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A STOCHASTIC MODEL FOR ESTIMATION OF REPAIR RATE FOR SYSTEM OPERATING UNDER PERFORMANCE BASED LOGISTICS

STOCHASTYCZNY MODEL DO SZACOWANIA INTENSYWNOŚCI NAPRAW DLA SYSTEMU DZIAŁAJĄCEGO W WARUNKACH LOGISTYKI WYDAJNOŚCIOWEJ

Performance Based Logistics (PBL) concept has an aim to improve the system availability and it has been extensively researched in the recent years. These researches showed that inventory level does not impact system availability as much as component reliability and repair time in repairable system operating under PBL contract. Based on that, in this paper, we propose a new stochastic model for determination of annual repair rate for critical aircraft components in such system in order to achieve desired availability. The result obtained could be used for planning of base stock level and capacity of repair facilities.

Keywords: *repair rate, availability, stochastic model, performance based logistic.*

Koncepcja Logistyki Opartej na Wydajności (Performance Based Logistics, PBL), której celem jest poprawa gotowości systemów, została w ostatnich latach szeroko zbadana. Badania te wykazały, że w przypadku systemów działających w warunkach PBL, poziom zapasów nie wpływa na gotowość systemu w tak dużym stopniu jak niezawodność elementów składowych oraz czasy napraw. Opierając się na tej obserwacji, w niniejszym artykule proponujemy nowy model stochastyczny do określania rocznej intensywności napraw krytycznych elementów samolotu tworzących system tego typu. Model ten pozwala na osiągnięcie pożądanej gotowości. Uzyskany model może być wykorzystany do planowania bazowego poziomu zapasów oraz przepustowości zakładów remontowych.

Słowa kluczowe: *intensywność napraw, gotowość, model stochastyczny, logistyka oparta na wydajności.*

1. Introduction

The concept of Performance Based Logistics (PBL) originates from the military aircraft industry. It refers to acquiring cost-effective weapon system support. PBL is a strategy which has an aim to improve the performance and to lower the total operating cost of the complex system (especially in aviation and defense industry) during the post production phase of their life-cycle [21].

This system was succeeded by numerous commercial companies [20, 5]. In practice, the principle works as follows – e.g. when servicing aircraft engines under the PBL contract, the maintenance and service are not charged by the number of used spare parts, the number of repairs or activities, but the number of flight hours that the engine's operator makes instead [17].

The availability is one of the crucial criteria in the PBL contracts [28] and can be largely affected by factors such as reliability and number of spare parts. Control of all factors is vital for contract users. Many researchers have tried to provide mathematical models which will answer the question: How do supply management, parts reliability and maintenance affect the availability?

One of the most utilized, and at a later stage most perfected, is the METRIC model, as a first practical mathematic model from this area [23]. It is based on the Poisson distribution with the mean value estimated by a Bayesian procedure and uses "one-for-one" policy of filling out the storage and modeling the system on the basis of mean repair time rather than its distribution. Other models based on the METRIC model have appeared later, such as VARI-METRIC [24]

and MOD-METRIC model [15] which provided better results in simulations than the initial model.

Regarding the aspect of expenses and production order, this problem was examined in paper [2] by considering the failure rate as a function that depends on the number of machines and determining the optimum supplies for expensive, critical parts with low demand. This topic was further elaborated in paper [3] but in conditions of limited capacity related to repairs of spare parts and in paper [26] in relation to systems with the condition based maintenance strategy. Furthermore, the supplies of repairable spare parts in the case of non-stationary Poisson demand have been examined in paper [12] with the goal to minimize system's expenses. Similarly, with minor corrections, this problem was also examined in papers [27, 25] with the goal to reduce the delivery time, delays and transport costs. The aforesaid models proved to be far superior in relation to the original Sharebrook's METRIC model.

Kang et al. examined systems for inventory management under the PBL contracts [7]. They have developed a methodology which determines the system's availability based on reliability of its components/parts and maintenance possibilities. They concluded that mean time to failures (MTBF), number of spare parts and mean time to repair (MTTR) have the greatest impact on availability. Some modifications of this model are presented in papers [8, 9, 10, 18 and 19].

This problem was examined further, by relaxing assumptions such as fixed repair rate, fixed failure rate and infinitive repair in paper [14]. The results that were achieved in this research proved that the level of supplies of reparable spare parts does not affect the system

availability as much as reliability and repair rate. The authors advise to focus on the component reliability and repair system efficiency to improve system availability.

Based on the aforesaid research, the model presented in this paper observes the repair rate as a stochastic process and has an aim to determine this parameter for preferred level of availability. The need for stochastic modeling of repairable systems has been justified and explained in paper [1].

2. Model for assessment of expected time to repair

In this paper we are observing system that alternates between two states – system is operative at certain time and non-operative otherwise. In the literature, this approach is known as alternating renewal process [4]. We assumed that at the start system is operative. It remains in that state for a period of time T (failure time), then it stops operating for time R (repair time) and after being repaired system is back in operative state. The duration of the renewal cycle is $T+R$. We also assumed that perfect repair has been carried out at the constant rate after which system behaves the same as the new one. In this case we are observing the system in which failure time has Rayleigh distribution and the goal is to determine the repair rate in order to optimize performance of the system i.e. to determine repair rate for desired level of system availability in the case when mean time between failures (MTBF) is known. MTBF is reliability measure of repairable system and includes only operational time between failures and not the repair time. Main purpose of PBL contracts is to optimize system availability. Steady state availability is often used availability measure in repairable system and according to definitions is equal to:

$$A = \lim_{t \rightarrow \infty} A(t). \quad (1)$$

According to key renewal theorem the limited probability that system is available can be expressed as ratio of the mean of period when system is operative and mean of the period which represents one renewal cycle [22]:

$$\lim_{t \rightarrow \infty} A(t) = \frac{E[T]}{E[T] + E[R]}, \quad (2)$$

where E is the expected value operator. Based on this relation is derived a well known formula for availability:

$$A = \frac{MTBF}{MTBF + MTTR}. \quad (3)$$

MTTR is mean time to repair i.e. expected time needed to repair a failed component. If there exists probability density function $p(t)$, than the $MTBF$ can be defined as:

$$MTBF = \int_0^{\infty} tp(t)dt. \quad (4)$$

Since we assumed that failure time has Rayleigh distribution with probability density function (shortly written PDF) $p(t) = \frac{2t}{x} \exp\left(-\frac{t^2}{x}\right)$, $t > 0$ [6] where the distribution parameter x is determined by relation $E(t^2) = x$, the previous equation is:

$$MTBF = \int_0^{\infty} \frac{2t^2}{x} \exp\left(-\frac{t^2}{x}\right) dt. \quad (5)$$

By solving the previous integral we obtained following equation:

$$MTBF = \frac{1}{2} \sqrt{\pi x}. \quad (6)$$

The rate of repair can be observed as a reciprocal value of MTTR [16]. So, in order to simplify further calculation we introduce the changes:

$$u = 1 / MTBF = 2 / \sqrt{\pi x}, \quad (7)$$

where such defined u denotes the failure rate and:

$$\mu_r = 1 / MTTR,$$

where μ_r denotes repair rate.

According to (7) Rayleigh's random variable x is:

$$x = \frac{4}{\pi u^2} \quad (8)$$

and the availability formula (3) can now be expressed as:

$$A = \frac{\mu_r}{u + \mu_r}. \quad (9)$$

Equation (9) will be further used in order to determine repair rate for desired level of availability in case when MTBF is known. We could use eq. (3) and define availability through MTBF and MTTR. In that case we would observe MTTR as a stochastic process and we would determine its characteristic PDF and other parameters for certain predefined availability but, according to our opinion observing $1/MTTR$, rate of repair, as stochastic process, is more significant for the entire repair process planning and managing.

Due to complexity of process of estimating the components' failure rate in relation to time, as well as a stochastic nature of the observed process, the parameter x could also be considered as a random variable that changes significantly slower than random variable t described with the Rayleigh's model. Since we already know from the stochastic theory [13], for cases when the changes of variable t are described by the Rayleigh's model and when $x \approx t^2$, slow changes of variable x can be described with the stochastic process with exponential distribution with the use of:

$$p_x(x) = \frac{\exp\left(-\frac{x}{x_0}\right)}{x_0}, \quad x > 0, \quad (10)$$

where $x_0 = E(x)$.

Since the goal of this paper is to determinate the repair rate for desired level of availability in case when MTBF is known and we already expressed Rayleigh's random variable x in (8), than the following transformation is applicable:

$$p(u) = p_x\left(\frac{4}{\pi u^2}\right) |J|, \quad (11)$$

where $|J|$, Jacobian transformation of random variable x , is stated in (12):

$$|J| = \left| \frac{dx}{du} \right| = \frac{8}{\pi u^3}. \quad (12)$$

By replacing (12) into (11), we get

$$p(u) = \frac{8}{u^3 \pi x_0} \exp\left(-\frac{4}{u^2 \pi x_0}\right). \quad (13)$$

Now, based on (9) the repair rate μ_r can be presented as $\mu_r = \frac{Au}{1-A}$ with PDF function:

$$p(\mu_r) = p_u\left(\frac{Au}{1-A}\right)|J|, \quad (14)$$

where Jacobian transformation is:

$$|J| = \frac{du}{d\mu_r} = \frac{A}{1-A}. \quad (15)$$

According to previous, PDF function of repair rate can be stated as:

$$p(\mu_r) = \frac{8A^2}{(1-A)^2 \mu_r^3 \pi x_0} \exp\left(\frac{-4A^2}{(1-A)^2 \mu_r^2 \pi x_0}\right). \quad (16)$$

This is a major contribution of this paper, exact mathematical characterization of MTTR random process. By using this PDF expression an exact modeling of repair rate process can be obtained by generating exact repair rate sample values for corresponding values of availability and MTBF. In such way, simulation of repair rate process through generating its samples could serve for dynamical prediction of system performances.

Now, let present cumulative distribution function (shortly written CDF) of repair rate as:

$$F(\mu_r) = \int_0^{\mu_r} p(\mu_r) d\mu_r = 1 - \exp\left(\frac{-4A^2}{(1-A)^2 \mu_r^2 \pi x_0}\right) \quad (17)$$

With the use of inverse sampling $y = F(\mu_r)$, the inverse CDF is

$F^{-1}(\mu_r) = y^{-1}$ and repair rate samples μ_r can be expressed as:

$$\mu_r = \sqrt{\left(-\frac{(1-A^2)\pi x_0}{4A^2} \ln(1-F^{-1}(\mu_r))\right)} = \sqrt{\left(-\frac{(1-A^2)\pi x_0}{4A^2} \ln(1-y)\right)}, \quad (18)$$

where y is uniformly distributed in interval $[0, 1]$. By introducing change $U = 1 - y$, (18) can be reduced to:

$$\mu_r = \sqrt{\left(-\frac{(1-A^2)\pi x_0}{4A^2} \ln U\right)}$$

where U is uniformly distributed in interval $[0, 1]$.

Further, based on the equation (16), we can determine the expected repair rate of component $\bar{\mu}_r$ in relation to the preferred level of availability as:

$$\bar{\mu}_r = \int_0^{+\infty} \mu_r p(\mu_r) d\mu_r \quad (19)$$

After replacing (16) into (19) the previous expression is reduced to:

$$\bar{\mu}_r = \frac{2A}{(1-A)\sqrt{\pi}\sqrt{x_0}}. \quad (20)$$

This measure that characterizes MTTR random process is in that way for the first time expressed as the function of availability and MTBF and can be observed as their function.

3. Numerical results

In order to verify our model we are using data that originate from [7, 14], where the system of unmanned aerial vehicles consisting of four air vehicles, two ground-control stations, modular mission payloads, data links, remote data terminals and an automatic landing subsystem were observed. The concept of an unmanned aerial vehicle (UAV) is not new but it has not been utilized in civilian sector due to the insufficient level of reliability of current solutions that leads to high probability of failure occurrence [11].

The following critical repairable components: aircraft's engine, propeller and avionics, are taken into consideration. Known parameters examined in the system are:

- Each aircraft has 120 flight hours per month, i.e. 1440 (120*12) flight hours per year.
- Mean time between failures (MTBF is 750 flight hours for the aircraft engine, 500 for the propeller and 1000 for avionics.
- Based on that it is possible to determine the MTBF as follows:
 - for the aircraft engine $MTBF_e = 750 / 1440$
 - for the aircraft propeller $MTBF_p = 500 / 1440$
 - for the avionics $MTBF_a = 1000 / 1440$

Based on the model presented in previous section, a numerical analysis was conducted with the goal to calculate the annual expected time for repair in order to acquire availability of $A=0.85$, $A=0.9$, $A=0.95$ by emphasizing the stochastic nature of this process. A similar analysis can also be conducted for other values of parameter A .

Fig.1 represents the probability of engine's repair rate depending on time for cases when it is expected that availability of this component is 85%, 90% and 95%. Likewise, Fig.2 provides data related to the propeller, while Fig.3 refers to system's avionics.

Fig. 4 provides graphics data on repair rate in relation to availability on annual level. The availability parameter was set in interval $[0.5 - 1]$ and according to previous equations we can calculate the annual repair rate for aircraft engine, propeller and avionics. It can be seen that the repair rate increases with the increase of required level of availability. Complete set of graphic data is presented in the following table:

Table1. Level of annual repair rate in relation to availability

Availability	Propeller	Electronics	Engine
0.50	1.92	2.88	1.44
0.55	2.35	3.52	1.76
0.60	2.88	4.32	2.16
0.65	3.57	5.35	2.67
0.70	4.48	6.72	3.36
0.75	5.76	8.64	4.32
0.80	7.68	11.52	5.76
0.85	10.88	16.32	8.16
0.90	17.28	25.92	12.96
0.95	36.48	54.72	27.36
0.97	62.08	93.12	46.56

4. Conclusion

According to previous PBL studies, the base stock level does not influence system availability as much as repair rate and reliability, so in this paper we proposed a model for determination of expected repair rate on annual level by observing it as a stochastic process, for the first time. The presented model can be used for estimation of other significant maintainability parameters. Further, by setting the availability parameter at required values and assuming the base stock level was fixed at some constant value, we can determine the repair rate, not just for critical components mentioned in this paper, but for any other repairable system that meets the accepted assumptions. By using this PDF expression an exact modeling of repair rate process can be obtained by generating exact repair rate sample values for corresponding values of availability and MTBF. In such way simulation of repair rate process through generating its samples could serve for dynamical prediction of system performances. This modeling procedure could be used for planning implementation of new service stations or increasing any other capacity required for repairable spare parts servicing in order to increase repair system efficiency. The potential area of further research is optimization of cost that these modifications could bring, such as optimizing the number of repair stations.

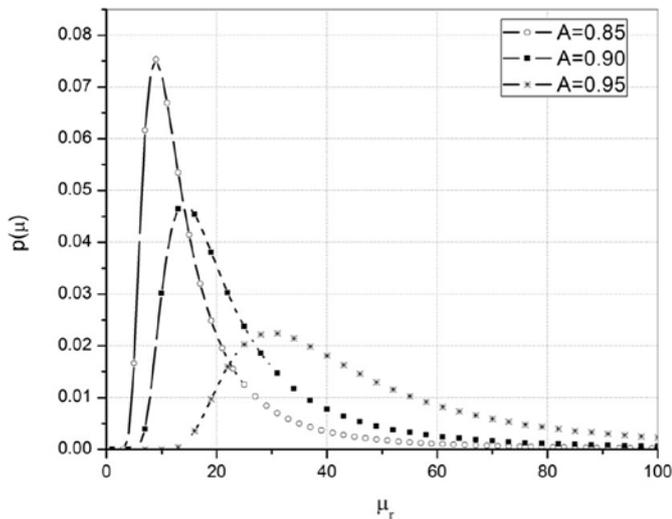


Fig. 1. PDF of engine repair rate

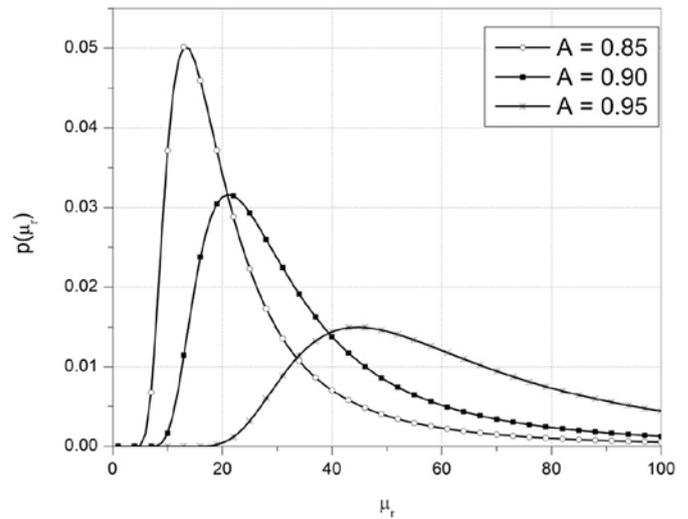


Fig. 2. PDF of propeller repair rate

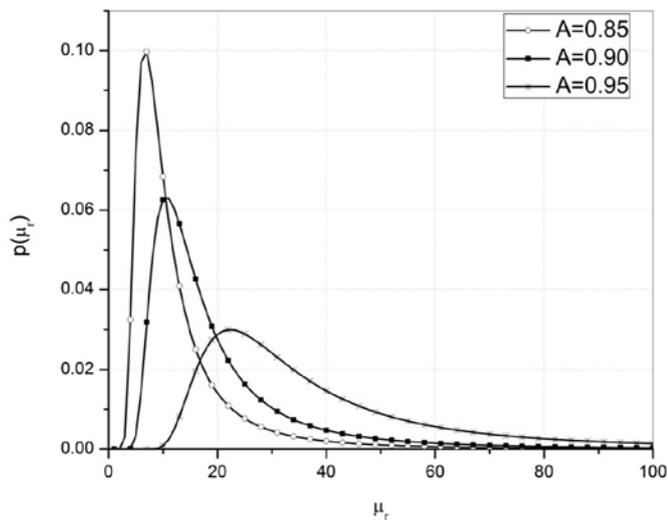


Fig. 3. PDF of aircraft's avionics repair rate

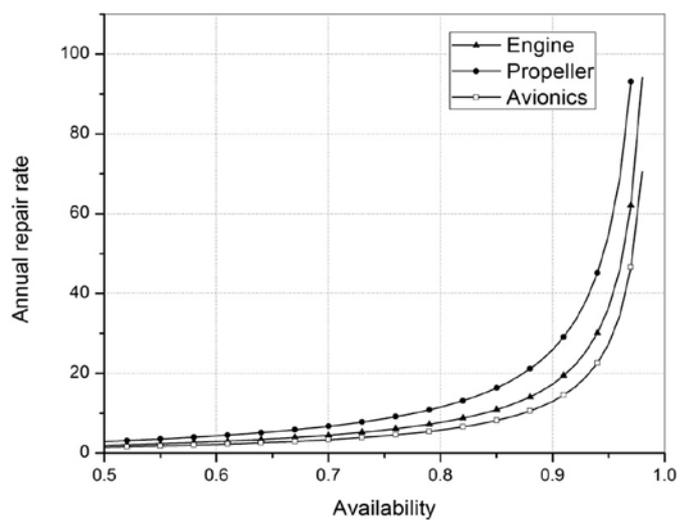


Fig. 4. Annual level of repair rate in relation to availability

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