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SYSTEM RELIABILITY OPTIMIZATION: A FUZZY MULTI-OBJECTIVE GENETIC ALGORITHM APPROACH

OPTYMALIZACJA NIEZAWODNOŚCI SYSTEMU: METODA ROZMYTEGO ALGORYTMU GENETYCZNEGO DO OPTYMALIZACJI WIELOKRYTERIALNEJ

System reliability optimization is often faced with imprecise and conflicting goals such as reducing the cost of the system and improving the reliability of the system. The decision making process becomes fuzzy and multi-objective. In this paper, we formulate the problem as a fuzzy multi-objective nonlinear program. A fuzzy multi-objective genetic algorithm approach (FMGA) is proposed for solving the multi-objective decision problem in order to handle the fuzzy goals and constraints. The approach is able flexible and adaptable, allowing for intermediate solutions, leading to high quality solutions. Thus, the approach incorporates the preferences of the decision maker concerning the cost and reliability goals through the use of fuzzy numbers. The utility of the approach is demonstrated on benchmark problems in the literature. Computational results show that the FMGA approach is promising.

***Keywords**: System reliability optimization, multi-objective optimization, genetic algorithm, fuzzy optimization, redundancy.*

Często spotykanym problemem w optymalizacji niezawodności systemu są niedokładnie określone i sprzeczne cele, takie jak zmniejszenie kosztów systemu przy jednoczesnej poprawie jego niezawodności. Proces podejmowania decyzji staje się wtedy rozmyty i wielokryterialny. W niniejszej pracy, sformułowaliśmy ten problem jako rozmyty wielokryterialny program nieliniowy (FMOOP). Zaproponowaliśmy metodę rozmytego wielokryterialnego algorytmu genetycznego (FMGA), która pozwala rozwiązać wielokryterialny problem decyzyjny z uwzględnieniem rozmytych celów i ograniczeń. Podejście to jest uniwersalne, co pozwala na rozwiązania pośrednie, prowadzące do rozwiązań wysokiej jakości. Metoda uwzględnia preferencje decydenta w zakresie celów związanych z kosztami i niezawodnością poprzez wykorzystanie liczb rozmytych. Użyteczność FMGA wykazano na przykładzie wzorcowych problemów z literatury. Wyniki obliczeń wskazują, że podejście FMGA jest obiecujące.

***Słowa kluczowe**: Optymalizacja niezawodności systemu, optymalizacja wielokryterialna, algorytm genetyczny, optymalizacja rozmyta, nadmiarowość.*

1. Introduction

System reliability optimization is a very important subject matter in industry. Reliable systems are essential for sustainable productivity and competitiveness in modern industry [22, 24–25, 31]. To maximize productivity, industrial systems, such as manufacturing systems, must be available and operational as much as possible. Nevertheless, since industrial systems consist of a number of components, the ultimate probability of system survival directly depends on the characteristics of the constituent components. Hence, system failure is inevitable. As such, it is essential to enhance system reliability through suitable reliability optimization methods, so as to improve the overall system productivity. Developing effective methods for system reliability enhancement is imperative.

The ever-increasing need for highly reliable systems necessitates the search for improved methods for system reliability optimization. In system reliability design, two typical approaches can be used to enhance system reliability: (i) adding redundant components in the subsystems of the system, and (ii) increasing the reliability of the components that constitute the system.

Industrial systems are designed under several restrictions, including cost, weight, and volume of the resources. With limited resources, the major aim is to find a trade-off between reliability and other resource constraints [22]. One of the feasible ways is to maximize system reliability via redundancy and component reliability choices, a problem called reliability-redundancy allocation problem [24]. However, in designing a highly reliability system, the main problem is to find a trade-off between reliability enhancement and resource consumption. This calls for an application of a suitable multi-criteria approach. Various multi-criteria programming approaches and multi-criteria solution approaches have been applied on different problems in the literature [1–3, 23].

In the real world, system reliability optimization problems are inundated with a number of uncertainties and difficulties. This is due to the reasons that: (i) the management goals and the constraints are often characterised with some imprecision or vagueness; (ii) the coefficients or parameters as understood by the decision maker may be characterized with some vagueness; and, (iii) the available historical data, collected under specific conditions, are often imprecise and vague. In addition, variability and changes in the manufacturing processes that produce the components of the systems lead to uncertainties in component reliability. Probabilistic approaches, which essentially deal with uncertainty arising from randomness, cannot adequately address inherent uncertainties in the data. While probabilistic approaches deal with uncertainties arising from randomness, fuzzy approaches seek to address uncertainties that arise from vagueness of human judgment and imprecision due to system complexity [4–6, 13–15, 27]. As a result, the concept of fuzzy reliability is more promising [7–9, 28].

Bellman and Zadeh [5] introduced the fuzzy optimization approach that utilizes aggregation operators for combining fuzzy goals and fuzzy decision space. Since the inception of the fuzzy optimization approach, a number of methods and applications have been proposed to solve optimization problems that involve vagueness and ambiguity [12, 20, 21, 30]. These approaches treat parameters (coefficients) as fuzzy numerical data. Apart from the fuzziness of the system reliability problem, the presence of conflicting, nonlinear and ambiguous objectives further complicates the problem. In such a fuzzy environment, with multiple objectives, simultaneous reliability maximization and cost minimization calls for a cautious trade-off approach. Thus, finding the optimal solution is almost impossible. Metaheuristic and other intelligent methods are a potential application method for such complex problems [11, 10, 26]. Therefore, the most appropriate procedure is to cautiously find a set of solutions that satisfy the decision maker's expectations to the highest possible degree. Clearly, this calls for an interactive fuzzy multi-objective optimization approach which incorporates the preferences and expectations of the decision maker, allowing for human (expert) judgment. Iteratively, it becomes possible to obtain the most satisfactory solution in a fuzzy environment.

In view of the above issues, the purpose of this paper is to address the problem of system reliability optimization in a fuzzy environment characterized with multiple conflicting objectives. Therefore, our specific objectives are as follows:

(1) to develop a fuzzy multiple-objective nonlinear programming model for the reliability optimization problem;
(2) to use an aggregation method to transform the fuzzy model to a single-objective optimization problem; and,
(3) to use a global metaheuristic optimization method to obtain a set of acceptable solutions.

In our current study, we develop a fuzzy multi-objective genetic algorithm (FMGA) which utilizes a fuzzy theory based method to evaluate the objective functions represented as membership functions. We use the max-min operator to aggregate the membership functions of the objective functions while incorporating the decision maker's judgment. In this respect, we define our notations and assumptions as follows.

Nomenclature:

m	the number of subsystems in the system
n_i	the number of components in subsystem i, $1 \le i \le m$
n	$\equiv(n_1, n_2, \ldots, n_m)$, the vector of the redundancy (number of redundant components) allocation for the system
r_i	the reliability of each component in subsystem i, $1 \le i \le m$
r	$\equiv(r_1, r_2, \ldots, r_m)$, the vector of the component reliabilities for the system
q_i	$=1 - r_i$, the failure probability of each component in subsystem i, $1 \le i \le m$
$R_i(n_i)$	$=1-q_i^{n_i}$, the reliability of subsystem i, $1 \le i \le m$
R_s	the system reliability
g_i	the i^{th} constraint function
w_i	the weight of each component in subsystem i, $1 \le i \le m$
v_i	the volume of each component in subsystem i, $1 \le i \le m$
c_i	the cost of each component in subsystem i, $1 \le i \le m$
V	the upper limit on the sum of the subsystems' products of volume and weight
C	the upper limit on the cost of the system
W	the upper limit on the weight of the system
b	the upper limit on the resource

Assumptions:

(1) The availability of the components is unlimited;
(2) The weight and product of weight and square of the volume of the components are deterministic;
(3) The redundant components within the individual subsystems are identical;
(4) Failures of individual components are independent;
(5) All failed components will not damage the system and are not repaired.

2. System reliability optimization

The system reliability optimization problem is a maximization problem subject to multiple non-linear constraints. In this connection, the problem can be expressed as a mixed integer nonlinear programming problem. In this study, we present a reliability redundancy problems commonly found in the literature, with a particular emphasis on the series system [22, 24]. The series system reliability problem consists of five subsystems as reported in the literature as shown in Fig. 1.

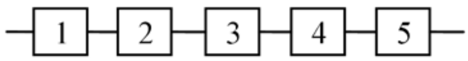

Fig. 1. The series system

Following our notation proposed in section 1, the system reliability optimization problem can be formulated as a nonlinear mixed integer program:

$$\text{(P1)}\quad \text{Max}\, f(\mathrm{r},\mathrm{n}) = \prod_{i=1}^{m} R_i(n_i)$$

Subject to:

$$g_1(\mathrm{r},\mathrm{n}) = \sum_{i=1}^{m} w_i v_i^2 n_i^2 \le V$$

$$g_2(\mathrm{r},\mathrm{n}) = \sum_{i=1}^{m} \alpha_i \left(-1000/\ln r_i\right)^{\beta_i} \left(n_i + \exp(n_i/4)\right) \le C$$

$$g_3(\mathrm{r},\mathrm{n}) = \sum_{i=1}^{m} w_i n_i \exp(n_i/4) \le W$$

$$0 \le r_i \le 1,\quad n_i \in Z^+,\quad 1 \le i \le m$$

where, r_i, and n_i, are the reliability and the number of components in the i[th] subsystem respectively; $f(\cdot)$ is the objective function for the overall system reliability; $g(\cdot)$ is the constraint function; m si the number of subsystems. The primary goal is to determine the number of components and their reliability in each subsystem so that the overall system reliability is maximized. Thus, the problem falls in the category of constrained non-linear mixed integer optimization problems. The next section presence the proposed fuzzy multi-objective optimization approach, based on genetic algorithm.

3. Fuzzy multi-objective optimization approach

In a fuzzy environment, the objective goal, the constraints and the consequences of the decision taken are inherently imprecise. Thus, in practice, the decision maker seeks to consider a trade-off between reliability, cost, weight and volume. For instance, a common approach may be to simultaneously maximize reliability and minimize cost. In this connection, the multi-objective formulation is obtained by transforming constraints to objective functions, such that reliability and other costs functions can be optimized jointly. This is achieved through the use of membership functions for the objective functions. This makes the approach more applicable and adaptable to the real life human decision process. Therefore, the fuzzy multi-objective optimization problem (FMOOP) can generally be represented as follows;

$$
\begin{aligned}
&\text{(P2) Min } \tilde{f}(x)\\
&\text{Subject to:}\\
&g_z(x) \lesssim \text{ or } \equiv \text{ or } \gtrsim 0 \quad z = 1,2...,p\\
&x_q^l \le x_q \le x_q^u \qquad q = 1,2,...,Q
\end{aligned}
$$

where, $x = (x_1, x_2, ..., x_Q)^T$ is a vector of decision variables that optimize a vector of fuzzy objective functions, $\tilde{f}(x) = \{\tilde{f}_1(x), \tilde{f}_2(x), ..., \tilde{f}_d(x)\}$ over the decision space X; $\tilde{f}_1(x), \tilde{f}_2(x), ..., \tilde{f}_d(x)$ are d individual objective functions; x_q^l and x_q^u are lower and upper bounds on decision variable x_q, respectively. Here, we use the symbol "~" to denote a fuzzy function or operator.

3.1. Membership functions

The notion of fuzzy set theory permits gradual assessment of membership, defined in terms of a suitable membership function that maps to the unit interval [0,1]. To date, several membership functions, such as Generalized Bell, Gaussian, Triangular and Trapezoidal have been used to represent fuzzy membership in a several applications. Though various functions can be used, it has been shown that linear membership functions can provide equally good quality solutions with much ease [29]. The triangular and trapezoidal membership functions have widely been recommended [8, 12, 29]. In this study, therefore, we use linear functions to define the fuzzy membership functions of the objective functions.

In this study, we assume that an expert user has a range of acceptable feasible values of each objective functions. However, we further assume that there is a lower and upper limit to that range of acceptable objective function values, as specified by the expert user. Let m_t and M_t denote the minimum and maximum acceptable values of each objective function $\tilde{f}_t(x)$, $t = 1,2,...,h$, where h is the number of objective functions. Further, let μ_{f_t} denote the membership function corresponding to the objective function f_t. Then, the membership function corresponding to minimization and maximization of specific objective functions can be defined in terms of degree of satisfaction. Fig. 2 illustrates the linear membership functions, both for minimization as well as for maximization problems. We define the membership functions for both situations.

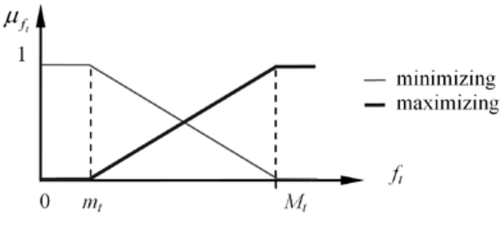

Fig. 2. Fuzzy membership function for $f_t(x)$

When the objective is concerned with minimization, the linear membership function can be formulated as in the following expression:

$$
\mu_{f_t}(x) = \begin{cases} 1 & f_t(x) \le M_t \\ \dfrac{M_t - f_t(x)}{M_t - m_t} & m_t \le f_t(x) \le M_t \\ 0 & f_t(x) \ge M_t \end{cases} \qquad (1)
$$

where, m_t and M_t denote the minimum and maximum acceptable feasible values of each objective function. Clearly, the function $\mu_{f_t}(x)$ is monotonically decreasing in $f_t(x)$. On the other hand, when the objective is about maximization, the membership function can be defined as follows:

$$
\mu_{f_t}(x) = \begin{cases} 1 & f_t(x) \ge M_t \\ \dfrac{f_t(x) - m_t}{M_t - m_t} & m_t \le f_t(x) \le M_t \\ 0 & f_t(x) \ge m_t \end{cases} \qquad (2)
$$

It can be seen from this analysis that $\mu_{f_t}(x)$ is a monotonically increasing function of $f_t(x)$. The next step is to formulate the corresponding crisp model. The use of fuzzy evaluation in FMGA allows the algorithm to accept inferior which would otherwise be infeasible when using conventional crisp formulation. The advantage of this approach is that it makes the algorithm robust enough to cope with any infeasibility. Allowing the FMGA to pass through inferior solutions gives the algorithm speed and flexibility, which ultimately improves the search power of the approach.

3.2. Corresponding crisp model

In practice, it is desirable to consider the imprecise management or decision maker's preferences in our formulation. Therefore, to incorporate the decision maker's preferences and to enhance the interactive flexibility of the model, a set of user-defined weights $w = \{w_1, w_2, \ldots, w_h\}$ are introduced. We convert the multi-objective system reliability optimization problem into a single objective optimization problem [14]:

$$
\begin{aligned}
&\text{(P3) Max } \left(\frac{\lambda_1(x)}{w_1} \wedge 1\right) \wedge \left(\frac{\lambda_2(x)}{w_2} \wedge 1\right) \wedge ... \wedge \left(\frac{\lambda_h(x)}{w_h} \wedge 1\right)\\
&\text{Subject to:}\\
&\lambda_t(x) = \mu_{f_t}(x) \qquad w_t \in [1,0) \; t = 1,...,h\\
&x_q^l \le x_q \le x_q^u \qquad q = 1,...,Q
\end{aligned}
$$

where, $\mu_{f_t}(x) = \{\mu_{f_1}(x), \mu_{f_2}(x), ..., \mu_{f_h}(x)\}$ is a set of fuzzy regions that satisfy the objective functions; x is a vector of decision variables, λ_t denotes the degree of satisfaction of the t^{th} objective, w_t denotes the weight of the t^{th} objective function as suggested by the expert judgment of the user or decision maker, and the symbol "∧" is the aggregate min operator or the intersection operator. For instance, the expression $(\lambda_1(x)/w_1) \wedge 1$ gives the minimum between 1 and $\lambda_1(x)/w_1$. Though the values of $\lambda_1(x)$ are in the range [0,1], the value of $\lambda_1(x)/w_1$ may exceed 1, howbeit, by the min operator the final value of $(\lambda_1(x)/w_1) \wedge 1$ will always lie in [0,1]. We use a metaheuristic approach to solve problem P3.

3.3. Genetic algorithm approach

Genetic Algorithm (GA) is a stochastic global optimization technique that attempts to evolve a population of candidate solutions by giving preference of survival to quality solutions, whilst allowing some low quality solutions to survive in order to maintain diversity in the population [16, 18]. Each candidate solution is coded into a string of digits, called chromosomes. New offspring are obtained from probabilistic genetic operators, such as selection, crossover, mutation, and inversion [16]. A comparison of new and old (parent) candidates is done based on a given fitness function, retaining the best performing candidates into the next population. Thus, characteristics of candidate solutions are passed from generation to generation through probabilistic selection, crossover, and mutation. The general flow of the GA approach is presented in Fig 3. The metaheuristic is represented as an iterative procedure consisting of sub-procedures: initialization, evaluation, selection, crossover, and mutation.

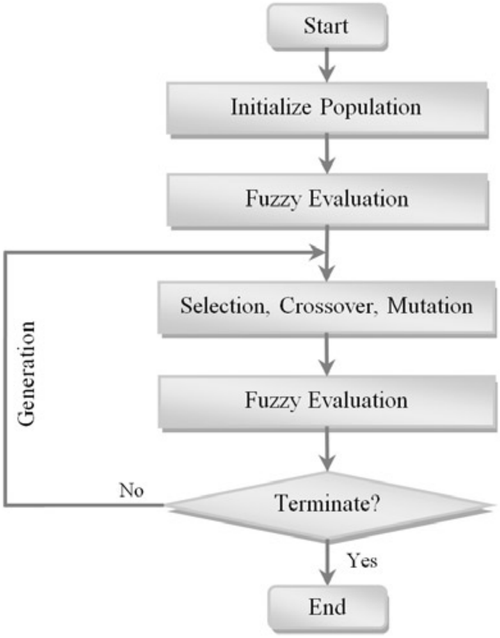

Fig. 3. Fuzzy multi-objective genetic algorithm approach

3.4. Genetic encoding scheme

In our FMGA implementation for the system reliability problem, the genetic chromosome uses the variable vectors *n* and *r*. Thus, we use a real-coded genetic encoding scheme, where the integer variable n_i is coded as a real variable and transformed to the nearest integer value upon evaluating the objective function.

3.5. Initialization and evaluation

In the initialization procedure, an initial population of the desired size, *pop*, is generated randomly from the solution space. FMGA then computes the objective function for each string according to the objective function represented in model P3. The value of the objective function is always in the range [0,1].

3.6. Selection and recombination

A number of selection strategies exist in literature [13]. In this study, we adopted the remainder stochastic sampling without replacement. By this strategy, each chromosome *j* is selected and stored in the mating pool according to the expected count e_j, represented by the expression;

$$e_j = \frac{f_j}{\sum_{j=1}^{pop} f_j / pop} \quad (3)$$

where, f_j is the objective function value of the j^{th} chromosome. Each chromosome receives copies equal to the integer part of e_i, that is, $[e_i]$, while the fractional part is treated as success probability of obtaining additional copies of the same chromosome into the mating pool. The crossover operator is then applied to selected parent chromosomes for the purpose of exchanging genetic information between the selected chromosomes, thereby producing new offspring. Here, we use the arithmetic crossover operator as in [26] to define a linear combination of two chromosomes. A crossover probability of 0.42 was assumed in this application. For instance, let p_1 and p_2 denote the selected parents, and α represent a random value in the range [0,1], then the resulting offspring, q_1 and q_2, are given by the following expression:

$$\begin{aligned} q_1 &= \alpha p_1 + (1-\alpha) p_2 \\ q_2 &= (1-\alpha) p_1 + \alpha p_2 \end{aligned} \quad (4)$$

3.7. Mutation operator

As generations proceed, the population converges to a common solution, which may lead to result in pre-mature convergence. To curb premature convergence, and to maintain population diversity, a mutation operator is applied to every new chromosome, at a very low probability. In our application, we used a uniform mutation with a mutation probability of 0.032.

3.8. Replacement

In every generation, new offspring are created. The new offspring may be better or worse than the preceding generation. As such, the non-performing individuals are replaced with better ones using a replacement strategy. According to Goldberg (1979) [16], some of the replacement strategies found in the literature include probabilistic replacement, crowding strategy, and elitist strategy. In this application, we a combination of these strategies was implemented.

3.9. Termination criteria

Two termination conditions are used to stop the FMGA iteration, that is, when the number of generations exceeds the preset maximum iterations, or when the average improvement in the fitness of the best solution over specific generations is less than a small number, which is assumed to be 10^{-6} in this application. The maximum generations was set at 500.

3.10. Overall FMGA procedure

The overall structure of the FMGA for the system reliability problems consists of all the procedures discussed in the previous sections; that is, initialization, selection, evaluation, crossover, mutation, replacement, and termination. Fig. 4 presents the pseudo-code of the algorithm.

Algorithm 1: Pseudo code for FMGA

```
1: randomly generate initial population
Repeat
  2: evaluation of fitness, objective: f(x), x = (x1, x2,…,xh)
  3: selection strategy
  4: crossover and mutation
  5: replacement
  6: advance population; oldpop = newpop
Until (termination criteria is satisfied)
```

Fig. 4. Pseudo code for the overall FMGA procedure

The next section presents the comparative results of our FMGA computations based on the benchmark problems found in the literature [17, 19, 24, 31].

4. Numerical experiments

To evaluate the usefulness of our proposed FMGA for solving mixed integer reliability problems, the series reliability system illustrated in P1 will be solved using the approach. We use the parameter values in [19] and to define the specific instances of this problems as shown in Tables 1.

Table 1. Basic data used in series system

i	$10^5\alpha_i$	β_i	$w_i v_i^2$	w_i	V	C	W
1	2.330	1.5	1	7	110	175	200
2	1.450	1.5	2	8	110	175	200
3	0.541	1.5	3	8	110	175	200
4	8.050	1.5	4	6	110	175	200
5	1.950	1.5	2	9	110	175	200

The parameters of the FMGA were set as follows: The crossover and mutation were set at 0.45 and 0.035, respectively. A two-point crossover was used in this application. The population size was set to 20. The maximum number of generations or iterations was set at 150. This implies that the termination criterion is either limited to a maximum number of iterations or to the order of the relative error set at 10^{-6}, whichever comes earlier. Specifically, whenever the best fitness f^* at iteration t is such that $|f_t - f^*| < \varepsilon$ is satisfied, then three best solutions are selected; where ε is a small number equal to 10^{-6}. The FMGA was implemented in JAVA, and the program was run 25 times, while selecting the best 3 solutions out of the converged population.

The FMOOP provided by formulation (P3) is used to solve benchmark problems in [19]. A fuzzy region of satisfaction is constructed for each objective function, that is, objective functions corresponding to system reliability, cost, volume, and weight, which are denoted by λ_1, λ_2, λ_3, and λ_4, respectively. By using the constructed membership functions together with their corresponding weight vectors, we obtain an equivalent crisp optimization formulation for our problem:

$$\text{(P4) Max } \left(\frac{\lambda_1(x)}{\omega_1}\wedge 1\right)\wedge\left(\frac{\lambda_2(x)}{\omega_2}\wedge 1\right)\wedge\left(\frac{\lambda_3(x)}{\omega_3}\wedge 1\right)\wedge\left(\frac{\lambda_4(x)}{\omega_4}\wedge 1\right)$$

Subject to :

$$\lambda_t(x) = \mu_{f_t}(x) \qquad t = 1,\dots,4$$

$$0.5 \le r_i \le 1-10^{-6} \qquad r_i \in [0,1]$$

$$1 \le n_i \le 10 \qquad n_i \in Z^+$$

$$0.5 \le R_s \le 1-10^{-6} \qquad R_s \in [0,1]$$

The weight set $\omega = \{\omega_1, \omega_2, \omega_3, \text{and } \omega_4\}$ was selected in the range [0.2,1], where the values of the weights indicate the bias towards specific objectives as specified by the expert decision maker. In particular, the weight set ω = [1,1,1,1] implies that the expert user prefers that there should be no bias towards any objective goal, that is, there is no preference at all. Every other combination of weights implies that there is some bias towards one or more specific objectives, and the relative importance of objectives is ranked accordingly. For instance, with a weight set defined by ω = [1,0.5,0.5,0.5], the preference is biased towards the region that is closer to the objective corresponding to reliability than to the rest of the objectives that are equally ranked with weight value of 0.5. Therefore, the decision making process takes into account the decision maker's preferences and choices based on expert opinion. In addition, the FMGA approach is a useful decision support tool that can provide a set of good solutions in an interactive manner, rather than prescribe a single solution. Furthermore, the approach enables the decision maker to specify the minimum and maximum values of objective functions in terms of reliability, cost, volume, and weight, denoted by f_1, f_2, f_3, and f_4, respectively. Table 2 provides a list of the selected minimum and maximum values of the objective functions, for the series. This approach makes the FMGA algorithm more adaptable and flexible for addressing specific problem situations while accommodating the expert user's managerial preferences. Computational results and discussions are presented in the next section.

Table 2. Minimum and maximum feasible values of objective functions

	Series System			
	f_1	f_2	f_3	f_4
M_i	1	180	120	210
m_i	0.6	60	5	100

5. Results and Discussions

This section presents the comparative results of the numerical experiments. The best three FMGA solutions are compared with the results obtained by other algorithms in the literature, for the series, series-parallel and complex bridge systems. We compare our results with those in [19] and [31].

Table 3 shows the comparative numerical results in which the best three solutions of the problem are compared against solutions from the literature. The results indicate that the best three FMGA solutions are better than the solutions reported previously in [19, 31], particularly in terms of system reliability. In terms of cost, the solutions are slightly less than the previously reported solutions; the difference in cost is, however, not significant. Though there are a few exceptional instances where the cost of the FMGA are slightly higher with differences in the order of 10^{-6}, it can be seen that, overall, FMGA provides better solutions than the approaches reported previously. FMGA approach found high quality solutions, most of which are better than those previously recorded in the literature. In summary, the approach offers a number of practical advantages to the decision maker, including the following:

- FMGA addresses the imprecise and fuzzy characteristics of the system reliability optimization problem;
- The method address conflicting multiple objectives, giving a trade-off between the objectives;
- The approach accommodates the decision maker's preferences in its procedure;
- The method gives a population of alternative solutions for the decision maker, rather that prescribe a solution;
- The method is practical, flexible and easily adaptable to specific problem situations.

In view of the above advantages, FMGA is a potentially useful approach that can be ffurther developed into a decision support tool for optimizing practical industrial system reliability situations.

6. Conclusions

In the real world, decision makers concerned with system reliability optimization encounter problems of finding a judicious trade-off between maximizing reliability and minimizing cost to an acceptable degree of satisfaction. In such a fuzzy environment, the management goals and constraints are not known precisely. Moreover, the goals are often conflicting, which further complicates the reliability optimiza-

Table 3. Comparison of best-3 FMGA solutions with other algorithms

Best 3 FMGA Solutions				Wu et al. [31]	Hsieh et al.[19]
No.	$(r_i: n_i)$	$(r_i: n_i)$	$(r_i: n_i)$		$(r_i: n_i)$
1	(0.779401321:3)	(0.77940279:3)	(0.77939597:3)	(0.78037307:3)	(0.779427:3)
2	(0.871839015:2)	(0.87181554:2)	(0.87183716:2)	(0.87178343:2)	(0.869482:2)
3	(0.902877370:2)	(0.90287257:2)	(0.90288515:2)	(0.90240890:2)	(0.902674:2)
4	(0.711415792:3)	(0.71141514:3)	(0.71140318:3)	(0.71147356:3	(0.714038:3)
5	(0.787779580:3)	(0.78783097:3)	(0.78780147:3)	(0.78738760:3)	(0.786896:3)
R_s	0.931682387	0.931682384	**0.931682388**	0.9316800	0.93157800
C_s	175.0000000	175.0000000	175.0000000	174.99899	174.878546
W_s	192.4810818	192.4810818	192.4810818	192.48108	192.481082
V_s	83.00000000	83.00000000	83.00000000	83.000000	83.0000000
Note: Bold indicates the best FMGA solution					

tion problem. One most viable and useful option is to us a fuzzy satisficing approach that includes the preferences and expert judgments of the decision maker. We provided a multi-objective non-linear mixed integer program for addressing system reliability optimization problems. The fuzzy multi-objective model is transformed into a single-objective model which uses a fuzzy evaluation method. Genetic algorithm uses the fuzzy evaluation method to evaluate the fitness of individuals in each population at every generation. Numerical results demonstrate that the fuzzy multi-objective Genetic Algorithm approach is able to provide high quality solutions while accommodating the preferences of the user.

This study offers a useful contribution to decision makers in system reliability design. Contrary to single-objective approaches which seek to optimize system reliability only. FMGA provides a trade-off between management goals. At design stage, the information required for system reliability design is imprecise and incomplete. To that effect, the problem becomes ill-structured such that reliance on expert information is inevitable. Using FMGA, the vagueness and imprecision of the expert knowledge, at the design stage, can be addressed effectively while taking into account the multiple conflicting objectives. Furthermore, FMGA provides a population of good alternative solutions in an interactive manner, giving the decision maker a wide choice of practicable solutions and an opportunity to consider other practical factors that cannot be included in the formulation. Overall, FMGA is a useful platform for decision support for system reliability design when the parameters, the management goals, the design constraints, and the impact of the possible alternative actions are not precisely known.

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