Agricultural tractors are equipped with air braking systems to control and operate the braking systems of towed agricultural vehicles. This paper presents a mathematical model of a hydraulically actuated trailer brake control valve. The results of the statistical Kolmogorov-Smirnov test confirmed the consensus between the experimental and simulated pressure transients during testing the response time of a farm tractor's control circuit. The computer model developed in Matlab-Simulink can be used as a tool to analyze transient processes by using simulation methods in the process of designing the air braking systems of farm tractors.

**Keywords:** farm tractor, air braking system, hydraulically actuated trailer brake control valve, mathematical modelling, simulation.

### 1. Introduction

Currently, in farm tractors moving at speeds of up to 50–60 km·h\(^{-1}\), hydraulically actuated wet multi-disc brakes [24] are commonly used. In low- and medium-power tractors, simple and inexpensive hydraulic brake systems with no power assistance are preferred. These manually operated systems work at a low operating pressure and a small required brake cylinder volume [14]. However, in more powerful tractors, hydraulic systems powered by the tractor's hydraulic system are mainly used [15, 18]. These hydraulic brake systems allow the supply of the flow of fluid from the tractor hydraulic pump to the service brake cylinders, ranging from small to large in volume, at a greater pressure in a relatively short time. Apart from the service and parking brake systems, today's farm tractors are fitted with air brake systems to actuate the air braking systems of assembled machinery or trailers. Often, so-called combined systems [29] are used to operate with both the single- and dual-line air braking systems of towed vehicles.

A typical air braking system of an agricultural tractor consists of two major parts: an energy supply unit and a control device. The role of the energy supply unit is to purify and compress the air and to maintain the adequate pressure in the tractor and trailer reservoirs so that the required trailer braking performance is ensured. The control device permits a smooth gradual braking process for the tractor-trailer combinations. The co-operation of the tractor hydraulic braking system and the trailer air braking system is provided by a hydraulically actuated trailer brake control valve (Fig. 1).

The braking systems of agricultural vehicles driven on public roads must meet several specific requirements [3, 4] for braking performance, response time during rapid braking (up to 0.6 s) and braking compatibility with combination vehicles [27].

The behaviour of the braking system of a tractor and a trailer, including the unsteady states, may be predicted already at an early stage of design using simulation methods. The main benefit of the model-based design (MBS) is that it increases the speed and efficiency in testing new solutions, provides the possibility of confronting them with the applicable requirements, and detects any errors resulting from malfunctioning or mistakenly taken assumptions earlier than in the case of constructing physical prototypes [25].

In mathematical modelling of multi-circuit air braking systems, pneumatic elements of a discrete nature (such as line filters, fasteners) and even elements of parameters distributed in a continuous manner (e.g. pipes) are usually substituted with idealized elements in the form of a lumped volume and resistance [9, 13, 19]. This yields a mathematical model as a system of ordinary differential equations, which are able to be solved within a majority of software designed for numerical simulation, including object-oriented programs [12, 30].

The main difficulty in modelling the air braking system of farm tractors is the lack of appropriate mathematical models for some components and devices, particularly the trailer brake control valves. Most of the brake valve models known from the literature concern typical air brake valves used in commercial vehicles [6, 22, 28] and trailers [8, 10, 11, 21, 23].
This paper presents mathematical modelling of a trailer brake control valve hydraulically actuated by brake fluid pressure from the service hydraulic brake system of a farm tractor. The mathematical and computer model of this valve considers the heat exchange and the friction and inertia forces of movable elements, which are generally omitted in a modelling process [6, 22, 28]. The computer model implemented in Matlab-Simulink is used to test the response time of the control device of a typical tractor’s air braking system. For modelling the energy supply unit, the relationship described in [12] is used. The developed model can be used in a farm tractor design process for the dynamic calculation of the tractor’s air braking system.

2. Experimental setup for testing of the tractor pneumatic braking system

A simplified schematic diagram of the combined pneumatic braking system of a Pronar 1523A agricultural tractor [26] equipped with a hydraulic brake drive is shown in Fig. 1. The energy supply unit includes filter 1, air compressor 2, unload valve (governor) 3, and compressed air reservoir 4. Compressed air is also fed to the control device, which includes proportional brake valve 11 and inverse brake valve 7. The trailer control valve 11 is connected, via ports 41 and 42, with the hydraulic system of the tractor service brakes. When the brake pedal is depressed, the hydraulic control pressure will act on valve 11, causing this valve to open thus increasing the air pressure in the line with coupling head 10 that controls the dual-line trailer braking system. The trailer braking system supply line is connected to coupling head 9. For controlling the single-line trailer braking system, inversion valve 7 is used, which, upon the pressure increase in its control port 4 causes a pressure drop in the supply and control line with coupling head 8.

Fig. 1. Schematic diagram of a Pronar 1523A farm tractor’s combined air braking system coupled with the experimental control line response time testing setup: 1 – filter, 2 – FOS Polmo Łódź 607.23.931 compressor, 3 – Visteon 51 10 011 unloader valve, 4 – 20 dm³ air reservoir, 5 – drain valve, 6 – single pressure gauge, 7 – Visteon 45 10 016 inversion trailer control valve, 8 – “single-line” coupling head (black), 9 – “supply” coupling head (red), 10 – “brake” coupling head, 11 – Haldex 329 020 311 trailer brake control valve, 12 – tractor service brake hydraulic system, 13 – 385 cm³ air vessel, 14 – 2.5 m-long 13 mm-internal diameter pipe, 15 – pressure transducer, 16 – pedal force transducer, 17 – input/output adapter, 18 – computer with a measuring card.

The experimental setup for testing the response time of the tractor dual-line pneumatic system control device is distinguished in Fig. 1 with the grey background. Pressure in selected locations of the air brake system was measured with Danfoss type MBS 32 industrial pressure transducer 10 (with a pressure range of 0–10 bar; an output range of 0–10 V; and an accuracy class of 0.3%). CL 23-type extensometric brake pedal force sensor 16 complete with a CL10D type ZEP-WN industrial amplifier (with a measuring range of 0–1 kN; an output signal range of 0–10 V; and an accuracy class of 0.1%) was used for measuring the force on the brake pedal. The transducers were voltage supplied from input/output adapter 17. The output voltage signals from the force and pressure transducers were acquired from the input/output adapter using a (12 bit resolution) Senga MC1212 measuring card and then directly converted into force and pressure data with the use of integrated software installed on computer 18 designed for data collection during the run of the tests.

3. Modelling of the hydraulically actuated trailer brake control valve

In the control circuit of the Pronar 1523A tractor’s air system, a Haldex 329 020 311 brake valve [5] was used as trailer control valve 11 (Fig. 1), which has the same design as a Wabco 470 015 brake valve [29]. A schematic diagram of the structure of a typical hydraulically actuated valve is given in Fig. 2.

When the brake pedal is depressed, the pressure rises both in the hydraulic wheel brake cylinders of the tractor and in the hydraulic chambers $V_{41}$ and $V_{42}$ of the control valve coupled with the tractor’s twin (parallel) master cylinder. Under the influence of the brake fluid pressure, graduating piston 3 and the piston 1 move downwards, causing inlet head 2 to open. The compressed air from the inlet chamber $V_1$ flows into a variable-capacity outlet chamber $V_2$ as a mass flux $m_{1,2}$. The flux $m_2$ flowing out from the outlet chamber $V_2$ is routed to the control circuit of the trailer braking system, resulting in the activation of the trailer brakes. The air mass flux $m_1$ flows into a fixed-volume inlet chamber $V_1$ from the supply circuit.

During the brake release caused by the decrease in hydraulic control pressure, piston 1 raises again by the action of spring 5. Poppet valve 2 closes (thus cutting off the inlet chamber from the outlet system).
flow reversal), and then flows out to the atmosphere. The mass flux \( \dot{m}_{\text{out}} \) is given by:

\[
\dot{m}_{\text{out}} = -\frac{\dot{m}_{\text{out}}}{RT_m},
\]

where: \( \dot{m}_{\text{out}} \) – the fluid mass flux \( \text{kg·s}^{-1} \), \( R \) – gas constant for air, being equal to 288 \( [J·K^{-1}·kg^{-1}] \), \( T_m \) – air temperature \([K]\) upstream of the resistance, \( \sigma \) – the Saint-Venant–Wantzel function for the critical product \( \sigma \) of the pressure upstream and downstream of the resistance, respectively. The mass flux is given by:

\[
\dot{m}_{\text{in}} = \frac{\dot{m}_{\text{out}}}{\Psi_{\text{max}}(\sigma)},
\]

where: \( \Psi_{\text{max}} \) – maximum value of the Saint-Venant–Wantzel function. Instead of the dimensionless two-range Saint-Venant flow function \( \Psi(\sigma) \), given by:

\[
\Psi(\sigma) = \begin{cases} 
1 & \text{for } \sigma \leq \sigma_* \\
\frac{1}{\Psi_{\text{max}}} \left( \frac{2\kappa}{\kappa - 1} \right) \kappa^{\frac{1}{\kappa-1}} & \text{for } \sigma_* < \sigma \leq 1
\end{cases}
\]

a single-range hyperbolic function \([19, 20]\) with a constant parameter value of \( b = 1.13 \) typical of pneumatic air braking systems was used, as being more convenient for numerical computation and still sufficiently accurate:

\[
\Psi(\sigma) = \frac{b(1 - \sigma)}{b - \sigma}
\]

Using equations (3) and (5), the equations of mass fluxes flowing through the braking valve are obtained as:

\[
\dot{m}_{1-2} = \mu_1 A_1 \frac{p_1}{\Psi_{\text{max}}(\sigma)} \Psi(\sigma) \frac{\dot{m}_{\text{in}}}{RT_m}
\]

\[
\dot{m}_{2-3} = \mu_2 A_2 \frac{p_2}{\Psi_{\text{max}}(\sigma)} \Psi(\sigma) \frac{\dot{m}_{\text{out}}}{RT_m}
\]

The flow cross-section areas \( A_1 \) (during braking) and \( A_2 \) (during releasing) are dependent on the piston travel \( h_p \) and the distance \( h_v \) of head 2 from the valve seat and are given by:

\[
A_1 = \begin{cases} 
0 & \text{if } h_v \leq h_{ro} \\
\pi D_w (h_v - h_{ro}) & \text{if } h_{ro} < h_v \leq h_{wm}
\end{cases}
\]

\[
A_2 = \begin{cases} 
0 & \text{if } h_p \geq h_v - h_{po} \\
\pi D_{sw} \left( D_{sw}^2 - D_{ipv}^2 \right) & \text{if } h_{po} < h_v < h_{wm}
\end{cases}
\]

where: \( D_{sw} \) – stationary seat average diameter \([m]\) and inner diameter \([m]\), respectively; \( D_{ipv} \) – movable seat outer diameter \([m]\) (in...
the piston), \( h_{wp} \), \( h_{wm} \) – head 2 position [m] corresponding to the beginning of opening (allowing for plate seal deformation) and position [m] corresponding to the attaining of the maximum flow field value, respectively, \( D_{wp} \) – the average diameter [m] of the moveable seat, \( D_{1} \), \( D_{2} \) – the outer and the inner diameter [m], respectively, of the head sleeve, \( d_{s} \) – the screw diameter [m] of the parking brakes mechanism, \( h_{wp} \), \( h_{wm} \) – piston position [mm] corresponding to the beginning of opening the passage to the atmosphere (allowing for plate seal deformation) and position [mm] in which the flow field attains the maximum value, respectively.

The following relationship exists between the displacement \( h_{p} \) of control piston 1 and the displacement \( h_{sp} \) of head 2:

\[
h_{p} = \begin{cases} 
0 & \text{if } h_{p} \leq h_{o} \\
\frac{(h_{p} - h_{b})}{h_{p} - h_{o}} & \text{if } h_{p} > h_{o}
\end{cases}
\]  

(10)

where: \( h_{o} \) – the maximum distance (clearance) [m] between head 2 and piston 1 in the upper extreme position.

With the assumed simplifications, the mechanical elements of the brake valve can be considered as a dynamical system with one degree of freedom with variable mass, as described by the following equation of motion:

\[
m_{p} \frac{d^{2}h_{p}}{dt^{2}} = F_{H} + F_{P2a} + F_{P} + F_{v} + F_{f}
\]  

(11)

where: \( m_{p} \) – reduced mass [kg] of the elements moving together with piston 1, \( F_{H} \) – force [N] of the brake fluid acting on piston 3, \( F_{P2a} \) – pressure force [N] acting on piston 1, \( F_{P} \) – force of piston return spring 5 [N], \( F_{v} \) – force [N] of piston friction against the housing, \( F_{f} \) – force [N] of the pressure of head 2 acting on piston 1.

The reduced mass of the elements moving together with piston 1 is:

\[
m_{p} = \begin{cases} 
m_{p} + m_{b} + m_{p} + m_{sp} / 3 & \text{if } h_{p} \leq h_{o} \\
m_{p} + m_{b} + m_{p} + m_{sp} / 3 & \text{if } h_{p} > h_{o}
\end{cases}
\]  

(12)

where: \( m_{b} \) – mass [kg] of graduating piston 3, \( m_{p} \) – mass [kg] of control piston 1, \( m_{sp} \) – mass [kg] of head 2 with the sleeve guide, \( m_{sp} \) – mass [kg] of spring 4 clamping head 2, \( m_{sp} \) – mass [kg] of return spring 5, \( m_{sp} \) – mass [kg] of the brake fluid.

The forces of pressure acting on the pistons are:

\[
F_{H} = \frac{\pi D_{2}^{2}}{4} p_{41} + \frac{\pi D_{2}^{2} - D_{1}^{2}}{4} p_{42}
\]  

(13)

\[
F_{P2a} = \frac{\pi D_{1}^{2}}{4} p_{2a} - \frac{\pi D_{1}^{2} - D_{2}^{2}}{4} p_{2a} = -\frac{\pi D_{1}^{2} - D_{2}^{2}}{4} (p_{2a} - p_{2a})
\]  

(14)

where: \( p_{2a}, p_{42} \) – pressures [Pa] of the brake fluid in the control hydraulic chambers, \( p_{2a} \) – atmospheric pressure [Pa], \( p_{2} \) – air pressure [Pa] in chamber \( V_{2} \), \( D_{ap}, D_{bp} \) – diameters [m] of graduating piston 3 [m], \( D_{sp} \) – diameter [m] of piston 1.

The force of the pressure of return spring 5 acting on piston 1 is calculated from the relationship:

\[
F_{sp} = \left( F_{spo} + c_{sp} h_{sp} \right)
\]  

(15)

where: \( F_{spo} \) – preset force [N] of spring 5 for \( h_{p}=0 \); \( c_{sp} \) – stiffness [N·m⁻¹] of spring 5.

Assuming that the poppet valve has an unladen design (\( D_{sp} = D_{2} \)), the force of head pressure acting on the piston is described by the relationship:

\[
F_{v} = \left\{ \begin{array}{ll}
\frac{\pi D_{1}^{2} - D_{2}^{2}}{4} \left( p_{1} - p_{2} \right) & \text{if } h_{p} \leq h_{o} \\
\frac{\left( p_{1} - p_{2} \right)^{2} - \pi D_{1}^{2} - D_{2}^{2}}{4} & \text{if } h_{p} > h_{o}
\end{array} \right.
\]  

(16)

where: \( h_{o} \) – head guide kinematic friction force [N], \( c_{v} \) – stiffness [N·m⁻¹] of spring 4, \( k_{v} \) – viscous friction coefficient [N·s·m⁻¹].

The force of static and kinetic friction of the pistons against the housing is described using the Karnopp model [1] according to:

\[
F_{f} = \left\{ \begin{array}{ll}
\left( F_{vp} - F_{cp} \right) \cdot \min \left( F_{vp} / F_{dc} \right) & \text{if } \frac{dh_{p}}{dt} < 0 \\
\left( F_{sp} + k_{v} \cdot dh_{p} \right) & \text{if } \frac{dh_{p}}{dt} > 0
\end{array} \right.
\]  

(17)

The total kinetic friction force \( F_{v} \) and the kinetic friction force \( F_{f} \) of piston 1, as well as the kinetic friction force \( F_{f} \) of the guide of head 2, are described by the experimental relationships:

\[
F_{f} = \pi D_{p} \left( f_{s} + k_{s} \left( p_{2} - p_{42} \right) \right) + \pi D_{a} \left( f_{s} + k_{s} \left( p_{41} - p_{42} \right) \right) + \pi D_{b} \left( f_{s} + k_{s} \left( p_{41} - p_{42} \right) \right) + \pi D_{a} \left( f_{s} + k_{s} \left( p_{41} - p_{42} \right) \right) + \pi D_{b} \left( f_{s} + k_{s} \left( p_{41} - p_{42} \right) \right)
\]  

(18)

where: \( F_{f} \) – total external force [N], \( f_{s}, f_{s} \) – static friction force and kinetic friction force [N·m⁻¹], respectively, per unit piston perimeter, independent of the differential pressure across the piston, \( k_{s}, k_{s} \) – proportionality factors [m].

Based on the energy conservation law for open systems, excluding the kinetic and potential energy, the following equations for the change in the internal energy of the air in the control volumes \( V_{1} \) and \( V_{2} \) of the chambers are obtained:

\[
\frac{dU_{1}}{dt} = Q_{1} + H_{1} - H_{1-2}
\]  

(19)

\[
\frac{dU_{2}}{dt} = Q_{2} + W_{1-2} + H_{2} - H_{2-3}
\]  

(20)

where: \( U_{1} \) – internal energy [J] of the air in a given chamber, \( Q_{1} \) – rate of heat flow [W] exchanged with the environment, \( H_{1} \) – rate of enthalpy flow [W] reaching (+) or leaving (-) the chamber, \( W_{1} \) – rate of external work [W] done by the air in the chamber:

\[
U_{1} = m_{v} \cdot c_{v} T_{l} ; \quad H_{1} = m_{v} \cdot c_{v} T_{m} ; \quad W_{1} = -p_{1} \cdot V_{l} ; \quad Q_{1} = a_{l} \cdot A_{l} (T_{m} - T_{l})
\]  

(21)

where: \( c_{v}, c_{p} \) – specific heat capacity of the gas at constant volume and constant pressure: \( c_{v} = 717 \, [\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}] \), \( c_{p} = 1005 \, [\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}] \), \( T_{l} \) – abso-
lute temperature [K] of air in the i-th chamber, \( T_w \) – flow temperature [K] (for the flow leaving the chamber \( T_w = T_1 \)), \( \alpha_i \) – heat transfer coefficient [Wm\(^{-2}\)K\(^{-1}\)], \( p_1 \) – absolute pressure in the i-th chamber; \( A_i \) – heat transfer area [m\(^2\)], \( T_w \) – valve wall temperature [K] equal to ambient temperature \( T_e \).

After the differentiation of the internal energy and using Equation (2) and the Clapeyron equation in the differential form [16]:

\[
R \left( \frac{dm_{v1}}{dt} T_1 + m_{v1} \frac{dT_1}{dt} \right) = p_1 \frac{dp_1}{dt} + V_1 \frac{dp_1}{dt} \tag{22}
\]

the following differential equations for the variations of air pressure and temperature in individual valve chambers are obtained:

\[
\frac{dp_1}{dt} = \frac{1}{V_1} \left( \kappa - 1 \right) \left( \dot{Q}_1 + H_1 - H_{1-2} \right) \quad \dot{Q}_1 = \alpha_1 A_i \left( T_w - T_1 \right) \tag{23}
\]

\[
\frac{dT_1}{dt} = \frac{T_1}{p_1 V_1} \left[ V_1 \frac{dp_1}{dt} - RT_1 \left( m_{v1} - m_{v1-2} \right) \right] \tag{24}
\]

\[
\frac{dp_2}{dt} = \frac{1}{V_2} \left( \kappa - 1 \right) \left( \dot{Q}_2 + H_{1-2} - H_{2-3} \right) - \kappa \cdot p_2 \frac{dp_2}{dt} \tag{25}
\]

\[
\frac{dT_2}{dt} = \frac{T_2}{p_2 V_2} \left[ p_1 \frac{dp_1}{dt} + V_2 \frac{dp_2}{dt} - RT_2 \left( m_{v2} - m_{v2-3} \right) \right] \tag{26}
\]

Using the relationship between the mass flow and the volume flow of the brake fluid ( \( \dot{m}_v = \frac{m}{\rho} \) ) and after differentiation of mass in relationships (1):

\[
m = d \rho \cdot V + dV \cdot \rho \tag{27}
\]

the following equations for the volume flux are obtained by taking into account the fluid bulk modulus and housing compliance [7]:

\[
\pm \dot{m}_{41V} = \frac{\pi D_m^2}{4} \frac{dh_1}{dt} + \frac{V_{A1}}{B_2} \frac{dp_{A1}}{dt} \tag{27}
\]

\[
\pm \dot{m}_{42V} = \frac{\pi}{4} \left( \frac{D_m^2 - D_n^2}{4} \right) \frac{dh_2}{dt} + \frac{V_{A2}}{B_2} \frac{dp_{A2}}{dt} \tag{28}
\]

where: \( B_2 \) – effective bulk modulus [Pa].

### 4. Example tractor air braking system simulation

For the validation of the computer model of the trailer control brake valve, as well as the models of the other components, the experimental results of the response time testing of the Pronar 1523A farm tractor’s dual line air braking system (Fig. 1) were utilized. The response time of the tractor’s dual-line pneumatic system control circuit was determined according to the method specified in Regulation 13 of the ECE [4] based on the variations in the brake pedal force and pressure as measured at the end of a 2.5 m-long 13 mm-internal diameter line (an imitation of the trailer control line) connected to the brake coupling.

The computer models of most components of the energy supply unit and the control device were created in the form of S-function type graphical sub-systems written in the m-files of the Matlab-Simulink program, based on own algorithms and procedures [9, 12]. For modelling the pipes of the hydraulic brake system, the transmission line method, as described by Krus [17], was applied.

The actual time variations of the pedal force \( F_p \) (input signal) and the actual pressure variations (output signals) recorded during the experimental tests at the points in the system, as indicated in Fig. 1, including pressure \( p_{ce} \) in air reservoir 4, pressure \( p_{ce} \) in vessel 13 and pressure \( p_{ce} \) at the end of line 14 connected to brake coupling head 10, were input to the computer model in the form of From File-type source blocs.

Sample simulation (the solid lines) and experimental (the dotted lines) test results obtained during the testing of the response time of the Pronar 1523A air braking system’s control device are presented in Fig. 3. From the modelling viewpoint, i.e. the calculation of the response time from the theoretical pressure \( p_s \), it can be assumed that the developed model is sufficiently accurate. This is evidenced by the obtained values of the statistical indicators of agreement [31] between the time function graphs of the experimental pressure \( p_{ce} \) and the simulated pressure \( p_r \) at the end of the control line (the coefficient of determination \( R^2 > 98\% \) and the mean absolute percentage error MAPE=5%) and, above all, a small shift in time (less than 0.01 s) between both graphs at the pressure equal to 75% of the asymptotic pressure. The adequacy of the computer model was also confirmed by the results of the Kolmogorov-Smirnov test at a significance level of 5%. The obtained results of the validation of the tractor braking system computer model are also indirectly indicative of the suitability of the mathematical and computer trailer control brake valve model for simulation of the transient processes occurring in the braking systems of vehicles.

### 5. Summary

In the mathematical modelling of the trailer brake control valve, the actual course of the phenomena accompanying the operation of braking valves, usually omitted at the stage of creating a physical or a computer model, has been taken into account to a considerable extent. The differential equations of the variations in air pressure and temperature in individual valve chambers consider the heat exchange with the environment, while the differential equation of the piston motion takes into account the static-kinetic friction forces and the inertial forces. The discrete brake model equations can be used to formulate...
the mathematical and computer models of other braking valves of a similar design.

The obtained results of the Kolmogorov-Smirnov test, as well as the values of the statistical indicators $R^2$, MAPE, assessing the consistence between the experimental and the simulation pressure transients at the end of the control line during testing of the control circuit response time have confirmed the adequacy of the Pronar 1523A tractor’s air braking system computer model implemented in Matlab-Simulink.

The developed computer model can be used to study transient processes in the air braking system of agricultural tractors equipped with hydraulic brakes and to predict the dynamic properties of the air braking system of tractor-trailer units (speed and the synchrony of action) using simulation methods.

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