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CONSISTENCY ANALYSIS OF DEGRADATION MECHANISM IN STEP-STRESS ACCELERATED DEGRADATION TESTING

ANALIZA NIEZMIENNOŚCI MECHANIZMU DEGRADACJI W PRZYSPIESZONYCH BADANIACH DEGRADACJI Z OBCIĄŻENIEM STOPNIOWYM

Step-stress accelerated degradation testing (SSADT) has been used by many researchers for the reliability assessment of highly reliable products. Most of the previous works on SSADT assume that the degradation mechanism keeps unchanged during the accelerated degradation testing. However, some recent investigations have shown that degradation mechanisms may be different among various accelerated stress levels. For an accurate extrapolation of accelerated testing results to the ambient condition, the degradation mechanism at all accelerated stress levels should be the same. Taking the variation of the degradation mechanism into account, it is advisable to test the degradation mechanism consistency in a SSADT. This paper proposes a likelihood ratio test method for the consistency analysis of degradation mechanism in the SSADT. We first introduce the basic principle of the likelihood ratio test method. Then we describe the model for SSADT data and the parameter estimation method. Further, we propose a decision rule for the consistency analysis. The proposed method is illustrated and validated with examples on the consistency analysis of degradation mechanism in a SSADT of silicone rubbers.

Keywords: *step-stress accelerated degradation testing, degradation mechanism consistency, likelihood ratio test, rubber seals.*

Streszczenie Wielu badaczy wykorzystuje przyspieszone badania degradacji z obciążeniem stopniowym (ang. step-stress accelerated degradation testing, SSADT) do oceny niezawodności wysoce niezawodnych produktów. Większość wcześniejszych prac nad SSADT zakłada, że podczas badań przyspieszonych mechanizm degradacji pozostaje niezmienny. Jednak, najnowsze badania wykazały, że mechanizmy degradacji mogą różnić się w zależności od poziomu przyspieszonego obciążenia. Poprawna ekstrapolacja wyników badań przyspieszonych na warunki otoczenia wymaga aby mechanizm degradacji przy wszystkich poziomach obciążenia był taki sam. Biorąc pod uwagę zmienność mechanizmu degradacji, wskazane jest badanie stopnia (nie)zmienności mechanizmu degradacji w badaniach SSADT. W artykule zaproponowano metodę analizy niezmienności mechanizmu degradacji w badaniach SSADT opartą na teście ilorazu wiarygodności. W pierwszej kolejności, przedstawiono podstawową zasadę testu ilorazu wiarygodności. Następnie, opisano model dla danych SSADT i metodę estymacji parametrów. Ponadto zaproponowano regułę decyzyjną stanowiąca narzędzie do analizy niezmienności. Omawianą metodę zilustrowano i zweryfikowano na przykładzie analizy niezmienności mechanizmu degradacji w badaniach SSADT gumy silikonowej.

Słowa kluczowe: *przyspieszone badania degradacji z obciążeniem stopniowym, niezmienność mechanizmu degradacji, test ilorazu wiarygodności, uszczelki gumowe.*

1. Introduction

Due to the strong market competition, many newly developed products are highly reliable and long-lifetime, such as light emitting diodes and silicone rubber seals. For these highly reliable products, it turns out to be impractical to make the reliability assessment within a feasible life testing time. Accelerated life testing (ALT) and accelerated degradation testing (ADT) have been widely used in industry to solve this problem. Hirose [11] used the ALT data to estimate the lifetime of insulation film at the normal stress. Cary [5] applied the ADT to the reliability evaluation of an integrated logic family. Tang [22] described the procedure for the reliability prediction of power supplies by nondestructive ADT data. Wang [25, 26] made a research on the lifetime prediction of self-lubricating spherical plain bearings based on the ADT, and he provided an optimal design of the test plan. In an ALT and ADT, the failure times and performance degradation data of samples at accelerated stress levels are recorded respectively. When the failure mechanism for an ALT and the degradation mechanism for an ADT are consistent at all stress levels, the accelerated test results can be extrapolated to estimate the product lifetime at the normal condition. In terms of the sample size and the amount of test time needed, SSADT is more efficient than other accelerated tests. Many reliability analysts recently employ the SSADT to assess the lifetime distribution of highly reliable products. Tseng et al. [23] presented an optimal test plan for the SSADT of carbon-film-resistors, and they performed a sensitivity analysis for the test plan. Cai and Liao [4, 15] established a SSADT model and a test plan for the degradation of light emitting diodes. In order to obtain an accurate lifetime prediction, the design of the test plan should guarantee the failure mechanism and degradation mechanism keep unchanged at all accelerated stress levels. But these researchers mainly focused on the exact statistical inference of test data. They guaranteed the consistent degradation mechanism based on the empirical assumption and standard specifications.

In fact, some investigations have reported the variation of the degradation mechanism over a wide range of stress levels. Tan and Singh [21] found the change of the degradation mechanism among various stress levels in the ADT of light emitting diodes, and they concluded the mechanism variation presented a challenge in the extrapolation of accelerated results to the normal condition. Patel and Skinner [20] reported the change of the underlying degradation process at high temperatures of an ADT of polysiloxane rubbers. Gillen et al. [7, 8, 14] showed the evidence for the variation of degradation mechanism over a sufficiently wide range of temperatures. They also obtained distinct lifetime prediction results based on the consistency analysis of the degradation mechanism.

Researchers should judge whether the degradation or failure mechanism changes at the accelerated stress levels, and guarantee that the mechanism is consistent for the reliability assessment of highly reliable products. This process is defined as the mechanism consistency analysis in the accelerated degradation or life testing. Heat is known to be one important factor that affects the product performance and failure. Most of accelerated tests use the elevated temperature stress for accelerating the failure and degradation process. The Arrhenius model is commonly used as the acceleration model for such an ALT and ADT. For the Arrhenius model, the consistency of the failure mechanism or degradation mechanism is equivalent to the activation energy consistency [13, 16, 17]. There are a few works on the consistency analysis of the failure mechanism in an ALT. Hu et al. [13] discussed the possible failure mechanism shifting in an ALT of electronic devices and packages. Guo et al. [9, 10] explored the decision rule for judging the failure mechanism consistency in an ALT. But the decision rule can not be directly applied to the case of an ADT, due to the difference in statistical models of the ALT and ADT. Gillen [3, 6] proposed a test method for examining the degradation mechanism consistency in an ADT. This method is a graphical method in essence, which tests the consistency by examining the linearity of the log-plot of degradation rates versus inverse temperatures. If the log-plot is a nearly straight line, the degradation mechanism is determined to be constant within the investigated temperature range. Otherwise, it changes at a sufficiently high temperature. This graphical method is simple and provides available supplements to a formal analysis, but it is subjective in practice and can not be applied to the consistency analysis of the degradation mechanism in a SSADT. In fact, a statistical test method is a formal analysis for the degradation mechanism consistency, and a decision rule should be proposed for the SSADT. Unfortunately, few literatures deal with the statistical test method of the degradation mechanism consistency in the SSADT.

The purpose of this paper is to develop a statistical test method for the consistency analysis of degradation mechanism in a SSADT. In statistical terms, it is intended to develop a statistical method for testing the null hypothesis of consistent activation energy against the alternative hypothesis of inconsistent activation energy in a SSADT. The rest of this paper is organized as follows. First the basic principle of the likelihood ratio test method for testing the activation energy consistency is introduced. Then a reliability model for SSADT data is established, followed by the maximum likelihood estimation (MLE) method of unknown parameters. Furthermore, a decision rule for identifying the degradation mechanism consistency is constructed and the test method is compared with the AIC method to show its validity. Examples are also given to illustrate how to make judgments on the degradation mechanism consistency in a SSADT.

2. Basic principle of the likelihood ratio test method

Suppose that the probability distribution function of a population Y is $f(y,\theta_1,\cdots,\theta_p)$, which depends on the vector parameter $\boldsymbol{\theta}=(\theta_1,\cdots,\theta_p)$. The activation energy E_i at each temperature stress level T_i is an unknown parameter, thus $E_i \in \boldsymbol{\theta}$ and we set $(\theta_h,\cdots,\theta_p)=(E_1,\cdots,E_i)$. As mentioned above, we are interested in testing the activation energy consistency in a SSADT. Therefore we have the following null hypothesis H_0 and alternative hypothesis H_1:

$$H_0 : E_i = E \text{ for all } i \text{ against } H_1 : E_i \neq E_j \text{ for } i \neq j$$

From the definition stated above, the activation energy is assumed to be constant in H_0 and hence the model for H_0 is called the activation energy fixed (E -fixed) model. In contrast, activation energy depends on the stress in H_1, and this case is called the activation energy free (E -free) model. Except for the activation energy parameter, other unkown parameters are the same for the E -fixed and E -free model. The E -free model is a more general model and has more independent parameters than the E -fixed model. Likelihood ratio (LR) test methods can compare models for two hypotheses, provided that one model is a special case of another model. Hirose [11] used the LR test method to determine whether the shape parameter of Weibull distribution changes in an ALT. We employ the LR test method to test the activation energy consistency in a SSADT.

The LR test method is based on the likelihood ratio statistic. A random sample $y_1, y_2, \cdots, y_n$ is considered from the population Y. The likelihood function associated with this sample is given by:

$$L(\boldsymbol{\theta}) = \prod_{i=1}^{n} f(y_1, y_2, \cdots y_n; \boldsymbol{\theta}) \tag{1}$$

Let $\hat{\boldsymbol{\theta}}_{H_0} = \arg\max L_0(\boldsymbol{\theta})$ denote the MLE for the E -fixed model, and $\hat{\boldsymbol{\theta}}_{H_1} = \arg\max L_1(\boldsymbol{\theta})$ denote the MLE for the E -free model. Then the likelihood ratio statistic λ is:

$$\lambda = \frac{\max\{L_0(\boldsymbol{\theta})\}}{\max\{L_1(\boldsymbol{\theta})\}} = \frac{L_0(\hat{\boldsymbol{\theta}}_{H_0})}{L_1(\hat{\boldsymbol{\theta}}_{H_1})} \tag{2}$$

Due to the use of more parameters, the E -free model will always fit better to the sample observations than the E -fixed model except they both fit well to the data. Hence the E -free model has the same or greater likelihood value than the E -fixed model. The likelihood ratio statistic λ satisfies:

$$0 \le \lambda \le 1 \text{ for } L_0(\boldsymbol{\theta}) \le L_1(\boldsymbol{\theta}) \tag{3}$$

If the E -fixed model is an appropriate model for data fitting as the E -free model, the likelihood ratio is large otherwise it is small. Thus the LR test method rejects the null hypothesis (E -fixed model) if the value of the likelihood ratio statistic is too small. The critical region or rejection region of a LR test is:

$$W = \{\lambda \le c\} \qquad 0 \le c \le 1 \tag{4}$$

where the critical value c depends on the statistical distribution of λ and the specified significance level β in the LR test.

After the construction of the degradation model and the estimation of unknown parameters, the specific decision rule for the consistency analysis of degradation mechanism is derived. The decision rule corresponding to the SSADT data is presented in the fourth part of this paper.

3. Model for SSADT data and parameter estimation

3.1. Model for SSADT data

Suppose that q temperature stress levels are employed in a SSADT. The performance characteristics of n testing units are measured at m_i specified time points $t_{ik}(k=1,2,\cdots,m_i)$ for each temperature stress $T_i(i=1,2,\cdots,q)$. Nondestructive inspections for the performance characteristic are conducted. The termination time for the degradation test at T_i is t_i ($t_{ik}<t_i$). When the terminatime time at T_i is reached, the temperature T_i will be increased to the temperature T_{i+1} until the highest temperature T_q is reached. Then the testing stress sequence of a q-step SSADT can be expressed as:

$$T=\begin{cases} T_1 & 0\le t<t_1 \\ \vdots & \vdots \\ T_q & t_{q-1}\le t<t_q \end{cases}$$

Let $L(t\mid T_i)$ denote the mean degradation path at a constant temperature T_i. Then the path can be modeled by:

$$L(t\mid T_i)=B\exp\left(-K_i t^{\alpha}\right) \tag{5}$$

where the parameter α is independent of the temperature stress and $0<\alpha\le 1$.The degradation model (5) is recommended by the industrial standard HGT 3087 for rubber seals [12]. Yu and Tseng [29] also used the model $B=1$ with $\alpha=0.5$ to describe the degradation path of a light emitting diode. For convenience of data fitting and parameter estimation, the linear transformation of (5) is used in this paper:

$$D(\tau\mid T_i)=\ln B-K_i\tau \tag{6}$$

where $\tau=t^{\alpha}$, and the parameter B denotes the initial value of the performance characteristic. The initial measurement data are generally standardized to be unity in the statistical analysis, thus B should be a constant approximating to unity. When α is fixed, obviously K_i can be interpreted as the degradation rate, which depends on the stress level T_i.

Similar to the previous cumulative exposure model for the step-stress ALT [19, 30], the degradation rate depends only on the current stress regardless of the accumulation history of the degradation process [24]. Let ω_{i+1} denote the equivalent duration time at the stress T_{i+1}, which yields the same cumalative degradation amout as the termination time τ_i at the stress T_i of the SSADT. For $i=1,2,\cdots,q-1$,

$$D(\omega_{i+1}\mid T_{i+1})=D(\omega_i+\tau_i-\tau_{i-1}\mid T_i) \tag{7}$$

where $\omega_1=0$, $\tau_i=t_i^{\alpha}$ and $\tau_0=0$.

Let $D_{SS}(\tau)$ denote the mean degradation path of a SSADT with a linear degradation model. The relationship between $D_{SS}(\tau)$ and $\{D(\tau\mid T_i)\}_{i=1}^{q}$ is:

$$D_{SS}(\tau)=\begin{cases} D(\tau\mid T_1) & 0\le\tau<\tau_1 \\ D(\tau-\tau_1+\omega_2\mid T_2) & \tau_1\le\tau<\tau_2 \\ \vdots & \vdots \\ D(\tau-\tau_{q-1}+\omega_q\mid T_q) & \tau_{q-1}\le\tau<\tau_q \end{cases} \tag{8}$$

The graphical illustration for the transformation from $D_{SS}(\tau)$ to $\{D(\tau\mid T_i)\}_{i=1}^{3}$ is shown in Figure 1.

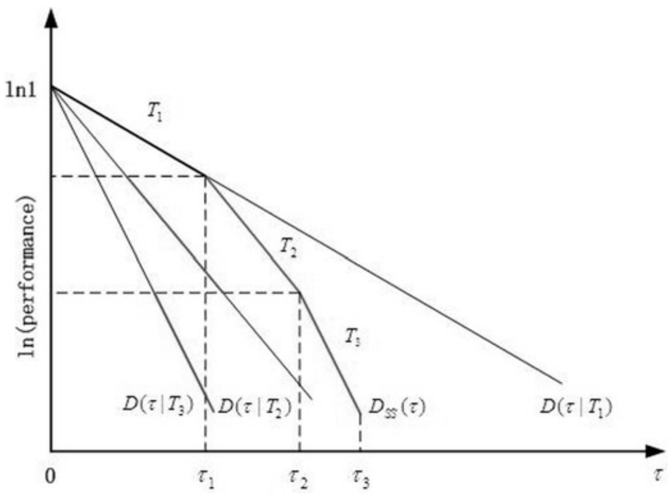

Fig. 1. Transformation from the SSADT path $D_{SS}(\tau)$ to ADT path $\{D(\tau\mid T_i)\}_{i=1}^{q}$ with $q=3$

From (6) and (7), for $i=1,2,\cdots,q-1$, the equivalent duration time ω_{i+1} is given by:

$$\omega_{i+1}=\frac{K_i}{K_{i+1}}(\tau_i-\tau_{i-1}+\omega_i) \tag{9}$$

Applying the iterative procedure to (9), the analytic expression for ω_{i+1} is:

$$\omega_{i+1}=\frac{\sum_{l=1}^{i}K_l(\tau_l-\tau_{l-1})}{K_{i+1}} \tag{10}$$

From (8) and (10), the mean degradation path of a SSADT is:

$$D_{SS}(\tau)=\begin{cases} \ln B-K_1\tau & 0\le\tau<\tau_1 \\ \ln B-K_1\tau_1-K_2(\tau-\tau_1) & \tau_1\le\tau<\tau_2 \\ \vdots & \vdots \\ \ln B-\sum_{l=1}^{q-1}K_l(\tau_l-\tau_{l-1})-K_q(\tau-\tau_{q-1}) & \tau_{q-1}\le\tau<\tau_q \end{cases} \tag{11}$$

The observed sample degradation $\ln y_{ijk}(j=1,2,\cdots,n)$ of the unit j at the time t_{ik} is the mean degradation plus the unit-to-unit variability. Thus SSADT data $\ln y_{ijk}$ is modeled by:

$$\ln y_{ijk}=D_{SS}(\tau_{ik})+\varepsilon_{ijk} \tag{12}$$

where the unit-to-unit variability ε_{ijk} is commonly assumed to be s-independent of k and normally distributed $N(0,\sigma^2)$ [2, 18, 28]. From (12), the cumulative distribution for SSADT data y_{ijk} is:

$$F_Y(y_{ijk}) = \mathrm{P}\left(\ln Y \le \ln y_{ijk}\right) = \Phi\left(\frac{\ln y_{ijk} - D_{SS}\left(\tau_{ik}\right)}{\sigma}\right) \tag{13}$$

where $\tau_{ik} = t_{ik}^{\alpha}$.

For a decreasing degradation, a unit is considered to have failed when its performance characteristic degrades below the specified threshold value ζ. Therefore the reliability at time t for the normal stress level T_0 is:

$$R(t) = P(Y > \zeta) = 1 - \Phi\left(\frac{\ln\zeta - D(\tau \mid T_0)}{\sigma}\right) \tag{14}$$

3.2. Parameter estimation

According to the Arrhenius reaction rate theory, the relationship between the degradation rate K_i and temperature stress T_i can be formulated by:

$$K_i = \exp\left(A - \frac{E_i}{RT_i}\right) \tag{15}$$

where A is the intercept of the Arrhenius model, E_i is the activation energy at temperature T_i, and R is the gas constant.

In the E-fixed model, $E_i = E$. From (13) and (15), the log-likelihood function for SSADT data in the E-fixed model is given by:

$$\ln L_0(B,\alpha,A,E,\sigma) = \sum_{i=1}^{q}\sum_{j=1}^{n}\sum_{k=1}^{m_i}\left\{\frac{\left[\ln y_{ijk} - \ln B + G_{ik}\right]^2}{-2\sigma^2} - \ln\sigma - \frac{\ln(2\pi)}{2}\right\} \tag{16}$$

where:

$$G_{ik} = \exp\left(A - \frac{E}{RT_i}\right)(\tau_{ik} - \tau_{i-1}) + \sum_{l=1}^{i-1}\exp\left(A - \frac{E}{RT_l}\right)(\tau_l - \tau_{l-1})$$

Furthermore, the log-likelihood equations from (16) are:

$$\frac{\partial \ln L_0}{\partial B} = 0 \quad \frac{\partial \ln L_0}{\partial \alpha} = 0 \quad \frac{\partial \ln L_0}{\partial A} = 0 \quad \frac{\partial \ln L_0}{\partial E} = 0 \quad \frac{\partial \ln L_0}{\partial \sigma} = 0 \tag{17}$$

The unknown parameters for the E-fixed model are $\hat{\boldsymbol{\theta}}_{H_0} = (B,\alpha,A,E,\sigma)$. The MLEs of these parameters can be obtained by solving equations in (17) with the Newton-Raphson method. Then the maximum value of log-likelihood $\ln L_0(\hat{\theta}_{H_0})$ is derived from (16).

In the E-free model, $E_i \ne E$. The unknown parameters for the E-free model are $\hat{\boldsymbol{\theta}}_{H_1} = (B,\alpha,A,E_1,\cdots,E_q,\sigma)$. The increase in the amount of unknown parameters makes the corresponding MLE for E_i intractable. However, the desired $\ln L_1(\hat{\theta}_{H_1})$ can be calculated without deriving the estimation for E_i.

Let $\boldsymbol{\upsilon}_{H_1} = (B,\alpha,K_1,\cdots,K_q,\sigma)$. The log-likelihood equation satisfies:

$$\frac{\partial \ln L_1}{\partial E_i} = \frac{\partial \ln L_1}{\partial K_i}\cdot\frac{\partial K_i}{\partial E_i} = \frac{\partial \ln L_1}{\partial K_i}\cdot\frac{-K_i}{RT_i} = 0 \tag{18}$$

From (15) and (18), it is seen that $\ln L_1(\hat{\theta}_{H_1}) = \ln L_1(\hat{\upsilon}_{H_1})$. Thus the log-likelihood function in the E-free model is given by:

$$\ln L_1(B,\alpha,K_1,\cdots,K_q,\sigma) = \sum_{i=1}^{q}\sum_{j=1}^{n}\sum_{k=1}^{m_i}\left\{\frac{\left[\ln y_{ijk} - \ln B + H_{ik}\right]^2}{-2\sigma^2} - \ln\sigma - \frac{\ln(2\pi)}{2}\right\} \tag{19}$$

where:

$$H_{ik} = K_i(\tau_{ik} - \tau_{i-1}) + \sum_{l=1}^{i-1} K_l(\tau_l - \tau_{l-1})$$

Furthermore, the log-likelihood equations from (19) are:

$$\frac{\partial \ln L_1}{\partial B} = 0 \quad \frac{\partial \ln L_1}{\partial \alpha} = 0 \quad \frac{\partial \ln L_1}{\partial K_i} = 0 \quad \frac{\partial \ln L_1}{\partial \sigma} = 0 \tag{20}$$

Similarly, the maximum value of log-likelihood $\ln L_1(\hat{\theta}_{H_1})$ is obtained by solving equations in (20) with the Newton-Raphson method. The detailed expressions for the log-likelihood equations in (20) are:

$$\frac{\partial \ln L_1}{\partial B} = \frac{1}{\sigma^2 B}\sum_{i=1}^{q}\sum_{j=1}^{n}\sum_{k=1}^{m_i}\left(\ln y_{ijk} - \ln B + H_{ik}\right) \tag{21}$$

$$\frac{\partial \ln L_1}{\partial \alpha} = -\frac{1}{\sigma^2}\sum_{i=1}^{q}\sum_{j=1}^{n}\sum_{k=1}^{m_i}\left(\ln y_{ijk} - \ln B + H_{ik}\right)\cdot C_{ik} \tag{22}$$

$$C_{ik} = K_i(\tau_{ik}\ln t_{ik} - \tau_{i-1}\ln t_{i-1}) + \sum_{l=1}^{i-1} K_l(\tau_l \ln t_l - \tau_{l-1}\ln t_{l-1}) \tag{23}$$

$$\frac{\partial \ln L_1}{\partial \sigma} = \frac{1}{\sigma^3}\sum_{i=1}^{q}\sum_{j=1}^{n}\sum_{k=1}^{m_i}\left[\left(\ln y_{ijk} - \ln B + H_{ik}\right)^2 - \sigma^2\right] \tag{24}$$

$$\frac{\partial \ln L_1}{\partial K_i} = -\frac{1}{\sigma^2}\sum_{j=1}^{n}\sum_{k=1}^{m_i}\left(\ln y_{ijk} - \ln B + H_{ik}\right)\cdot\tau_{ik} - \frac{1}{\sigma^2}\sum_{g=i+1}^{q}\sum_{j=1}^{n}\sum_{k=1}^{m_i}\left(\ln y_{ijk} - \ln B + H_{gk}\right)\cdot\tau_i \tag{25}$$

The log-likelihood equations for the same parameters (B,α,σ) in (17) and (20) are similar. Specially, the expressions for the log-likelihood equations in (17) are calculated by applying (15) and constant activation energy as:

$$\frac{\partial \ln L_0}{\partial B} = \frac{\partial \ln L_1}{\partial B}\bigg|_{H_{ik}=G_{ik}} \tag{26}$$

$$\frac{\partial \ln L_0}{\partial \alpha} = \frac{\partial \ln L_1}{\partial \alpha}\bigg|_{H_{ik}=G_{ik}} \tag{27}$$

$$\frac{\partial \ln L_0}{\partial \sigma} = \frac{\partial \ln L_1}{\partial \sigma}\bigg|_{H_{ik}=G_{ik}} \tag{28}$$

$$\frac{\partial \ln L_0}{\partial A} = \sum_{i=1}^{q}\left(\frac{\partial \ln L_1}{\partial K_i}\cdot K_i\right)\bigg|_{H_{ik}=G_{ik}} \tag{29}$$

$$\frac{\partial \ln L_0}{\partial E} = \sum_{i=1}^{q}\left(\frac{\partial \ln L_1}{\partial K_i}\cdot \frac{-K_i}{RT_i}\right)\bigg|_{H_{ik}=G_{ik}} \tag{30}$$

4. Decision rule for the mechanism consistency analysis

In most cases, it is very difficult to determine the exact distribution of likelihood ratio statistic λ based on the specific distribution and hypothesis. Fortunately, the asymptotic distribution of the log-likelihood ratio statistic with large sample size has been determined by Wilks [27]. Let

$$\Lambda = -2\ln\lambda = -2\left[\ln L_0(\hat{\theta}_{H_0}) - \ln L_1(\hat{\theta}_{H_1})\right]$$

According to Wilks' theorem [27], the statistic Λ has an asymptotically chi-squared χ^2 distribution with v degrees of freedom $\chi^2(v)$. The degrees of freedom is equal to the difference in the number of independent parameters under H_0 and H_1, that is, $v = q-1$. According to (4), when $\lambda \le c$, the hypothesis H_0 is rejected. Thus when $\Lambda \ge -2\ln c$, the E-free model is much more appropriate than the E-fixed model. Taking the significance level β and the distribution of Λ into account, it can be concluded that:

$$-2\ln c = \chi^2_{1-\beta}(q-1)$$

where $\chi^2_{1-\beta}(q-1)$ denotes the $100(1-\beta)th$ percentile from the chi-squared distribution $\chi^2(q-1)$.

Therefore, the decision rule for judging the degradation mechanism consistency is

1) If $0 \le \Lambda < \chi^2_{1-\beta}(q-1)$, the null hypothesis is not rejected. The activation energy keeps unchanged, and the degradation mechanism remains consistent in the SSADT;
2) If $\Lambda \ge \chi^2_{1-\beta}(q-1)$, the null hypothesis is rejected at the β s-significance level. The activation energy depends on the stress, and a change in the degradation mechanism occurs.

From the statistical analysis above, the major steps for the consistency analysis of the degradation mechanism in a SSADT can be summarized as:

1) Select the proper degradation model and acceleration model. For rubber seals and light emitting diodes, the transformed linear degradation model (12) and Arrhenius model (15) are adopted;
1) Make the null hypothesis H_0 for E-fixed model, and the alternative hypothesis H_1 for E-free model. Derive the log-likelihood functions corresponding to the SSADT data under H_0 and H_1;
2) Calculate the value of test statistic Λ and the critical value $\chi^2_{1-\beta}(q-1)$ in the LR test method;
3) Compare the computed test statistic with the critical value, and make the decision on the degradation mechanism consistency based on the decision rule.

A method for the verification of the mechanism consistency analysis may be identified. By performing the asymptotic analysis and considering the expectation of Kullback-Leibler divergence, Akaike proposed the Akaike information criterion (AIC) for statistical model selection [1]. In practice, the conclusion from AIC may provide a reference for the validity of the decision from LR test method. The AIC value of a statistical model can be calculated by:

$$\text{AIC} = -2\ln L(\hat{\theta}) + 2\gamma \tag{31}$$

where the parameter γ is the dimension of $\boldsymbol{\theta}$. The value of $-2\ln L(\hat{\boldsymbol{\theta}})$ is related to the goodness of fit of the model, while the value of 2γ is associated with the complexity of the model. The smaller the value of $-2\ln L(\hat{\boldsymbol{\theta}})$, the better fit is the model. The smaller the value of 2γ, the less complex is the model. Given some candidate models for data fitting, the desired model is the one with the minimum AIC value.

According to the definition of the AIC, it deals with the trade-off between the goodness fit of the model and the complexity of the model. In contrast, the LR test only considers the goodness of fit of the model. Note that the s-significance level is specified in the LR test. However, it is difficult to determine the s-significance level in the AIC unless some other complicated methods are employed.

If the E-fixed model is an appropriate model for data fitting as the E-free model, the difference in the maximum values of log-likelihoods for two models is small. Due to less independent parameters in the E-fixed model, the AIC value of the E-fixed model is smaller than that for the E-free model. In terms of the goodness of fit and the complexity of the model, the E-fixed model is better than E-free model in this instance. If the E-free model is much more appropriate for data fitting than E-fixed model, the difference mentioned above is large. The AIC value of the E-fixed model is much larger than that for the E-free model, so the E-fixed model is worse than E-free model at this stage. Thus the decision from LR test method should be in accordance with the conclusion from the AIC. We will demonstrate this point in the numerical examples.

5. Numerical examples

The E-fixed model and E-free model are compared in numerical example 1 and example 2 respectively. Note that the E-fixed model denotes the consistent degradation mechanism and the E-free model denotes the inconsistent degradation mechanism. Both examples use the simulation data with activation energy from a real test.

5.1. Example 1

The degradation mechanism remains consistent in a SSADT. Silicone rubber seals are typical highly reliable products, and they are susceptible to thermal oxidation. Due to the effcet of thermal oxidation, their mechanical performance degrade in the period of storage or service. The degradation trend can be characterized by monitoring the compression set cs. When the compression set exceeds the threshold 70%, a failure in rubber seals occurs. Predicting the lifetime of rubber seals by the ADT is a subject of ongoing interest for years. The original activation energy is reported in a previous study on thermal aging of vulcanized polysiloxane rubbers [20]. In the original data, the ADT is carried out at constant temperatures within a wide temperature range (298-488 K). The result based on the analysis of compression set data shows the change in the activation energy occurs. For temperatures above and below 423 K, the activation energies are 77 ± 45 kJ/mol and 22 ± 7 kJ/mol respectively. Thus example 1 uses the simulated SSADT data of 4 silicone rubber seals at step-up temperatures of 393 K, 408 K, and 423 K. The assumed model parameters are $B = 1.05$, $\alpha = 0.38$, $\sigma = 0.01$, $A = 6.46$ and $E = 27.12$ kJ/mol respectively. For degradation modeling, the measured compression set cs is converted into the performance degradation data y by $y = 1 - cs/100$. According to the degradation model in (12), the simulated sample SSADT degradation paths are obtained and shown in Figure 2.

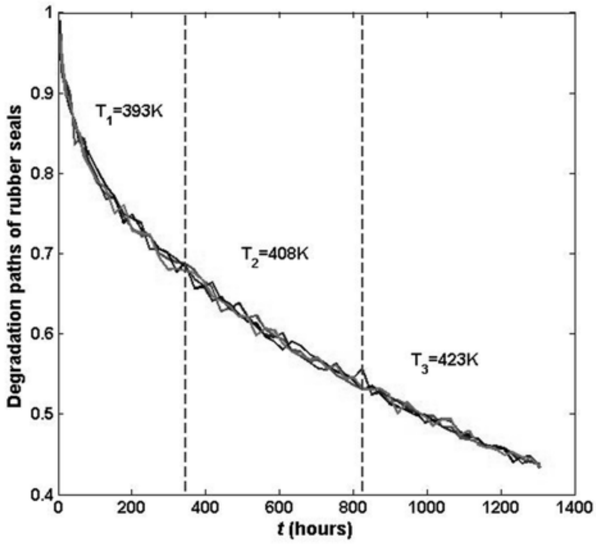

Fig. 2. Simulated SSADT degradation paths in example 1

By the numerical solution method, the MLEs of unknown parameters in the E-fixed model and E-free model are derived. To illustrate the goodness of fit of two models, these MLEs are compared with true values of unknown parameters. The comparison results for the E-fixed model and E-free model are presented in Tables 1 and 2 respectively. Note that the E-fixed model and E-free model are applied to the same set of data shown in Figure 2. From Tables 1 and 2, it is seen that both models fit well to the same set of data.

Table 1. Comparison between MLEs and true values for E-fixed model in example 1

Parameters	B	α	σ	A	E
MLE	1.044	0.390	0.01	6.20	26.40
True value	1.05	0.38	0.01	6.46	27.12

Further, the test statistic Λ is calculated as:

Table 2. Comparison between MLEs and true values for E-free model in example 1

Parameters	B	α	σ	K_1	K_2	K_3
MLE	1.047	0.385	0.01	0.154	0.210	0.276
True value	1.05	0.38	0.01	0.158	0.215	0.285

$$\begin{aligned}\Lambda &= -2\ln\lambda = -2\left[\ln L_0(\hat{\theta}_{H_0}) - \ln L_1(\hat{\theta}_{H_1})\right] \\ &= -2\times(767.391 - 767.468) = 0.154\end{aligned} \tag{32}$$

The small value of test statistic also suggests that both models are appropriate for data fitting. The number of temperature stress levels is 3 in example 1. Given the s-significance level $\beta = 5\%$, the critical value $\chi^2_{1-\beta}(q-1)$ satisfies $\chi^2_{0.95}(2) = 5.99 > \Lambda$. Thus the E-fixed model for the null hypothesis is not rejected, and the degradation mechanism is considered to be consistent in this SSADT based on the decision rule.

On the other hand, the AIC values of two models are calculated respectively as follows:

$$\mathrm{AIC}_{E\text{-fixed}} = -2\times 767.391 + 2\times 5 = -1524.8 \tag{33}$$

$$\mathrm{AIC}_{E\text{-free}} = -2\times 767.468 + 2\times 7 = -1520.9 \tag{34}$$

The AIC value of the E-fixed model is smaller than that of the E-free model. In terms of the goodness of fit and the complexity of the model, the E-fixed model is more appropriate than the E-free model. According to the AIC criterion, the degradation mechanism does not change in this SSADT.

5.2. Example 2

The degradation mechanism changes in a SSADT. As mentioned above, a change from 22 ± 7 kJ/mol to 77 ± 45 kJ/mol occurs at temperatures higher than 423 K. Thus example 2 uses the simulated SSADT data of 4 silicone rubber seals at step-up temperatures of 393 K, 408 K, and 428 K. The assumed model parameters are $B = 1.05$, $\alpha = 0.38$, $\sigma = 0.01$, $A = 6.46$, $E_1 = 27.12$ kJ/mol (393-423 K) and $E_2 = 120$ kJ/mol (423-428 K) respectively. The simulated sample SSADT degradation paths in example 2 are shown in Figure 3.

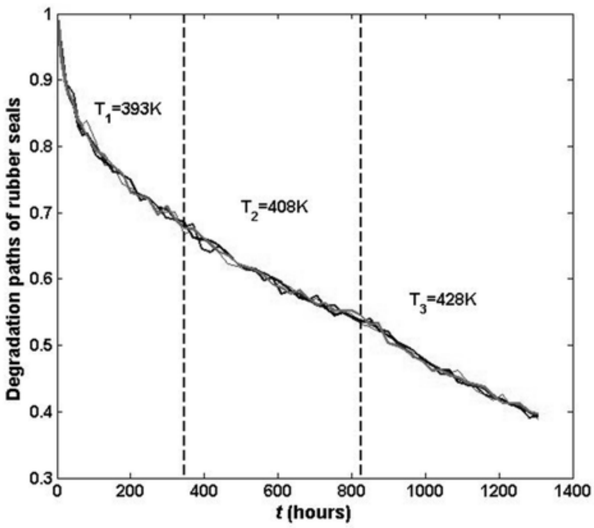

Fig. 3. Simulated SSADT degradation paths in example 2

Similarly, the MLEs of unknown parameters are derived and the comparison with true values for the E -fixed model and E -free model are presented in Tables 3 and 4 respectively. Note that example 1 and example 2 differ in the goodness of fit of E -fixed model to the test data. In example 1, the MLEs approach to the true values for the E -fixed model. However, the disparity between MLEs and true values are obvious for the E -fixed model in example 2. In addition, the E -free model fits well to the test data in both examples.

Table 3. Comparison between MLEs and true values for E -fixed model in example 2

Parameters	B	α	σ	A	E
MLE	1.106	0.304	0.01	12.96	47.43
True value	1.05	0.38	0.01	6.46	27.12; 120

Table 4. Comparison between MLEs and true values for E -free model in example 2

Parameters	B	α	σ	K_1	K_2	K_3
MLE	1.054	0.376	0.01	0.160	0.220	0.438
True value	1.05	0.38	0.01	0.158	0.215	0.425

Further, the test statistic Λ is calculated as:

$$\begin{aligned}\Lambda &= -2\ln\lambda = -2\left[\ln L_0(\hat{\theta}_{H_0}) - \ln L_1(\hat{\theta}_{H_1})\right] \\ &= -2\times(763.770 - 782.848) = 38.16\end{aligned} \quad (35)$$

As the critical value satisfies $\chi^2_{0.95}(2) = 5.99 < \Lambda$, the E -fixed model for null hypothesis is rejected at the 5% s-significance level. It can be concluded that the degradation mechanism changes in this SSADT by the decision rule.

In addition, the AIC values of two models are:

$$\text{AIC}_{E\text{-fixed}} = -2\times 763.770 + 2\times 5 = -1517.5 \quad (36)$$

$$\text{AIC}_{E\text{-free}} = -2\times 782.848 + 2\times 7 = -1551.7 \quad (37)$$

Note that the goodness of fit dominates the comparison of AIC values in example 2, whereas the complexity of model is the dominating factor in example 1. As the AIC value of the E -free model is smaller than that of the E -fixed model, the E -free model is more appropriate than the E -fixed model in example 2. According to the AIC criterion, a change in the degradation mechanism occurs in this SSADT.

In both examples, the decision from the LR test method accords with the conclusion from the AIC, and the decision agrees with the original setting of degradation mechanism consistency.

6. Conclusion

The variation of the degradation mechanism exists in the accelerated testing of some products such as rubber seals and light emitting diodes. The confident extrapolation of accelerated results requires the consistent degradation mechanism, thus the degradation mechanism consistency in the SSADT must be examined for the reliability assessment. In this paper, we make new contributions by proposing a statistical test method with an exact decision rule for the consistency analysis of degradation mechanism in a SSADT. First we point out that the difference in activation energies among various stress levels represents the change in the degradation mechanism. The basic principle of the LR test method is depicted. By means of a model similar to the cumulative exposure model in a SSALT, we establish a transformed linear degradation model for SSADT data. Then the derivation of MLEs for unknown parameters in two distinct models is discussed. Finally we propose a specific decision rule for the mechanism consistency analysis by the LR test method. The AIC criterion is introduced to verify the availability of the decision from the LR test method.

The proposed method is applied to the numerical examples with the consistent failure mechanism and inconsistent failure mechanism. The obtained statistical analysis in two examples indicates that the change of the degradation mechanism can be identified by the proposed method. In both cases, the judgment derived from the decision rule is in accordance with the conclusion from the AIC and the original setting of parameters, which verifies the proposed method.

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