Article citation info:
KOŁODZIEJ P, BORYGA M. Frequency analysis of coupling with adjustable torsional flexibility. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2014; 16 (2): 325–329.

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# FREQUENCY ANALYSIS OF COUPLING WITH ADJUSTABLE TORSIONAL FLEXIBILITY

# ANALIZA CZĘSTOŚCIOWA SPRZĘGŁA O REGULOWANEJ PODATNOŚCI SKRĘTNEJ*

*The article presents the frequency analysis of a flexible coupling allowing changes of torsional flexibility. The authors derived a relationship for coupling flexibility considering geometric and material parameters. Coupling flexibility change is executed in such a manner that the quotient of extortion frequency and natural frequency of the system is higher than 1.4. Oscillation parameters for selected values of torsional flexibility were calculated for extortion frequencies approximating natural frequencies and after flexibility change.*

***Keywords****: couplings flexibility, torsional stiffness, frequency analysis, amplitude characteristics.*

*W pracy przedstawiono analizę częstościową sprzęgła podatnego umożliwiającego zmianę sztywności skrętnej. Wyprowadzono zależność na sztywność sprzęgła uwzględniając parametry geometryczne i materiałowe. Zmiana sztywności sprzęgła dokonuje się tak aby iloraz częstości wymuszenia i częstości drgań własnych układu był większy od 1,4. Obliczono parametry drgań dla wybranych wartości współczynnika sztywności skrętnej przy częstościach wymuszenia bliskich częstości drgań własnych oraz po zmianie sztywności.*

***Słowa kluczowe****: sprzęgło podatne, sztywność skrętna, analiza częstościowa, charakterystyka amplitudowa.*

## 1. Introduction

The development of technology in machine design and exploitation extorts the necessity to choose the most favourable construction solutions and increasing exploitation velocity in production processes. Considering the results of the above-mentioned activities, one should conclude that the increase of movement velocity causes the increase of dynamic strains. Naming the relations with the term machine “dynamicity”, one should consider the characteristics of its mechanical state, i.e. the values of oscillation amplitudes of the construction as a whole and its individual elements and sub-assemblies. High values of dynamic strains have negative impact on durability, reliability, precision of work, shape errors and positioning precision [1]. The oscillations during the work of driving systems strained with changeable moment depend on the amplitude value and the coercion frequency, mass inertia moments of the drive elements, torsional stiffness and damping.

Mechanical systems with changeable flexibility are used as elements of coupling constructions, shafts and vibration eliminators. An example of such a torsional vibration eliminator using a changeable flexibility of neoprene rings, playing a role of springs and dampers, is presented in the works by Slavick and Bollinger’s [13]. Stiffness change results from axial relocation of countersunk screw causing the increase or decrease of pressure onto the rings. The range of neoprene flexibility changes allows adjusting the eliminator with a plate, weighting 20 kg, to resonance frequency of milling machine spindle used for device milling. In Kowal's publications [8–10] the changes of couplings or transmission shaft flexibility using cylindrical or disc push springs in packages are done with screw thread mechanisms. According to the author a coupling is a system effectively limiting dynamic strains during the set work and starting the drive system. Module construction made of disc springs allows building systems with different static characteristics. Filipowicz [2–5] and Filipowicz, Kuczaj [6, 11] presented new solutions of flexible systems and conducted theoretical and practical analysis of the double-acting couplings. The researchers concluded that the constructions allow obtaining torsion angles of a few degrees, transforming significant values of torque and easing momentary overloads. Moreover, they noticed that defining the most favourable characteristics of couplings (for the types of the machines) requires selecting the sets and systems of springs and threat mechanism parameters, what is possible as early as at designing stage.

The aim of the present work is to define the influence of torsional flexibility of the designed flexible coupling to the parameter of extorted normal oscillations. The contents of the article is the following. Chapter 2 presents the method of solving the equations of extorted, deadened vibrations. Chapter 3 explains how to adjust the torsional stiffness of coupling. Chapter 4 contains the description of coupling and calculating its torsional stiffness. In chapter 5 the results of analyses for ten selected values of torsional stiffness values are presented.

## 2. Equations of extorted vibrations and their solutions

Differential equation of extorted vibrations can be presented as illustrated in [14]:

$$\mathbf{B}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{H}\sin\omega t \tag{1}$$

where: $\mathbf{B}$, $\mathbf{C}$, $\mathbf{K}$ – inertia, damping and stiffness matrices respectively, $\mathbf{H}$ – vector, amplitude of extorting force, $\mathbf{q}$ – vector of generalised coordinates, $\omega$ – frequency of extorted vibrations [rad/s], $t$ – time [s].

Filling the stationary solution to the vibration equation you obtain a system of algebraic equations in which the unknown is the vector of complex amplitudes $\bar{\mathbf{a}}$ of set extorted vibrations [14]:

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

$$(\mathbf{K}-\omega^2\mathbf{B})\bar{\mathbf{a}}+i\omega\mathbf{C}\bar{\mathbf{a}}=\mathbf{H} \qquad (2)$$

where: $i=\sqrt{-1}$ – imaginary unit.

In the case of flexible coupling, system of equations has a form of

$$\begin{bmatrix} J_s & 0 \\ 0 & J_m \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}+\begin{bmatrix} c & -c \\ -c & c \end{bmatrix}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}+\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}=\begin{bmatrix} 0 \\ M \end{bmatrix}e^{i\omega t} \qquad (3)$$

where: $J_s$ – mass moment of inertia in the rotor and rotating elements of active part of coupling reduced to engine rotor axis [kg·m$^2$], $J_m$ – machine inertia moment reduced to the rotation axis of output shaft coupling [kg·m$^2$], $c$ – viscous damping coefficient [N·m·s/rad], $k$ – torsional stiffness [N·m/rad], $M$ – load torque coupling, [N·m], $\theta_1$ – angle tilt of active part of coupling [rad], $\theta_2$ – angle tilt of passive part of coupling [rad].

Using formula (2) and the transformations, one obtains:

$$\begin{bmatrix} k-J_s\omega^2+ic\omega & -(k+ic\omega) \\ -(k+ic\omega) & k-J_m\omega^2+ic\omega \end{bmatrix}\begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \end{bmatrix}=\begin{bmatrix} 0 \\ M \end{bmatrix} \qquad (4)$$

Having solved the obtained expressions into real and imaginary parts, one obtains:

$$\bar{a}_1=A_1(\omega)+iB_1(\omega)=\frac{M(k\alpha-\beta)}{[\alpha^2+\beta(J_s+J_m)]\omega^2}+i\frac{M\gamma J_m}{\alpha^2+\beta(J_m+J_s)} \qquad (5)$$

$$\bar{a}_2=A_2(\omega)+iB_2(\omega)=\frac{M(\delta\alpha-\beta)}{[\alpha^2+\beta(J_s+J_m)]\omega^2}+i\frac{M\gamma J_s}{\alpha^2+\beta(J_m+J_s)} \qquad (6)$$

where: $\alpha=\omega^2 J_s J_m-k(J_s+J_m)$, $\beta=\omega^2c^2(J_s+J_m)$, $\gamma=\omega c J_s$, $\delta=k-\omega^2 J_s$.

The module of amplitude and the phase angle is calculated with the formulas [14]:

$$a_i(\omega)=\sqrt{[A_i(\omega)]^2+[B_i(\omega)]^2} \qquad (7)$$

$$\phi_i(\omega)=arctg\frac{B_i(\omega)}{A_i(\omega)} \qquad (8)$$

## 3. The selection of coupling torsional stiffness

Characteristic equation allows defining natural vibration frequencies $\omega_0$. The equation has the form:

$$\det(\mathbf{K}-\omega_0^2\mathbf{B})=0 \qquad (9)$$

In case of flexible coupling the equation has a form

$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}-\omega_0^2\begin{bmatrix} J_s & 0 \\ 0 & J_m \end{bmatrix}=0 \qquad (10)$$

The calculated roots of equation (10) are:

$$\omega_0=0 \text{ and } \omega_0=\sqrt{\frac{k(J_s+J_m)}{J_s J_m}} \qquad (11)$$

The coupling has positive effect on the dynamic properties of the system provided the following condition is satisfied [12]:

$$\frac{\omega}{\omega_0}>\sqrt{2} \qquad (12)$$

The torsional stiffness $k$ must be adjusted in such a manner to make the coupling work at supercritical conditions, passing through resonance in the initial period of starting-up, when the dynamic torque is not yet excessively high. Placing the calculated value of main oscillations (11) into the relation (12) and making transformations one obtains the relation for the required torsional stiffness $k$:

$$k<\frac{J_s J_m\omega^2}{2(J_s+J_m)} \qquad (13)$$

## 4. Torsional stiffness of a flexible coupling

### 4.1. Coupling description

The analysis of problems concerning the influence of flexible elements stiffness on the values of dynamic loads in drive systems, brings to a conclusion that it is an effective method of reducing negative interactions to use the devices whose construction allows obtaining an adaptable (within a certain range) value of torsional stiffness. Still, the flexibility would be changeable independently from drive movement parameters and external load. Considering the above information one should conclude that the task is satisfied by the coupling with in-built mechanism of fluent change of torsional stiffness [7], whose scheme of actions is presented in figure 1.

In the above figures the authors presented the rules governing the torsional flexibility coupling. Stiffness change of the system requires blocking the active length of flat spring connecting active and passive discs. The blockade is possible due to the linear movement of driving disc attached to an shaft spline. Maximum coupling flexibility is obtained for springs' active length is the highest ($L_{max}$) and minimum is the lowest ($L_{min}$).

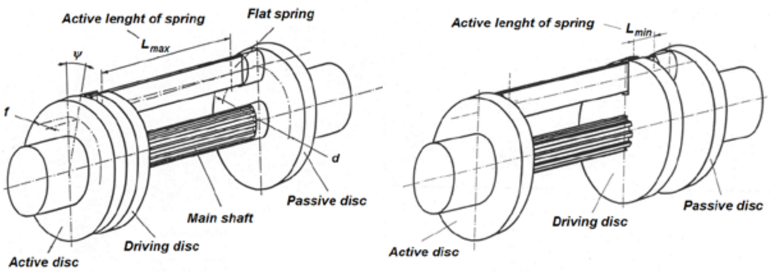


*Fig. 1. The scheme of actions of flexible coupling*

Figure 2 presents the construction of the designed coupling [7]. In the input shaft 10 brush runs 12 are attached to power drive 11 via toothed gear transmission (wheels 13 and 14) and lead-screw 15. The lead screw which can turn in active disc 16 and support plate is used to control the swashplate 18. Axial movement of the swashplate along the shaft spline 17 causes decrease of active length of flat springs 20 limiting the angle of turn between active 16 and passive 19 discs. Flat springs 20 are fixed in the covers 21, which can rotate in freely in discs 16 and 19.

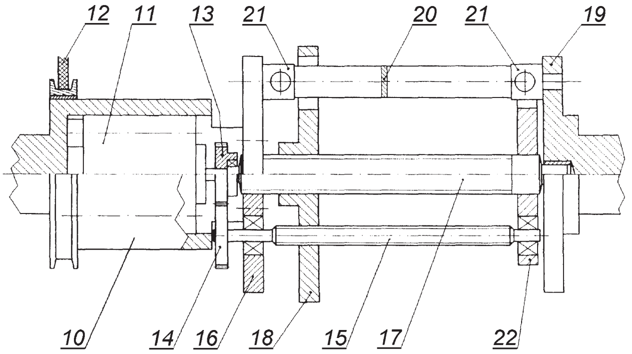


*Fig. 2. Flexible coupling*

### 4.2. Torsional stiffness calculations

Deflection of flat spring is:

$$f = \frac{FL^3}{3EJ_x} \tag{14}$$

where: $F$ – load force onto the spring [N], $L$ – active length of the spring [mm], $E$ – Young's modulus [MPa], $J_x$ – axial moment of inertia spring [mm$^4$].

The force acting on one spring can be calculated from the formula:

$$F = \frac{2M}{nd} \tag{15}$$

where: $d$ – spring distribution diameter [mm], $n$ – the number of springs.

For small angles it can be formed as:

$$f = \frac{d}{2} tg\psi \tag{16}$$

where: $\psi$ – relative angle of coupling torsion of discs [rad]

Comparing the relations (14) and (16) and using formula (15) you obtain:

$$\frac{d}{2} tg\psi = \frac{2ML^3}{3ndEJ_x} \tag{17}$$

For small angles expressed in radians $tg\psi \approx \psi$ thus appropriate transformation of formula (17) for the right units, you can obtain the relation for torsional stiffness *k:*

$$k = \frac{M}{\psi} = \frac{3nd^2EJ_x}{4000L^3} \tag{18}$$

Figure 3 presents graph of the torsional stiffness coefficient of coupling $k$ depending on active spring length $L$, wherein axis of $k$ is logarithmic. The graph was obtained for the following coupling parameters: $n$ = 4, $d$ = 100 mm, $J_x$ = 5.625 mm$^4$, $E$ = 2.1·10$^5$ MPa, $L$ = (5÷105) mm.

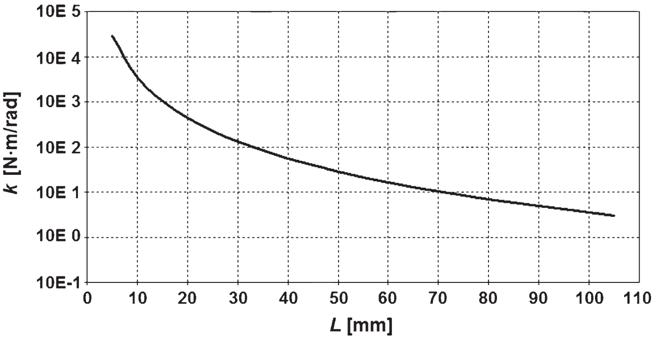


*Fig. 3. Graph presenting torsional stiffness k with relation to active length of spring L*

## 5. Results of the analysis

The calculations were made for ten selected values of torsional stiffness $k$ within the regulation range. Column 1 in table 1 contains the frequencies of natural oscillation ($\omega_0$) determined from the relations (11, 5, 6) and the values of angular transformations amplitudes of active ($a_1$) and passive ($a_2$) discs. The calculations employed the values of construction parameters of coupling presented in the previous chapter. Moreover, it was assumed that $J_s$ = 0.03 kg·m$^2$, $J_m$ = 10 kg·m$^2$, $c$ = 0.2 N·m·s/rad and $M$ = 3.5 N·m. Column 2 presents the values of torsional stiffness determining from the relation (13), with consideration for condition (12).

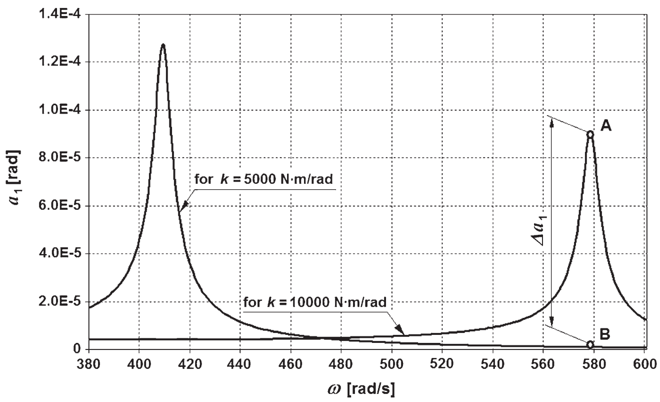


*Fig. 4. The graph of amplitude $a_1$ in relation to extortion ω frequency for torsional stiffness k = 10000 N·m/rad and k = 5000 N·m/rad*

Figures 4 and 5 present the results of simulations for a selected calculation case presented in table 1. Figure 4 presents a graph of amplitude $a_1$ with relation to extortion frequency $\omega$ . Initially, the system was near the resonance spot A (torsional stiffness $k$ = 10000 N·m/rad whereas extortion frequency was $\omega$ = 578 rad/s and was approximating the natural frequency $\omega_0$). At that point, the value of amplitude was $a_1$ = 9.017·10$^{-5}$ rad. The change of torsional stiffness from 10000 N·m/rad to $k$ = 5000 N·m/rad caused system transformation to point

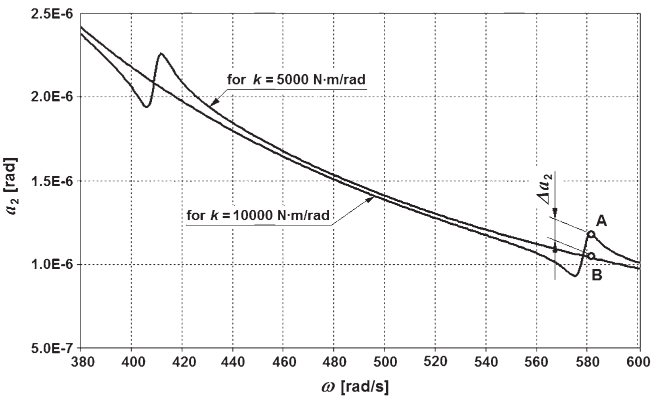


*Fig. 4. Graph of amplitude a1 in relation to extortion ω frequency for torsional stiffness k = 10000 N·m/rad and k = 5000 N·m/rad*

*Table 1. The values of torsional stiffness k and amplitude of angular displacement of $a_1$ and $a_2$ coupling discs*

| 1 | | | | 2 | | |
|---|---|---|---|---|---|---|
| $k$ | $\omega_0$ | $a_1$ | $a_2$ | $k$ | $a_1$ | $a_2$ |
| [N·m/rad] | [rad/s] | [rad] | [rad] | [N·m/rad] | [rad] | [rad] |
| 25000 | 914.239 | $5.682 \cdot 10^{-5}$ | $5.090 \cdot 10^{-7}$ | 12500 | $4.181 \cdot 10^{-7}$ (0.74%) | $4.202 \cdot 10^{-7}$ (82.56%) |
| 10000 | 578.215 | $9.017 \cdot 10^{-5}$ | $1.177 \cdot 10^{-6}$ | 5000 | $1.046 \cdot 10^{-6}$ (1.16%) | $1.050 \cdot 10^{-6}$ (89.28%) |
| 5000 | 408.860 | $1.274 \cdot 10^{-4}$ | $2.254 \cdot 10^{-6}$ | 2500 | $2.023 \cdot 10^{-6}$ (1.59%) | $2.078 \cdot 10^{-6}$ (92.16%) |
| 2500 | 289.107 | $1.806 \cdot 10^{-4}$ | $4.370 \cdot 10^{-6}$ | 1250 | $4.015 \cdot 10^{-6}$ (2.22%) | $4.145 \cdot 10^{-6}$ (94.86%) |
| 1250 | 204.430 | $2.549 \cdot 10^{-4}$ | $8.537 \cdot 10^{-6}$ | 625 | $8.455 \cdot 10^{-6}$ (3,32%) | $8.435 \cdot 10^{-6}$ (98.80%) |
| 630 | 144.553 | $3.606 \cdot 10^{-4}$ | $1.666 \cdot 10^{-5}$ | 312,5 | $1.665 \cdot 10^{-5}$ (4.62%) | $1.669 \cdot 10^{-5}$ (100.21%) |
| 315 | 102.623 | $5.101 \cdot 10^{-4}$ | $3.329 \cdot 10^{-5}$ | 157,5 | $3.435 \cdot 10^{-5}$ (6.73%) | $3.374 \cdot 10^{-5}$ (101.34%) |
| 160 | 73.139 | $7.199 \cdot 10^{-4}$ | $6.542 \cdot 10^{-5}$ | 80 | $6.596 \cdot 10^{-5}$ (9.16%) | $6.586 \cdot 10^{-5}$ (100.66%) |
| 80 | 51.717 | $1.036 \cdot 10^{-3}$ | $1.335 \cdot 10^{-4}$ | 40 | $1.414 \cdot 10^{-4}$ (13.64%) | $1.349 \cdot 10^{-4}$ (101.03%) |
| 60 | 31.670 | $1.208 \cdot 10^{-3}$ | $1.794 \cdot 10^{-4}$ | 30 | $3.700 \cdot 10^{-4}$ (30.62%) | $1.816 \cdot 10^{-4}$ (101.21%) |

$B$ and decreasing the amplitude down to $a_1 = 1.046 \cdot 10^{-6}$ rad (over 86 times lower than in point A).

Figure 5 presents the graph of amplitude $a_2$ with relation to extortion frequency $\omega$. In this case the decrease of $a_2$ is not as significant as for $a_1$. The decrease of torsional stiffness $k$ = 10000 N·m/rad to $k$ = 5000 N·m/rad caused a decrease of amplitude $a_2$ from z $1.177 \cdot 10^{-6}$ rad down to $1.050 \cdot 10^{-6}$ rad (points A and B).

## 6. Conclusions

On the basis of the discussions conducted in the framework of the present work, the following conclusions can be drawn:

- The presented construction solution of flexible coupling (with control system) may constitute a system for constant control and vibration compensation in mechanical systems.
- The change of torsional stiffness with relation to active spring length is strongly non-linear. For the assumed geometric and material parameters the torsional stiffness varies from 30.6 N·m/rad, in case of maximum spring length $L_{max}$ = 105 mm up to 283500 N·m/rad for minimum active spring length $L_{min}$ = 5 mm.
- Linear change of torsional stiffness $k$ generates non-linear change of angular displacement of coupling active disc $a_1$. The amplitude $a_1$ decreases and ranges from 0.74% to 30.62% of $a_1$ amplitude before the change, wherein increase of over 10% applies the values of torsional stiffness $k$ over 80 N·m/rad.
- The change of $k$ has smaller influence on angular displacement of passive disc plate $a_2$. For $k$ values more than 1250 N·m/rad amplitude $a_2$ decreases in the range of 82.56% to 98.80% of the amplitude value before the change and for $k$ less than 630 N·m/rad, $a_2$ increase to 101%.

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