the braking torque rise time resulting from the inertia of the brake control mechanism (\(t_{\text{sh}} \approx 0.2\) s) does not depend on the normal wheel load; this time was taken into account in the simulation process as a constant component of the braking force rise time \(t = t_{\text{sh}} + t_{\text{nh}}\),
- in the period from \(t = 0\) to \(t = t_{\text{nh}}\) the rotational wheel motion is uniformly retarded, i.e. the angular wheel velocity \(\omega\) linearly changes from the initial value of \(\omega = \omega_0\) (defined by the wheel rolling velocity \(v_0\) and the dynamic tyre radius \(r_d\)) to a value of \(\omega \approx 4/5 \omega_0\) (Figure 3),
- during the \(t_{\text{nh}}\) period, the longitudinal tangential reaction transmitted by the tyre is linearly rising from \(F_x = 0\) to the limiting value \(F_{x,\text{max}}\), following the formula \(F_x(t) = \frac{F_{x,\text{max}}}{t_{\text{nh}}} t\) (Figure 3),
- the limiting value of the longitudinal tangential reaction is limited by the sliding tyre-road adhesion force defined by the formula \(F_{x,\text{max}} = \mu_2 F_z\) (Figure 3),
- the value of the sliding tyre-road adhesion coefficient \(\mu_2\) depends on the normal wheel load \(F_z\) according to the relations shown in Figure 4.

![Fig. 3. Simplified description and parametrization of the dynamic wheel braking process](image)

![Fig. 4. Impact of the normal wheel load \(F_z\) on the sliding tyre-road adhesion coefficient values \(\mu_2\) (measurement results obtained in laboratory conditions, with the road surface being represented by a steel drum with a smooth surface)](image)
The increase in the longitudinal reaction $F_x$ rise time $t_{nh}$ caused by a growth in the normal wheel load may be estimated on the grounds of an analysis of the dynamics of the rotational wheel motion during dynamic braking. A schematic diagram of the forces and torques acting on the braked wheel has been presented in Figure 5 [2].

![Fig. 5. Forces and torques acting on a vehicle wheel during the braking process.](image)

**Notation:**
- $M_h$ – wheel braking torque
- $I$ – total moment of inertia of the wheel complete with the rotating elements connected to it
- $Q$ – wheel loading force, i.e. the part of the vehicle weight that is carried by the wheel
- $F_x$ – normal road reaction acting on the wheel
- $F_x$ – longitudinal road reaction acting on the wheel
- $\omega$ – angular wheel velocity
- $\ddot{x}$ – acceleration in the translational wheel motion
- $F_r$ – wheel pushing force, i.e. the part of the vehicle inertia force that acts on the wheel during the braking process

Based on the schematic diagram in Figure 5, the equation of vehicle wheel dynamics in the rotational motion has been formulated as follows:

$$I \ddot{\omega} = -M_h - F_z e + F_x r_d$$ (3)

It may seem that the angular wheel deceleration during the braking process and, thus, the time of reaching the limiting value of the longitudinal reaction $F_x$ are linearly related to the normal wheel load $F_z$. In the conditions of braking, however, the tyre is deformed in longitudinal direction by the longitudinal reaction $F_z$ in accordance with the sense of the reaction force. Therefore, the above equation should be supplemented with an expression representing the impact of the longitudinal tyre deflection. The circumferential elasticity characteristics of present-day tyres are almost linear. Hence, for rough calculation purposes, the longitudinal tyre deflection values, starting from $u_e = 0$, may be expressed in a simplified form as [5, 10]:

$$u_e(t) = \frac{\mu_u}{c_o} \left( \frac{F_z}{c_o} - \frac{F_x}{c_o} \right)$$ (4)

where:
- $c_o$ – circumferential tyre stiffness.

With this supplement, the wheel dynamics equation will take the form:

$$I \ddot{\omega} = -M_h - F_z e + F_x r_d - F_z r_d$$ (5)

Additionally, the following supplementary assumptions may be made:
- For the wheel freely rolling, the shift $e$ of the normal reaction is connected with the wheel rolling resistance coefficient by a relation:

$$e = f_r r_d$$ (6)

- The longitudinal tyre deflection $u_e$ is linearly rising with the value of the longitudinal tangential reaction $F_x$ in accordance with a relation:

$$u_e(t) = \frac{\mu_u}{c_o} t = \frac{F_x}{c_o} t$$ (7)

In consequence, equation (5) will take the form:

$$I \ddot{\omega} = -M_h - F_z \left( f_r r_d - \frac{\mu_u F_x}{c_o} \right) + \frac{\mu_u F_z r_d}{t_{nh}}$$ (8)

By further transformations of this equation, with taking into account the simplifying assumptions adopted previously, the longitudinal reaction $F_x$ rise time $t_{nh}$ may be determined:

$$\frac{d\omega}{dt} = -M_h - F_z \left( f_r r_d - \frac{\mu_u F_x}{c_o} \right) + \frac{\mu_u F_z r_d}{t_{nh}}$$

$$\int_{0}^{\frac{4\mu u}{5}} \frac{d\omega}{dt} = \frac{t_{nh}}{c_o} \left( \frac{\mu u}{c_o} \right) + \frac{\mu u F_z r_d}{t_{nh}}$$

$$\frac{I \mu u}{5} = M_h t_{nh} + F_z \left( f_r r_d - \frac{\mu u F_x}{2c_o} \right) - F_z \left( F_z - \frac{\mu u F_z}{2c_o} \right)$$

$$\frac{I \mu u}{5} = t_{nh} \left( M_h + F_z f_r r_d - F_z^2 \frac{\mu u}{2c_o} - F_z \frac{\mu u F_z}{2c_o} \right)$$

$$t_{nh} = \frac{I \mu u}{5} \left( M_h + F_z f_r r_d - F_z^2 \frac{\mu u}{2c_o} - F_z \frac{\mu u F_z}{2c_o} \right)$$

Results of the estimation of the time $t_{nh}$ of rise in the longitudinal reaction $F_x$ to the limiting value $F_{x, max}$ have been shown in Figure 6.

The value of the moment of inertia of a wheel with a tyre 275/70R22.5 complete with a hub and brake disc and the braking torque value were assumed on the grounds of test results given in other publications [7, 6].

Equation (13) shows a relation between the longitudinal reaction $F_x$ rise time $t_{nh}$ and many factors that characterize the conditions of wheel motion during the braking process. In the equation, direct dependence can be seen between the longitudinal reaction $F_x$ rise time $t_{nh}$ and the initial wheel rolling velocity $v_0$ (connected with the angular wheel velocity $\omega_0$), although the impact of an increase in the initial wheel rolling velocity $v_0$ is partly compensated by the influence of a
decline in the tyre-road adhesion coefficient value $\mu_2$. In effect, the simulation results have revealed that the time $t_{nh}$ of rise in the wheel braking force $F_x$ to the limiting value $F_{x,\text{max}}$ is nonlinearly growing with an increase in the normal wheel load $F_z$ (Figure 6). The nonlinearity of this relation is particularly strong within the range of high values of the normal wheel load. Here, the impact of the circumferential flexibility of the tyre can be clearly seen as this flexibility, through the circumferential tyre stiffness $c_{\theta}$ taken into account in equation (13), can significantly lengthen the estimated value of the time $t_{nh}$ especially at high values of the longitudinal reaction $F_x$, which are fostered by high values of the normal wheel load $F_z$.

The values of the longitudinal reaction $F_x$ rise time $t_{nh}$ are not very big. However, it has been shown that the time $t_{nh}$ is considerably lengthened under the influence of growth in the normal wheel load $F_z$. Such an elongation causes the vehicle braking force rise time $t_{nh}$ to be lengthened, too. Thus, it may contribute to a lengthening of the vehicle stopping distance in accordance with the equation presented previously (2).

3. Evaluation of the impact of vehicle mass on the stopping distance

The normal load on vehicle wheels varies with changes in vehicle mass. Significant changes in vehicle mass may especially occur in the case of motor trucks, whose load capacity may even be twice as high as the unladen vehicle mass.

The results of experimental testing of motor truck tyres and calculation results were used for the simulation of the process of emergency braking of a motor truck with varying mass. Apart from the assumptions presented previously, the following simplifying assumptions had been adopted before a computing application was prepared:
- the vehicle moves on four wheels with comparable characteristics,
- the vehicle mass is uniformly distributed among individual road wheels,
- the driver starts the emergency braking process at the instant when a hazardous situation is noticed ($t = 0$),
- each vehicle wheel is subjected to a braking torque $M_h$ of identical maximum value determined by the capacity of the wheel brake mechanism and its control system,
- the braking intensity is limited by the sliding tyre-road adhesion coefficient $\mu_2$, whose value is determined at the beginning of the braking process for the initial vehicle velocity $v_0$ and depends on the normal load on each vehicle wheel, with the value of this load remaining unchanged during the braking process (in the simplified model adopted),
- the sliding tyre-road adhesion coefficient is identical for each wheel and is determined by vehicle weight and initial braking velocity.

![Fig. 6](image_url) Impact of the normal wheel load $F_z$ and rolling velocity $v_0$ on the time $t_{nh}$ of rise in the wheel braking force $F_x$ to the limiting value $F_{x,\text{max}}$ (results obtained from simulations, for data typical of a medium-capacity motor truck)

![Fig. 7](image_url) Example set of results of calculating the physical quantities that characterize the process of emergency braking of a vehicle ($v_0 = 60 \text{ km/h}$, $m = 8000 \text{ kg}$): a) braking torque acting on wheels, b) braking deceleration, c) vehicle velocity, d) distance travelled, e) Vehicle velocity vs distance travelled.
the dynamic tyre radius is identical for each wheel,
any changes in the normal load on vehicle axles during the braking process were not taken into account,
typical values of the driver reaction time and the braking system response time were adopted [20, 4]; however, these values as constants do not have any impact on the phenomena observed.

A spreadsheet making it possible to carry out the calculations planned was prepared. The simulation of emergency braking of a vehicle was based on equation (2) and on the modelling data described in Section 2. However, time histories of the vehicle velocity and distance travelled were obtained from iterative calculations, with determining (in predefined time intervals) successive values of the physical quantities that characterize the course of the braking process, including:
- braking deceleration $a_h$,
- vehicle velocity $v$,
- distance travelled $s$.

Pursuant to the assumptions adopted, each set of results was obtained for specific values of the sliding tyre-road adhesion coefficient $\mu_2$, with the vehicle mass $m$ and initial braking velocity $v_0$ being taken into account.

The calculations were carried out for the following options:
- initial vehicle velocity $v_0 = 30, 60, \text{ and } 90 \text{ km/h}$;
- vehicle mass $m = 4000, 8000, \text{ and } 12000 \text{ kg, i.e. unladen, half-laden, and fully laden mass (MAM), respectively}$;
- road slope angle $\alpha = 0^\circ$ (horizontal road).

An example set of calculation results has been presented in Figure 7.

From the point of view of the analysis carried out, the greatest importance is attached to the curve additionally plotted to represent the vehicle velocity $v$ as a function of the distance travelled $s$, shown in Figure 7e.

Based on the example of the calculation results summarized in Figure 8, changes in the vehicle mass can be seen to have a definite impact on the quantities that characterize the vehicle braking process.

According to expectations, the calculation results showed that the raising of the vehicle mass, which means an increase in the normal load on each vehicle wheel, resulted in:
- time shift (delay) of the beginning of the braking phase (Figure 8a);
- reduction in the vehicle braking intensity and lengthening of the braking time (Figures 8a, 8b);
- lengthening of the vehicle stopping distance (Figure 8c).

However, the most conspicuous effects of an increase in the vehicle mass can be seen in Figure 8d. At a relatively low initial vehicle velocity ($v_0 = 60 \text{ km/h}$), the raising of the vehicle mass from the unladen to the half-laden and fully laden (MAM) value caused

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**Fig. 8.** Impact of vehicle mass on changes in the physical quantities that characterize the course of the emergency braking process ($v_0 = 60 \text{ km/h}$): a) braking deceleration, b) vehicle velocity, c) distance travelled, d) vehicle velocity vs distance travelled
the vehicle stopping distance to be extended by about 4 m and 9 m, respectively.

These stopping distance elongation values are comparable with, respectively, the width of a typical pedestrian crossing and a half of the overall length of a typical tractor-semitrailer unit. On the other hand, conclusions of particular importance from the point of view of vehicle safety and reconstruction of a road event can be drawn from examining the results presented in Figure 8d. The calculation results have shown that at the place where the unladen vehicle \((m = 4\,000\,\text{kg})\) would stop, the velocity of the vehicle being half-laden \((m = 8\,000\,\text{kg})\) and fully laden \((m = 12\,000\,\text{kg})\) would be, approximately, over 20 km/h and over 30 km/h, respectively. In spite of moderate initial vehicle velocity, these residual velocity values are high enough for a possible collision between the vehicle and a pedestrian or another object to bring about very serious effects.

The tests revealed that the changes in the vehicle mass had an insignificant impact on the time of starting the braking process (Figure 9).

On the other hand, the increase in the vehicle mass caused an elongation of the time of rise in the braking force, especially at low values of the initial braking velocity \(v_0\), according to the calculation results presented in Figure 6. Moreover, changes in the vehicle mass markedly affected the braking deceleration values and, in consequence, the braking and stopping distances achieved.

The impact of vehicle mass on the result of emergency braking in the conditions of various initial braking velocity \(v_0\) has been summarized in Figure 10. At each initial braking velocity, an increase in the vehicle mass considerably lengthens the stopping distance. Simultaneously, it can be seen that the velocity of a fully laden vehicle \((m = 12\,000\,\text{kg})\) at the place where an unladen vehicle \((m = 4\,000\,\text{kg})\) would come to a halt may range from about 25 km/h to even 40 km/h, depending on the initial braking velocity.

With the measurement results being used as an example, the following has been shown (Figure 11):
- the increase in the vehicle stopping distance caused by a growth in the vehicle mass is the highest at high initial braking velocities \(v_0\);  
- in relative terms, the raising of the vehicle mass from the unladen to the half-laden and fully laden (MAM) value may cause the vehicle stopping distance to be lengthened even by more than 20%.

4. Closing conclusions

The simulation tests carried out have shown that the raising of the vehicle mass may considerably lengthen the emergency stopping distance of a vehicle in result of:
- delay in the start of the braking process;  
- reduction in the braking intensity.

Moreover, it has been shown that the vehicle loaded with a cargo may still move with a considerable velocity at the instant when the unladen vehicle would have come to a halt. These conclusions are important from the point of view of safety of vehicle motion. Simultaneously, they show that significant changes in tyre properties, such as those indicated here, must be taken into account in the process of analysis and reconstruction of a road event.

The research work under consideration is worth continuing, in both its experimental and model simulation part. It has been shown that the phenomena of changes in the processes observed are rooted in the pneumatic tyre properties highlighted in the experiments. However, some simplifying assumptions were made in the simulation tests, which included a simplified model of friction between the pneumatic tyre when locked up and the road surface, with the adhesion coefficient value remaining constant during the whole braking process. It is presumed that the impact of the growth in the vehicle mass on the elongation of the vehicle stopping distance would be found stronger if the following factors were taken into account in the tests:
- real changes in the sliding tyre-road adhesion coefficient that occur with changes in the sliding velocity;  
- reduction in the tyre-road adhesion coefficient during the significantly extended braking time.

These issues may define the main directions for further research.
References


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