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COMPARISON STUDY OF HEAVY HAUL LOCOMOTIVE WHEELS' RUNNING SURFACES WEARING

BADANIA PORÓWNAWCZE ZUŻYCIA POWIERZCHNI BIEŻNYCH KÓŁ LOKOMOTYW DUŻEJ MOCY

The service life of railway wheels can differ significantly depending on their installed position, operating conditions, re-profiling characteristics, etc. This paper compares the wheels on two selected locomotives on the Iron Ore Line in northern Sweden to explore some of these differences. It proposes integrating reliability assessment data with both degradation data and re-profiling performance data. The following conclusions are drawn. First, by considering an exponential degradation path and given operation condition, the Weibull frailty model can be used to undertake reliability studies; second, among re-profiling work orders, rolling contact fatigue (RCF) is the principal reason; and third, by analysing re-profiling parameters, both the wear rate and the re-profiling loss can be monitored and investigated, a finding which could be applied in optimisation of maintenance activities.

Keywords: *reliability analysis; locomotive wheels; frailty; re-profiling; wear; Markov Chain Monte Carlo.*

Resurs kół pociągu może być znacząco różny w zależności od ich miejsca zamontowania, warunków pracy, charakterystyk związanych z reprofilacją, itp. W artykule, porównano koła dwóch wybranych lokomotyw kursujących na Linii Rud Żelaza w północnej Szwecji, aby zbadać niektóre ze wspomnianych różnic. Zaproponowano możliwość łączenia danych pochodzących z oceny niezawodności z danymi degradacyjnymi oraz danymi z reprofilacji. Przeprowadzone badania pozwalają wyciągnąć następujące wnioski. Po pierwsze, krzywa wykładnicza degradacji oraz zadane warunki pracy można wykorzystać w celu przeprowadzenia badań niezawodności z użyciem modelu Weibulla z efektami losowymi (tzw. "frailty model"); po drugie, główną przyczyną zlecania reprofilacji kół jest zmęczenie toczne (RCF); po trzecie, analiza parametrów reprofilacji pozwala na monitorowanie i badanie zarówno szybkości zużycia kół, jak i ubytku materiału podczas reprofilacji, co może mieć zastosowanie w optymalizacji czynności obsługowych.

Słowa kluczowe:*analiza niezawodności; koła lokomotywy; efekty losowe; reprofilacja; zużycie; markowowska metoda Monte Carlo.*

1. Introduction

The service life of different railway wheels can vary greatly. Take a Swedish railway company, for example. For the wheels of its 26 locomotives, statistics show that from 2010 to 2011, the longest mean time between re-profiling was around 59 000 kilometres and the shortest was about 31 000 kilometres. The large difference can be attributed to the non-heterogeneous nature of the wheels; each differs according to its installed position, operating conditions, re-profiling characteristics, etc. [6, 7, 14, 17, 23].

One common preventive maintenance strategy (used in our study) is re-profiling wheels after they run a certain distance. Re-profiling reduces the wheel's diameter; once the diameter is reduced to a pre-specified length, the wheel is replaced by a new one. Seeking to optimise this maintenance strategy, researchers have examined wheel degradation data to determine wheel reliability and failure distribution [6, 7, 23]. However, most studies cannot solve the combined problem of small data samples and incomplete datasets while simultaneously considering the influence of several covariates [14].

In addition, most reliability studies are implemented under the assumption that individual lifetimes are independent and identically distributed (i.i.d). In reality, sometimes Cox proportional hazard (CPH) models cannot be used because of the dependence of data within a group. For instance, because they have the same operating conditions, the wheels mounted on a particular bogie may be dependent. Modelling dependence in multivariate survival data has received considerable attention, especially in cases where the datasets comprise inter-related subjects of the same group [1, 20]. A key development in modelling such data is to consider frailty models, in which the data are conditionally independent. When frailties are considered, the dependence within subgroups can be considered an unknown and unobservable risk factor (or explanatory variable) of the hazard function. In this paper, we consider a gamma shared frailty, first discussed by Clayton [4] and Oakes [16] and later developed by Sahu et al. [20], to explore the unobserved covariates' influence on the wheels on the same bogie. We also adopt the Weibull hazard model to determine the distribution of the wheels' lifetime; the validity of this model has been established by Lin et al. [14].

Besides the degradation analysis, re-profiling information is a key source of data to evaluate the wheels' performance. As Fröhling and Hettasch [8] note, the "loss of material during re-profiling because of hollow or flange wear" is a significant element in the integrated data processing of the wheel-rail interface management. Even so, related studies remain limited.

To fill this gap in the literature, this paper compares the wheels on two selected locomotives on the Iron Ore Line in northern Sweden, taking an integrated data approach to reliability assessment by considering both degradation data and re-profiling data.

The remainder of the paper is organised as follows. Section 2 describes the background of the comparison study, by introducing the Iron Ore Line, as well as the degradation data and re-profiling parameters for the locomotive wheels being studied, along with their operating conditions. Section 3 presents the degradation analysis using a Weibull frailty model; the analysis considers the wheels' location in the bogies and their operating conditions as covariates and uses Markov Chain Monte Carlo (MCMC) methods. Sections 4 to 6 comprise the comparison study; the three sections compare the re-profiling work orders, the specified re-profiling parameters (the wheel diameters, the flange thickness, the radial run-out, and the lateral run-out), and the wear rate of the wheels, respectively. Each section is accompanied by a discussion. Section 7 offers conclusions and makes suggestions for future study.

2. Study Background

This section gives background information on the Iron Ore Line. It also introduces the degradation data and the re-profiling parameters for the locomotive wheels being studied, along with their operating conditions.

Fig. 1. Geographical location of Iron Ore Line (Malmbanan)

2.1. Iron Ore Line (Malmbanan)

The Iron Ore Line (Malmbanan) is the only existing heavy haul line in Europe; it stretches 473 kilometres and has been in operation since 1903. As Fig. 1 shows, it is mainly used to transport iron ore and pellets from the mines in Kiruna (also Malmberget, close to Kiruna, in Sweden) to Narvik Harbour (Norway) in the northwest and Luleå Harbour (Sweden) in the southeast. The track section on the Swedish side is owned by the Swedish government and managed by Trafikverket (Swedish Transport Administration), while the iron ore freight trains are owned and managed by the freight operator (a Swedish company). Each freight train consists of two IORE metric tonnes with axle loads of 30 tonnes. The trains operate in harsh conditions, including snow in the winter and extreme temperatures ranging from - 40 °C to + 25 °C. Because carrying iron ore results in high axle loads and there is a high demand for a constant flow of ore/pellets, the track and wagons must be monitored and maintained on a regular basis. The condition of the locomotive wheel profile is one of the most important aspects to consider.

2.2. Degradation data and re-profiling parameters

We use the degradation data from two selected heavy haul cargo locomotives (denoted as locomotive 1 and locomotive 2), collected from October 2010 to January 2012. The selection criteria are discussed in Section 2.3. Each locomotive is studied separately, and $n = 2$. For each locomotive, see Fig.2, there are two bogies (incl., Bogie I, Bogie II); and each bogie contains six wheels. The installed position of a wheel on a particular locomotive is specified by the bogie number (I, II-number of bogies on the locomotive), a wheel-set number (1, 2, 3-number of wheel-sets for each bogie, shown as "Axel" in Fig.2) and

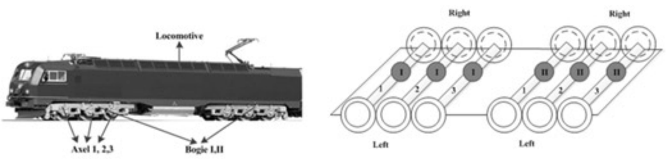


Fig. 2. Wheel positions specified in this study

the position of the wheel-set (right or left) where each wheel is mounted. For instance, the abbreviation I1H represents the wheel installed in the first bogie, the first wheel-set and the right side.

The diameter of a new locomotive wheel in this study is about 1250 mm. Following the current maintenance strategy, a wheel's diameter is measured after it runs a certain distance. If it is reduced to 1150 mm, the wheel set is replaced by a new one. Otherwise, it is re-profiled (see Fig.3). Therefore, a threshold level for failure, denoted as y_0 , is defined as 100 mm (y_0 = 1250 mm – 1150 mm). The wheel's failure condition is assumed to be reached if the diameter reaches y_0 . The dataset includes the diameters of all locomotive wheels at a given inspection time, the total running distances corresponding to their "mean time between re-profiling", and the wheels' bill of material (BOM) data, from which we can determine their positions.

The type of measurement tool is SIEMENS SINUMERIK (see

Fig. 3. One locomotive wheel under re-profiling and the measurement tool

Fig.3). During the re-profiling process, the re-profiling parameters include but are not limited to: 1) the diameters of the wheels; 2) the flange thickness; 3) the radial run-out; 4) the lateral run-out.

2.3. Comparison of the operating conditions

In this study, both locomotive 1 and locomotive 2 are operating on the Iron Ore Line (Malmbanan). In Fig.4, the horizontal axle represents the different working intervals; "Nrv-Kmb" represents the route from Narvik to Kiruna, while "Kmb-Nrv" represents the route from Kiruna to Narvik. Those intervals make up the whole Iron Ore Line (Malmbanan). The longitudinal axle of Fig. 4 (a) represents the proportion (%) of operating in each working interval (%); the longitudinal axle of Fig. 4 (b) represents the running distances (kilometers) in those intervals. For instance, the first blue bars (around 35% in (a) and about 40000 kilometres in (b)) represent that, locomotive 1 has been operated around 40000 kilometres in the interval between "Nrv" and "Kmb". And these 40000 kilometres are almost 35% of locomotive 1's total running distances.

As seen in Fig.4, during the period in question (from October 2010 to January 2012), the total running distance for locomotive 1 is 101035 kilometres and for locomotive 2, 81302 kilometres. About 70% of the locomotives' workload is between Narvik and Kiruna. There is no substantial difference between the running routes, but it seems that locomotive 1 works harder than locomotive 2, because the former runs 24% farther. As there is not a big difference between the topographies, we assume that the only difference in operating conditions is the total running distance.

3. Degradation analyses with the Weibull frailty model

In this section, we propose the Weibull frailty model for analysing the wheels' degradation data, using a MCMC computation scheme.

Before continuing, it should be pointed that Lin et al. [5] have used the Bayesian Exponential Regression Model, Bayesian Weibull Regression Model (easily transferred to an Extreme-Value Regression Model) and Bayesian Lognormal Regression Model, separately, to analyze the lifetime of locomotive wheels using degradation data and taking into account the position of the wheel. Their results show

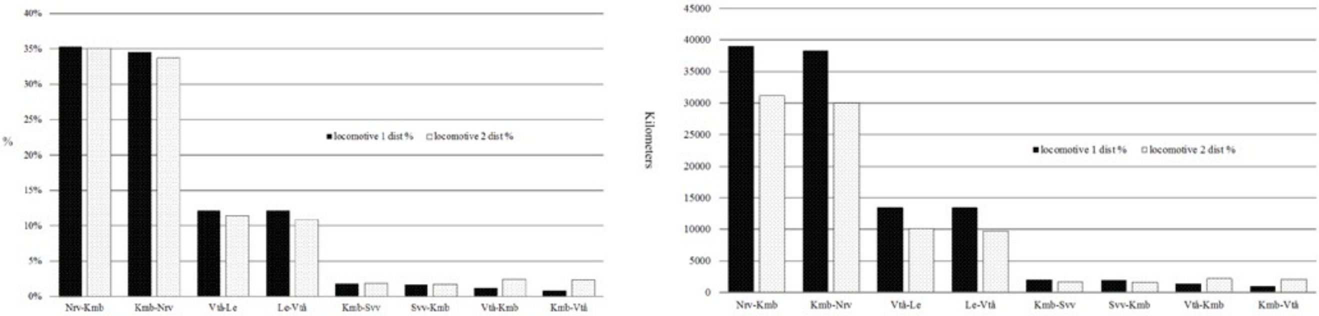


Fig. 4. Comparison of the operating conditions for locomotive 1 and locomotive 2

that "the performance of the Weibull Regression Model is close to the Log-normal Regression Model, which could also be a suitable choice under specified situations." As the Weibull Regression Model is more acceptable to engineers and the differences between the Weibull Regression and Lognormal Regression Models are quite small, we choose the former model in this comparative study.

3.1. Weibull frailty model

Most reliability studies are implemented under the assumption that individual lifetimes are independent and identically distributed (i.i.d). In reality, at times, Cox proportional hazard (CPH) models cannot be used because of the dependence of data within a group. For instance, because they have the same operating conditions, the wheels mounted on a particular bogie may be dependent. Modelling dependence in multivariate survival data has received considerable attention, especially in cases where the datasets comprise inter-related subjects of the same group [1, 20]. A key development in modelling such data is to consider frailty models, in which the data are conditionally independent.

Frailty models were first considered by Clayton [4] and Oakes [16] to handle multivariate survival data. In their models, the event times are conditionally independent according to a given frailty factor, which is an individual random effect. As discussed by Sahu et al.[20], the models formulate different variabilities and come from two distinct sources. The first source is natural variability, which is explained by the hazard function; the second is variability common to individuals of the same group or variability common to several events of an individual, which is explained by the frailty factor.

Assume the hazard function for the j^{th} individual in the i^{th} group is:

$$h_{ij}(t)=h_0(t)\exp(\mu_i+\mathbf{x}_{\mathbf{ij}}'\beta)\ . \qquad (1)$$

In equation (1), μ_i represents the frailty parameter for the i^{th} group. If $\omega_i=\exp(\mu_i)$, the equation can also be written as:

$$h_{ij}(t)=h_0(t)\omega_i\exp(\mathbf{x}_{\mathbf{ij}}'\beta)\ . \qquad (2)$$

Equation (1) is an additive frailty model, and equation (2) is a multiplicative frailty model. In both equations, μ_i and ω_i are shared by the individuals in the same group, and they are thus referred to as shared-frailty models and actually are extensions of the CPH model.

To this point, discussions of frailty models have focused on the forms of 1) the baseline hazard function and 2) the frailty's distribution. Representative studies related to the former include the gamma process for the accumulated hazard function [3, 21], Weibull baseline hazard rate [20], and the piecewise constant hazard rate [1] which is adopted in this paper due to its flexibility. Some researchers have examined finite mean frailty distributions, including gamma distribution [2, 4], lognormal distribution [15], and the like; others have studied non-parameter methods, including the inverse Gaussion frailty distribution [11], the power variance function for frailty [5], the positive stable frailty distribution [10, 19], the Dirichlet process frailty model [19] and the Levy process frailty model [9]. In this paper, we consider the gamma shared frailty model, the most popular model for frailty.

From equation (2), suppose the frailty parameters ω_i are independent and identically distributed (i.i.d) for each group and follow a gamma distribution, denoted by $Ga(\kappa^{-1},\kappa^{-1})$. Therefore, the probability density function can be written as:

$$f(\omega_i)=\frac{(\kappa^{-1})^{\kappa^{-1}}}{\Gamma(\kappa^{-1})}\cdot\omega_i^{\kappa^{-1}-1}\exp(-\kappa^{-1}\omega_i)\ . \qquad (3)$$

In equation (3), the mean value of ω_i is 1, where κ is the unknown variance of ω_i s. Greater values of κ signify a closer positive relationship between the subjects of the same group as well as greater heterogeneity among groups. Furthermore, as $\omega_i>1$, the failures for the individuals in the corresponding group will appear earlier than if $\omega_i=1$; in other words, as $\omega_i<1$, the predicted lifetimes will be greater than those found in the independent models.

Suppose $\boldsymbol{\omega}=(\omega_1,\omega_2,\cdots,\omega_n)'$; then:

$$\pi(\boldsymbol{\omega}|\kappa)\propto\prod_{i=1}^{n}\omega_i^{\kappa^{-1}-1}\exp(-\kappa^{-1}\omega_i)\ . \qquad (4)$$

Denote the j^{th} individual in the i^{th} group as having lifetime $\mathbf{t_{ij}}=(t_{11},t_{12},\cdots,t_{nm_1})'$, where $i=1,\cdots,n$ and $j=1,\cdots,m_i$. Suppose the j^{th} individual in the i^{th} group has a 2-parameter Weibull distribution $W(\alpha,\gamma)$, where $\alpha>0$ and $\gamma>0$. Then, the p.d.f. is $f(t_{ij}|\alpha,\gamma)=\alpha\gamma t_{ij}^{\alpha-1}\exp(-\gamma t_{ij}^{\alpha})$, and the c.d.f. $F(t_{ij}|\alpha,\gamma)$ and the reliability function $R(t_{ij}|\alpha,\gamma)$ are $F(t_{ij}|\alpha,\gamma)=1-\exp(\gamma t_{ij}^{\alpha})=1-R(t_{ij}|\alpha,\gamma)$. Meanwhile, the hazard rate function can be written as:

$$h_0(t_{ij}|\alpha,\gamma)=\gamma\alpha t_{ij}^{\alpha-1}\ . \qquad (5)$$

Based on the above discussions (equation (2), (3), and (5)), the Weibull frailty model with gamma shared frailties can be written as

$$h(t_{ij} \left| \mathbf{x_{ij}}, \omega_i \right.) = \gamma \alpha \omega_i t_{ij}^{\alpha-1} \exp(\mathbf{x'_{ij}} \boldsymbol{\beta}) \ . \quad (6)$$

In equation (6), $\omega_i \sim Ga(\kappa^{-1}, \kappa^{-1})$.

In reliability analyses, the lifetime data are usually incomplete, and only a portion of the individual lifetimes are known. Right-censored data are often called Type I censoring, and the corresponding likelihood construction problem has been extensively studied in the literature [12, 13]. Suppose the j^{th} individual in the i^{th} group has lifetime T_{ij} and censoring time L_{ij} . The observed lifetime $t_{ij} = \min(T_{ij}, L_{ij})$; therefore, the exact lifetime T_{ij} will be observed only if $T_{ij} \le L_{ij}$. In addition, the lifetime data involving right censoring can be represented by n pairs of random variables (t_{ij}, υ_{ij}) , where $\upsilon_{ij} = 1$ if $T_{ij} \le L_{ij}$ and $\upsilon_{ij} = 1$ if $T_{ij} > L_{ij}$. This means that υ_{ij} indicates whether lifetime T_{ij} is censored or not. The likelihood function is deduced as:

$$L(t) = \prod_{i=1}^{n} \prod_{j=1}^{m_i} [f(t_{ij})]^{\upsilon_{ij}} R(t_{ij})^{1-\upsilon_{ij}} \ . \quad (7)$$

If we denote $\lambda_{ij} = \exp(\mathbf{x'_{ij}} \boldsymbol{\beta} + \log \gamma + \log \omega_i)$, equation (6) becomes $h(t_{ij} \left| \lambda_{ij}, \alpha \right.) = \lambda_{ij} \alpha t_{ij}^{\alpha-1}$, the Weibull regression model with a gamma frailty $W(\alpha, \lambda_{ij})$.

If we denote the model's dataset as $D = (n, \omega, \mathbf{t}, \mathbf{X}, \upsilon)$, following equation (7), the complete likelihood function $L(\boldsymbol{\beta}, \gamma, \alpha \left| D \right.)$ for the individuals in the i^{th} group can be written as:

$$L(\boldsymbol{\beta}, \gamma, \alpha \left| D \right.) \propto (\gamma \alpha t_{ij}^{\alpha-1} \omega_i \exp(\mathbf{x'_{ij}} \boldsymbol{\beta}))^{\sum_{i=1}^{n} \sum_{j=1}^{m_i} \upsilon_{ij}} \exp(-\sum_{i=1}^{n} \sum_{j=1}^{m_i} \gamma t_{ij}^{\alpha} \exp(\mathbf{x'_{ij}} \boldsymbol{\beta}) \omega_i) \ . \quad (8)$$

Let $\pi(\cdot)$ denote the prior or posterior distributions for the parameters. Then, the joint posterior distribution $\pi(\omega_i \left| \boldsymbol{\beta}, \alpha, \gamma, D \right.)$ for gamma frailties ω_i can be written as:

$$\begin{aligned} \pi(\omega_i \left| \boldsymbol{\beta}, \alpha, \gamma, D \right.) &\propto L(\boldsymbol{\beta}, \gamma, \alpha \left| D \right.) \times \pi(\boldsymbol{\omega} \left| \kappa \right.) \\ &\propto (\gamma \alpha t_{ij}^{\alpha-1} \omega_i \exp(\mathbf{x'_{ij}} \boldsymbol{\beta}))^{\sum_{i=1}^{n} \sum_{j=1}^{m_i} \upsilon_{ij}} \exp(-\sum_{i=1}^{n} \sum_{j=1}^{m_i} \gamma t_{ij}^{\alpha} \exp(\mathbf{x'_{ij}} \boldsymbol{\beta}) \omega_i) \times \prod_{i=1}^{n} \omega_i^{\kappa^{-1}-1} \exp(-\kappa^{-1} \omega_i) \\ &\propto \omega_i^{\kappa^{-1} + \sum_{j=1}^{m_i} \upsilon_{ij} - 1} \exp\{-(\kappa^{-1} + \gamma \sum_{j=1}^{m_i} t_{ij}^{\alpha} \exp(\mathbf{x'_{ij}} \boldsymbol{\beta})) \omega_i\} \\ &\sim Ga\{\kappa^{-1} + \sum_{j=1}^{m_i} \upsilon_{ij}, \kappa^{-1} + \gamma \sum_{j=1}^{m_i} t_{ij}^{\alpha} \exp(\mathbf{x'_{ij}} \boldsymbol{\beta})\} \end{aligned} \quad (9)$$

Equation (9) shows that the full conditional density of each ω_i is a gamma distribution. Suppose γ has a gamma prior distribution, denoted by $\gamma \sim Ga(\rho_1, \rho_2)$. The full conditional density of γ is:

$$\begin{aligned} \pi(\gamma \left| \boldsymbol{\beta}, \alpha, \omega, D \right.) &\propto L(\boldsymbol{\beta}, \gamma, \alpha \left| D \right.) \times \pi(\gamma \left| \rho_1, \rho_2 \right.) \\ &\propto (\gamma \alpha t_{ij}^{\alpha-1} \omega_i \exp(\mathbf{x'_{ij}} \boldsymbol{\beta}))^{\sum_{i=1}^{n} \sum_{j=1}^{m_i} \upsilon_{ij}} \exp(-\sum_{i=1}^{n} \sum_{j=1}^{m_i} \gamma t_{ij}^{\alpha} \exp(\mathbf{x'_{ij}} \boldsymbol{\beta}) \omega_i) \times \prod_{i=1}^{n} \gamma^{\rho_1 - 1} \exp(-\rho_2 \gamma) \\ &\propto \gamma^{\rho_1 + \sum_{i=1}^{n} \sum_{j=1}^{m_i} \upsilon_{ij} - 1} \exp\{-(\rho_2 + \sum_{i=1}^{n} \sum_{m=1}^{n_i} t_{ij}^{\alpha} \exp(\mathbf{x'_{ij}} \boldsymbol{\beta}) \omega_i) \gamma\} \\ &\sim Ga\{\rho_1 + \sum_{i=1}^{n} \sum_{j=1}^{m_i} \upsilon_{ij}, \rho_2 + \sum_{i=1}^{n} \sum_{m=1}^{n_i} t_{ij}^{\alpha} \exp(\mathbf{x'_{ij}} \boldsymbol{\beta}) \omega_i\} \end{aligned} \quad (10)$$

Equation (10) also shows that the full conditional density of γ is a gamma distribution. The full conditional density of $\boldsymbol{\beta}$ and α can be given by:

$$\pi(\boldsymbol{\beta} \left| \eta, \omega, \gamma, D \right.) \propto \exp\{\boldsymbol{\beta}' \sum_{i=1}^{n} \sum_{j=1}^{m_i} \upsilon_{ij} x_{ij} - \gamma \sum_{i=1}^{n} \sum_{m=1}^{n_i} t_{ij}^{\alpha} \exp(\mathbf{x'_{ij}} \boldsymbol{\beta}) \omega_i\} \times \pi(\boldsymbol{\beta}) \quad (11)$$

$$\pi(\alpha \left| \boldsymbol{\beta}, \gamma, \omega, D \right.) \propto (\prod_{i=1}^{n} \prod_{j=1}^{m_i} t_{ij}^{\upsilon_{ij}})^{\alpha-1} \alpha^{\sum_{i=1}^{n} \sum_{j=1}^{m_i} \upsilon_{ij}} \exp\{-\gamma \sum_{i=1}^{n} \sum_{m=1}^{n_i} t_{ij}^{\alpha} \exp(\mathbf{x'_{ij}} \boldsymbol{\beta}) \omega_i\} \times \pi(\alpha) \ . \quad (12)$$

3.2. Comparison study of degradation analyses

3.2.1. Degradation path and lifetime data

From the dataset, we obtain 5 to 6 measurements of the diameter of each wheel during its lifetime (in the period October 2010 to January 2012). By connecting these measurements, we can determine a degradation trend. In their analyses of train wheels, most studies (e.g., [6], [7], [14]) assume a linear degradation path. In this study, the corresponding running distance (kilometres) is recognized as the lifetime. The degradation data of the wheels are tested with ReliaSoft Weibull++. The statistics of Ranks and MSE under different assumption of degradation path are compared, including Linear degradation path, Exponential degradation path, Power degradation path, Logarithmic degradation path, Gompertz degradation path, and Lloyd-Lipow degradation path. The results show that, an exponential degradation path is a better choice for the studied locomotive wheels. Meanwhile, the second choice is Linear degradation, and the third one is Power degradation. In Table 1, we list the lifetimes as their degradation reach to the threshold level (equal to 100mm).

Table.1. Statistics on degradation path and lifetime data

Locomotive	Bogie	Lifetime**	Locomotive 2	Bogie	Lifetime**
1	I	*159.00	2	I	205.47
1	I	*162.04	2	I	205.49
1	I	*159.04	2	I	207.51
1	I	*159.32	2	I	207.82
1	I	152.22	2	I	211.24
1	I	151.13	2	I	211.22
1	II	163.84	2	II	203.45
1	II	163.87	2	II	203.32
1	II	157.84	2	II	203.44
1	II	157.75	2	II	204.08
1	II	159.05	2	II	203.17
1	II	159.53	2	II	203.17

** Right-censored data; ** $\times 10^3$ kilometres*

Note: some lifetime data are right-censored (denoted by the asterisk in Table.1 However, we know the real lifetimes will exceed the predicted lifetimes.

Following the above discussion, a wheel's failure condition is assumed to be reached if the diameter reaches y_0. We adopt the linear path for all wheels and set $y_0 = y$. The lifetimes for these wheels are now easily determined and are shown in the "Lifetime" columns of Table 1. These lifetimes are the input of the Weibull frailty model, as discussed in Section 3.

3.2.2. Parameter Configuration

Following the discussion in 3.2.1, vague prior distributions are adopted in this paper as:

- Gamma frailty prior: $\omega_i \sim Ga$ (0.1, 0.1);
- Normal prior distribution: $\beta_0 \sim N$ (0.0, 0.001);
- Normal prior distribution: $\beta_1 \sim N$ (0.0, 0.001);
- Gamma prior distribution: $\alpha \sim Ga$ (0.1, 0.1);
- Gamma prior distribution: $\gamma \sim Ga$ (0.1, 0.1).

At this point, the MCMC calculations are implemented using the software WinBUGS [2]. We use a burn-in of 10,001 samples, along with an additional 10,000 Gibbs samples.

3.2.3. Results

Following the convergence diagnostics (incl., checking dynamic traces in Markov chains, time series, and comparing the Monte Carlo (MC) error with Standard Deviation (SD); see [22]), we consider the following posterior distribution summaries (see Table 3): the parameters' posterior distribution mean, SD, MC error, and the 95% highest posterior distribution density (HPD) interval.

In Table 2, $\beta_1 < 0$ means that the wheels mounted in the first bogie (as $x = 1$) have a shorter lifetime than those in the second (as $x = 2$). However, the influence could possibly be reduced as more data are obtained in the future, because the 95% HPD interval includes a 0 point. In addition, the heterogeneity of the wheels on the two locomotives is significant. Nevertheless, $\omega_1 < 1$ suggests that the predictive lifetimes for the wheels mounted on the first locomotive are shorter when the frailties are considered; however, $\omega_2 > 1$ indicates the opposite conclusion.

Table 2. Posterior distribution summaries

Parameter	mean	SD	MC error	95% HPD Interval
β_0	-0.2836	31.36	0.3449	(-60.69,61.4)
β_1	-0.1593	31.62	0.3085	(-63.2,62.71)
α	1.035	3.329	0.03368	(2.449E-16,10.36)
γ	0.9726	3.101	0.02904	(1.683E-15,9.277)
ω_1	0.9763	3.045	0.02924	(1.738E-16,9.595)
ω_2	1.029	3.261	0.0337	(9.718E-16,10.21)

By considering the random effects resulting from the natural variability (explained by covariates) and the unobserved random effects within the same group (explained by frailties), we can determine other reliability characteristics of lifetime distribution. The statistics on reliability $R(t)$ for the two wheels mounted in different bogies are:

- $h(t_1) = 0.97 \times 1.035 \times 0.9763 \times t_1^{0.035} \exp(-0.2836 + (-0.1593x))$
- $h(t_2) = 0.97 \times 1.035 \times 1.029 \times t_2^{0.035} \exp(-0.2836 + (-0.1593x))$

3.3. Discussion

The above results can be applied to maintenance optimisation, including lifetime prediction and replacement, preventative maintenance, and re-profiling. More specifically, determining reliability characteristics distributed over the wheels' lifetime could be used to optimise replacement strategies and to support related predictions for spares inventory. With respect to preventative maintenance, the wheels installed in different bogies should be given more attention during maintenance. Especially when the wheels are re-profiled, they should be checked starting with the bogies to avoid duplication of efforts. Last but not least, as the operating environments are similar for the two locomotives considered here, the frailties between bogies could be caused by the locomotives themselves, the status of the bogies or spring systems, and human influences (including maintenance policies and the lathe operator).

4. Comparison study on re-profiling work orders

This section compares the work orders for wheel re-profiling by date (denoted as "by date" in Fig.5) and the corresponding bogies' total number of kilometres in operation (denoted as "by kilometres" in Fig.6), separately.

In Fig.6, the work order statistics for re-profiling are listed by date. The number of the bar represents the type of work order reported in the system. For instance, number 1 means the reason for re-profiling is a high flange; number 3 represents the RCF problem; number 7 means the re-profiling is due to the dimension difference between wheels in a bogie; number 9 denotes a thick flange. The work orders have 14 categories for re-profiling: high flange, thin flange, RCF, unbalanced wheel, QR measurements, out-of-round wheel, dimension difference in between wheels in same bogie, vibrations, thick flange, cracks, remarks from measurement of the wheel by Miniprof, other defects, to plant for re-profiling, and hollowware. These categories are determined by the operator and are listed in Appendix A. Take Fig.5 (a) for example. By April 2010, the wheels of Locomotive 1 have been re-profiled 12. Eight times it was related to category 3 (RCF problem), and four times it was in category 7 (the dimension difference between wheels in a bogie).

In Fig.5 and Fig.6, the figures on the left side provide the statistics for locomotive 1, while those on the right are for locomotive 2. Note that in Fig.6, the work order statistics on re-profiling are listed by the corresponding bogies' total number of kilometres in operation on the reported date. In Fig.6(b), the wheels have run 87721 kilometres and been re-profiled 16 times, 12 times due to category 1 (high flange) and 4 times due to category 9 (thick flange).

It should be pointed out that since October 2010, new wheels have been mounted on both locomotives. However, the selected work orders are from the beginning of 2010; therefore, more re-profiling has been done on locomotive 1.

For locomotive 1, there are two failure modes: RCF and dimensional differences for wheels in the same bogie. The number of re-profiling work orders due to RCF is 64; the number due to dimensional differences for wheels in the same bogie is 8. Locomotive 2 shows three failure modes, high flange, RCF and thick flange. Again, the dominant failure mode is RCF with 38 re-profilings, followed by high flange with 12 re-profilings and thin flange with 4; see Fig. 5(b). Figs. 5(c) and (d) show the amount of material removed at each re-profiling for all wheels. Even here, the RCF failure dominates with more material lost in re-profiling. Figs. 5(e) and (f) show the mean cut deep for each re-profiling. The RCF failure mode has deeper cuts than other modes; the high flange failure mode has the smallest mean cut depth.

Fig. 6 shows the same information but uses the global traveling distance in kilometres (km). It should be pointed out that for Locomo-

Fig. 6. Work order statistics on re-profiling by kilometre

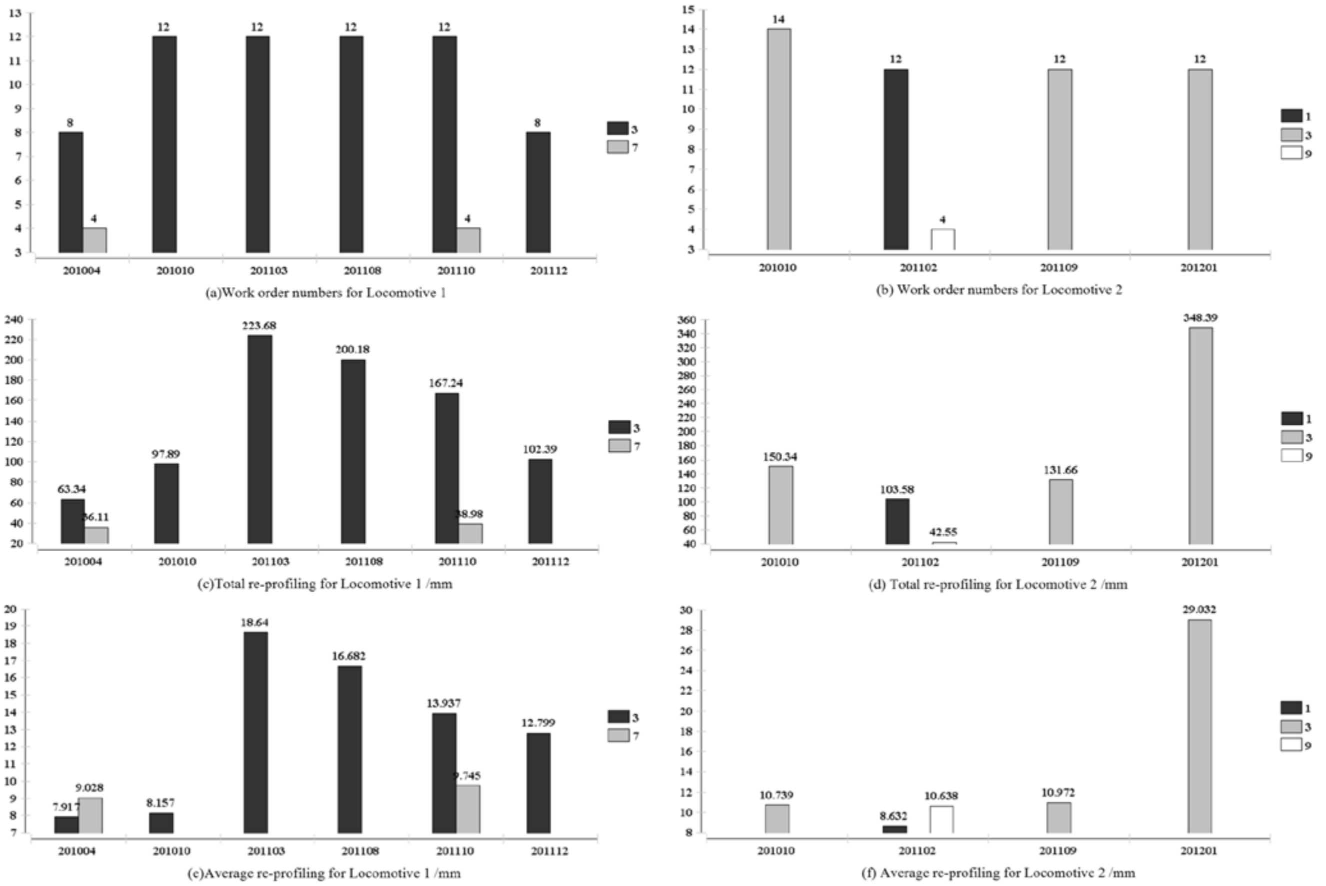


Fig. 5. Work order statistics on re-profiling by date

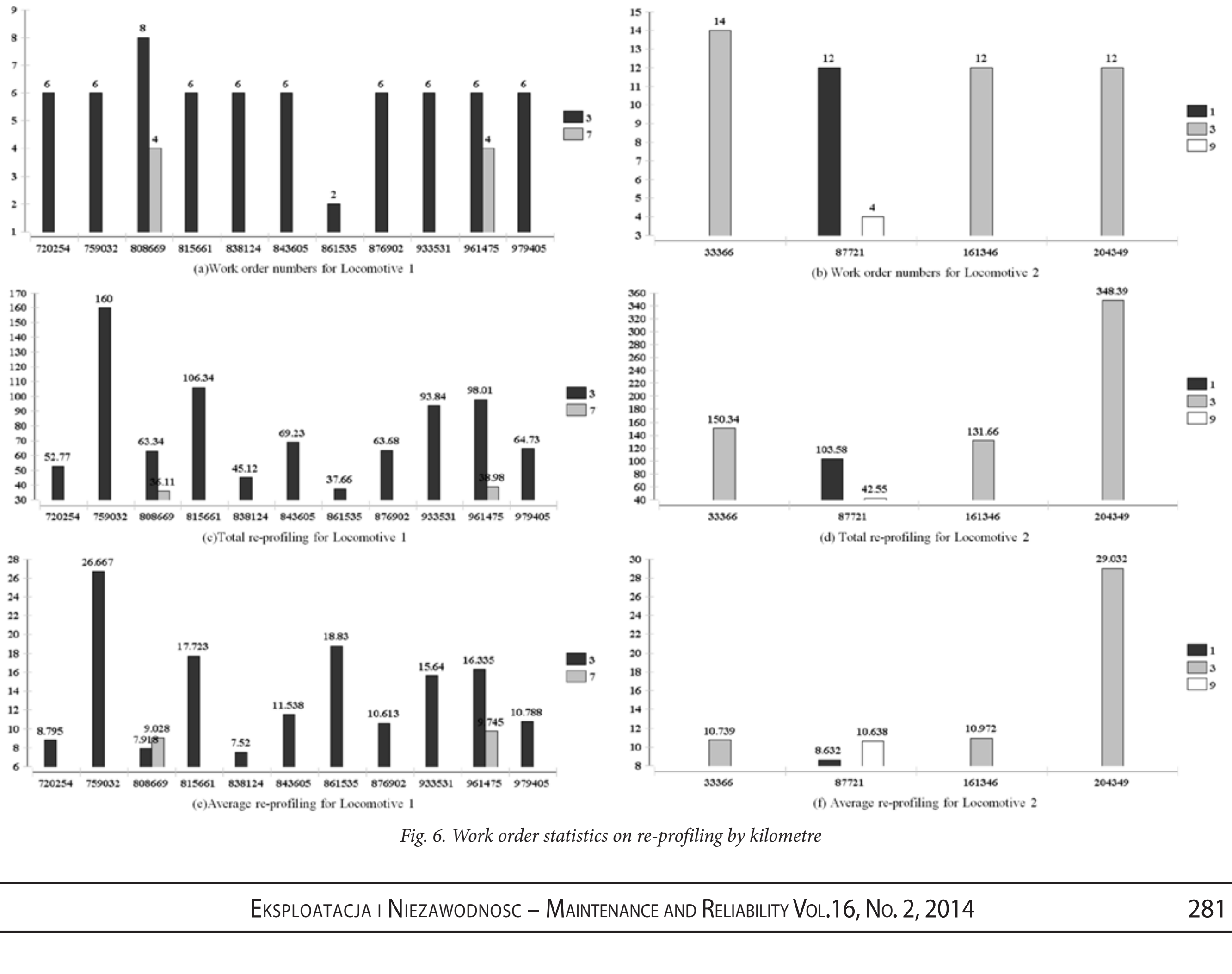


Fig. 6. Work order statistics on re-profiling by kilometre

tive 1, Fig. 6 has more bars on the left hand side because the wheelsets have been changed and the recorded kilometres are different.

Generally speaking, RCF is the main type of work order for both locomotives. What should also be pointed out is that in the work order statistics, natural wear and the amount of re-profiling are considered simultaneously. Yet the trends in the amount of re-profiling are different. For instance, for locomotive 1, there is a decreasing trend for new wheels, while locomotive 2 shows an increasing trend.

During this investigation, we discovered a number of problems in the work orders. For example, some reported data cannot be recognised (e.g., some wheels are apparently re-profiled twice on one date; some reported wheel diameters after re-profiling are even larger than before re-profiling).

We suggest applying related KPIs to monitor the re-profiling work and the wheel performance in the future.

5. Comparison study on re-profiling parameters

In this section, we compare the re-profiling parameters (the statistics before and after each re-profiling), including the diameter of the wheel (denoted as Rd), the flange thickness (denoted as Sd), the radial runout (denoted as Rr), and the axial runout (denoted as Rx).

5.1. Assessment of re-profiling parameters (Rd)

Starting in this section, we only include statistics by re-profiling date. In addition, due to the similarities of the wheels installed in the same bogie, we only list statistics for the chosen wheel within each bogie. The upper line represents the statistics obtained before re-profiling; the lower line represents statistics after re-profiling. Fig. 7 shows, the y-axle is the wheel diameter and the x-axle is the re-profiling date. For locomotive 1, the graphs start with the last re-profiling of an old wheel; step two is the first re-profiling with new wheels.

The wheels installed in the same bogie show similar trends in the before and after re-profiling statistics (denoted as Δ Rd). Δ Rd is decreasing for locomotive 1 and increasing for locomotive 2.

5.2. Assessment of re-profiling parameter (Sd)

Fig. 8 shows the statistics of the Sd for the selected wheels. Locomotive 1 is represented on the left hand side, with locomotive 2 on the right. For both, the flange thickness increases during winter and decreases in summer; this phenomenon is especially pronounced for locomotive 1 and the first bogie and first wheel-set. A reasonable explanation for this phenomenon (changes in flange thickness in winter and summer) is that, in winter time, the wheel treads have more wear compared with in the summer. Therefore, as the measurements are taken, the positions (from the wheel treads) where to measure the flange thickness are lower than in the summer. Considering the

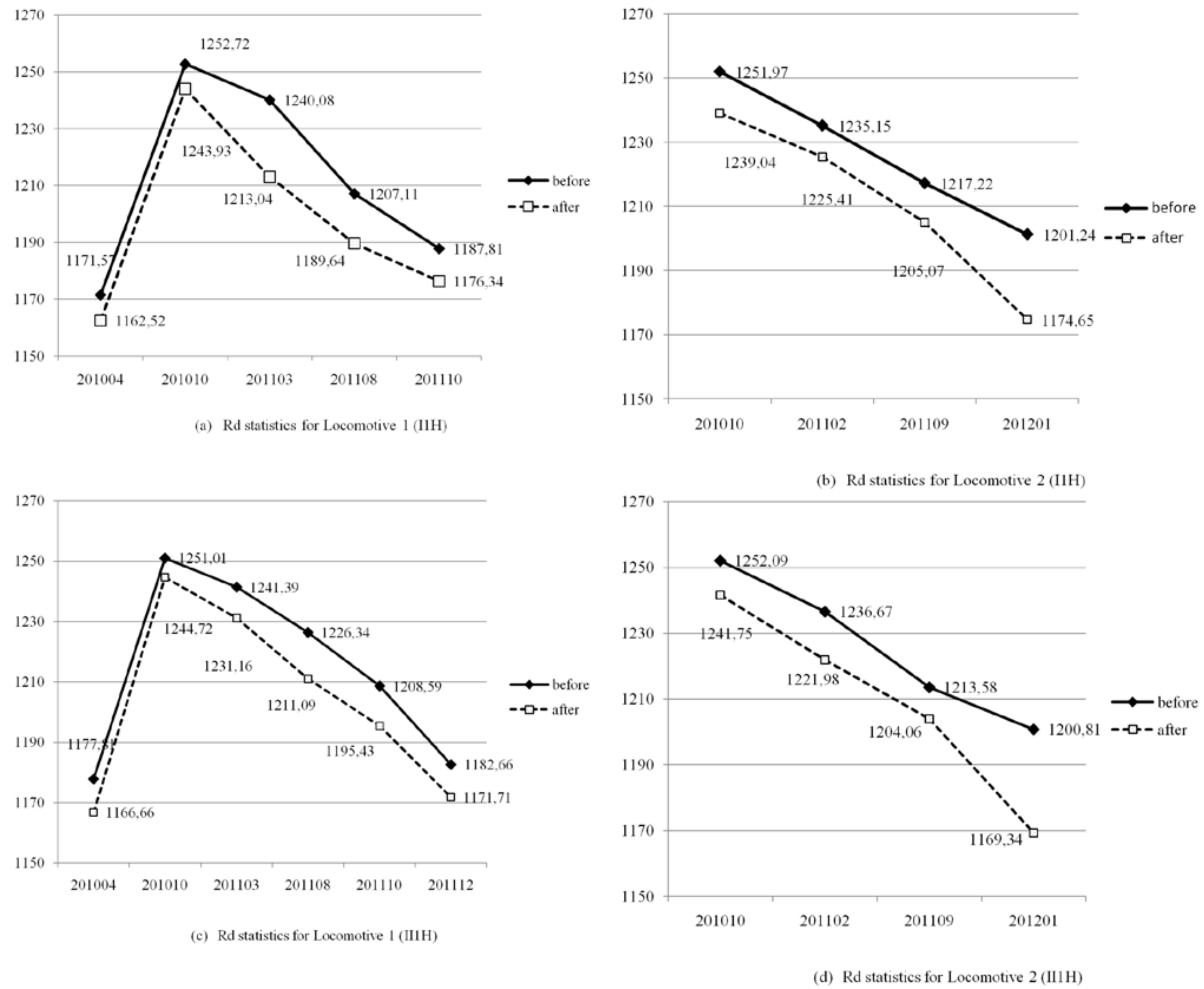


Fig. 7. Rd statistics by date (before and after re-profiling): one example (I1H & II1H)

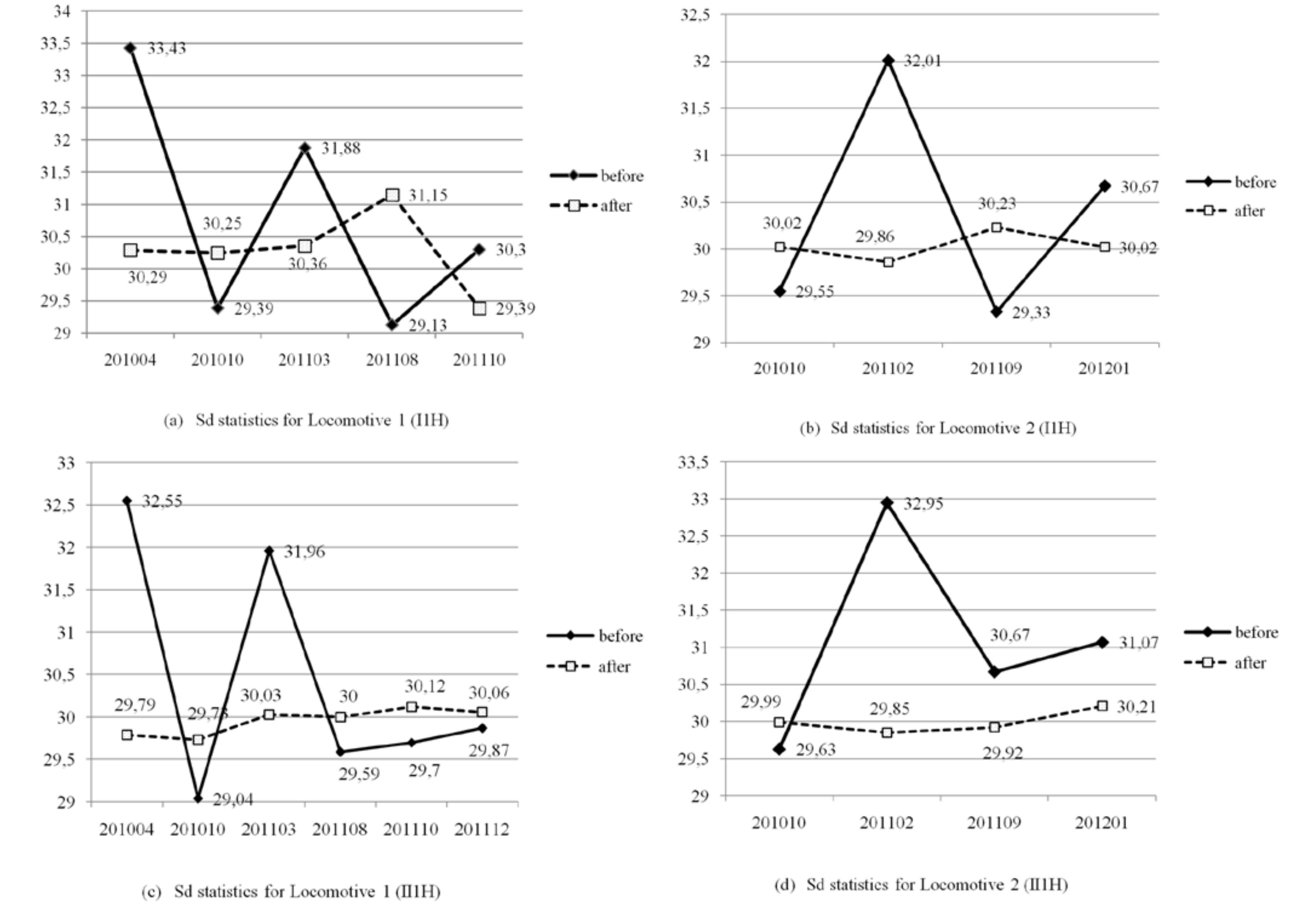


Fig. 8. Sd statistics by date (before and after re-profiling): one example (I1H & II1H)

wheels' geometry, it leads to the increasing of the flange thickness during wither and decrease in summer time.

Like the Rd statistics, the Sd statistics for the wheels installed in the same bogie are quite similar. The "after" statistics are stable. The "before" statistics are gradually becoming stable, which means the gap (denoted as Δ Sd) is decreasing.

Note that if we check the before and after statistics in different seasons, we see that the flange thickness decreases in summer and increases in winter; see Fig. 8 (a).

5.3 Assessment of re-profiling parameter (ΔRd, ΔSd, ΔRr, ΔRx)

In this section, we simultaneously consider the gaps of the four parameters discussed above: ΔRd, ΔSd, ΔRr, and ΔRx.

As discussed above, the statistics for the wheels installed in the same bogie are quite similar. Among these four parameters, the changing of ΔRd is the most obvious one, with ΔSd coming second. The changing of ΔRr and ΔRx are random and the amount is quite small compared to the first two parameters. Therefore, we suggest applying the first two parameters to monitor the wheels' re-profiling performance in the future.

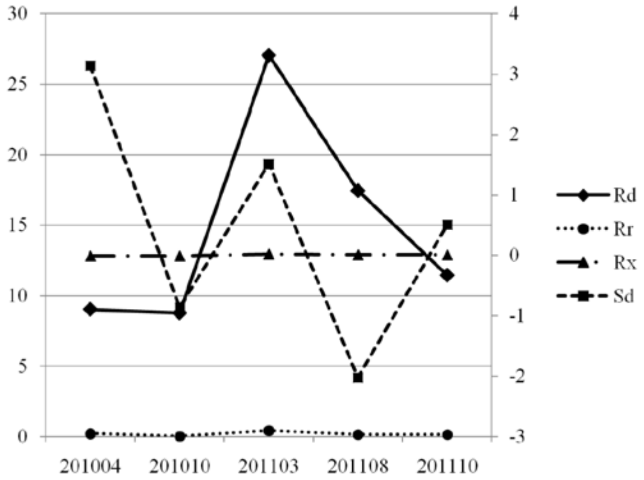
6. Comparison of wear rate

In this section, we compare the wheels' wear rates, shown in Tables 3 to 6. Table 3 shows locomotive 1, bogie 1 and the first wheel-set on the right side; Table 4 shows locomotive 1, bogie 2 and the first wheel-set on the right side; see Fig. 2 for the position of the bogies and wheel-sets. The number of re-profiling work orders is different between bogies: bogie 1 has 4 and bogie 2 has 5. The reason for the difference may be that bogie 1 was changed after the fourth re-profiling. The re-profiling at times 1 to 4 was done at the same time for both bogies, extending over 12 months.

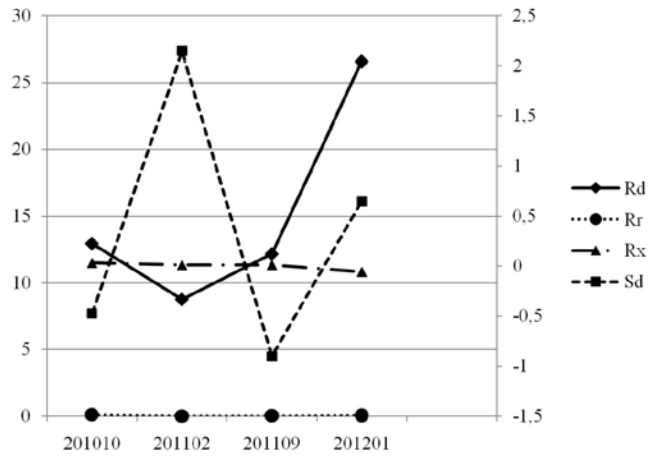
As for locomotive 1, Table 3 shows that it has been running for 123.351 km; the mean distance between re-profiling is 41.117 km. The distance after the last re-profiling for bogie 2 was only 17.930 km, less than half of the average distance for re-profiling numbers 1 to 4; see Table 4. Tables 3 and 4 also show the diameter of the wheel before and after re-profiling and the amount of material removed at each re-profiling. The mean amount of material removed during re-profiling for bogie 1 is 16.193 mm and for bogie 2, 11.176 mm. Remarkably, the amount of re-profiling for bogie 2, step 2 is 27.04 mm, much more than the others; as noted above, the mean is 16.193 mm. If we compare natural wear with artificial wear, the former is between 15 mm and 20% of the total wear. In addition, the total wear rate for locomotive 2, bogie 1, is 0.619 mm/1000 km; for bogie 2, it is 0.393 mm/1000km.

As mentioned, locomotive 1 and locomotive 2 have the same operating conditions (see Fig. 4 for the comparison), but the figures in Tables 5 and 6 show different results. Table 5 shows locomotive 2, the first bogie, the first wheel-set, and the right hand side wheel; Table 6 shows the second bogie, the first wheel-set, and the right hand side wheel. This locomotive has been re-profiled 4 times in 15 months; the mean distance between re-profiling is 56.990 km. The mean amount of material removed for re-profiling for bogie 1 is 15.10 mm; for bogie 2 it is 16.51 mm. The last re-profiling for the first bogie removed 26.59 mm and for the second bogie 31.47 mm. Finally, the total wear rate for locomotive 2, bogie 1, is 0.452 mm/1000 km and for bogie 2, 0.484 mm/1000km

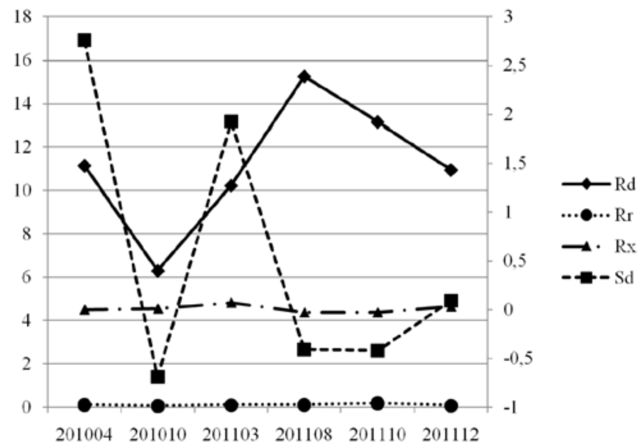
Explanatory comments for Tables 3, 4, 5, 6 include the following:



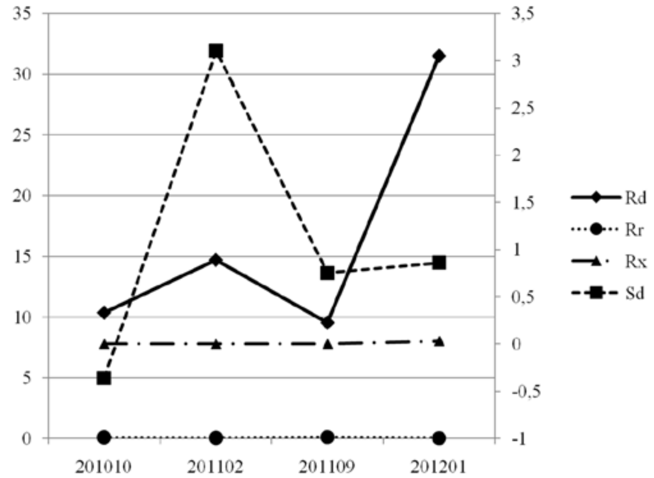
(a) Gap statistics for Locomotive 1 (I1H)



(b) Gap statistics for Locomotive 2 (I1H)



a. Gap statistics for Locomotive 1 (II1H)



b. Gap statistics for Locomotive 2 (II1H)

Fig. 9 Gap statistics by date (before and after re-profiling): one example (I1H & II1H)*

*) *Note: To make it more clearly, we adopted two axis here (left and right). For Rd and Rr, we adopted the axis on the left side; while for Rx and Sd, we adopted the axis on the righ side.*

Table 3. Statistics for wear rate: an example (locomotive 1, I1H)

Locomotive	1	Position	I1H		Total/Average
Number of re-profiling	1	2	3	4	4 times
Re-profiling date	201010	201103	201108	201110	12 months
Reported kilometres /1000km	720.254	759.032	815.661	843.605	/
Absolute kilometres /1000km	0	38.778	56.629	27.944	123.351
Diameters (before)/mm	1252.72	1240.08	1207.11	1187.81	/
Diameters (after)/mm	1243.93	1213.04	1189.64	1176.34	/
Re-profiling Amount/mm	8.79	27.04	17.47	11.47	64.77
Natural Wear/mm	0	3.85	5.93	1.83	11.61
Total Wear/mm	8.79	30.89	23.4	13.3	76.38
Re-profiling Amount %	1	0.875	0.747	0.862	0.848
Natural Wear %	0	0.125	0.253	0.138	0.152
WearRate_re-profiling	/	0.697	0.308	0.41	0.525
WearRate_Natural	/	0.099	0.105	0.065	0.094
WearRate_Total	/	0.797	0.413	0.476	0.619

Table 4. Statistics for wear rate: an example (locomotive 1, II1H)

Locomotive	2	Position	II1H			Total/Average
Number of re-profiling	1	2	3	4	5	5 times
Re-profiling date	201010	201103	201108	201110	201112	14 months
Reported kilometres /1000km	838.124	876.902	933.531	961.475	979.405	/
Absolute kilometres /1000km	0	38.778	56.629	27.944	17.93	141.281
Diameters (before)/mm	1251.01	1241.39	1226.34	1208.59	1182.66	/
Diameters (after)/mm	1244.72	1231.16	1211.09	1195.43	1171.71	/
Re-profiling Amount/mm	6.29	10.23	15.25	13.16	10.95	44.93
Natural Wear/mm	0	3.33	4.82	2.5	12.77	10.65
Total Wear/mm	6.29	13.56	20.07	15.66	23.72	55.58
Re-profiling Amount %	1	0.754	0.76	0.84	0.462	0.808
Natural Wear %	0	0.246	0.24	0.16	0.538	0.192
WearRate_re-profiling	/	0.264	0.269	0.471	0.611	0.318
WearRate_Natural	/	0.086	0.085	0.089	0.712	0.075
WearRate_Total	/	0.35	0.354	0.56	1.323	0.393

Table 5. Statistics for wear rate: an example (locomotive 2, I1H)

Locomotive	2	Position	11H		Total/Average
Number of re-profiling	1	2	3	4	4 times
Re-profiling date	201010	201102	201109	201201	15 months
Reported kilometres /1000km	33.366	87.721	161.346	204.349	/
Absolute kilometres /1000km	0	54.355	73.625	43.003	170.983
Diameters (before)/mm	1251.97	1234.15	1217.22	1201.24	/
Diameters (after)/mm	1239.04	1225.41	1205.07	1174.65	/
Re-profiling Amount/mm	12.93	8.74	12.15	26.59	60.41
Natural Wear/mm	0	4.89	8.19	3.83	16.91
Total Wear/mm	12.93	13.63	20.34	30.42	77.32
Re-profiling Amount %	1	0.641	0.597	0.874	0.781
Natural Wear %	0	0.359	0.403	0.126	0.219
WearRate_re-profiling	/	0.161	0.165	0.618	0.353
WearRate_Natural	/	0.09	0.111	0.089	0.099
WearRate_Total	/	0.251	0.276	0.707	0.452

Table 6. Statistics for wear rate: an example (locomotive 2, II1H)

Locomotive	2	Position	II1H		Total/Average
Number	1	2	3	4	4 times
Date	201010	201102	201109	201201	15 months
Reported kilometres /1000km	33.366	87.721	161.346	204.349	/
Absolute kilometres /1000km	0	54.355	73.625	43.003	170.983
Diameters (before)/mm	1252.09	1236.67	1213.58	1200.81	/
Diameters (after)/mm	1241.75	1221.98	1204.06	1169.34	/
re-profiling Amount/mm	10.34	14.69	9.52	31.47	66.02
Natural Wear/mm	0	5.08	8.4	3.25	16.73
Total Wear/mm	10.34	19.77	17.92	34.72	82.75
re-profiling Amount %	1	0.743	0.531	0.906	0.798
Natural Wear %	0	0.257	0.469	0.094	0.202
WearSpeed_re-profiling	/	0.27	0.129	0.732	0.386
WearSpeed_Natural	/	0.093	0.114	0.076	0.098
WearSpeed_Total	/	0.364	0.243	0.807	0.484

Table 7. Statistics for total wear rates

WearRate_total												
	11H	11V	12H	12V	13H	13V	21H	21V	22H	22V	23H	23V
Locomotive 1	0.619	0.607	0.614	0.605	0.542	0.533	0.393	0.404	0.467	0.467	0.467	0.472
Locomotive 2	0.452	0.439	0.448	0.448	0.449	0.448	0.484	0.482	0.568	0.575	0.487	0.476

- Absolute kilometres = the current reported kilometres – the previous reported kilometres;
- Re-profiling Amount = Diameters (before) - Diameters (after);
- Natural Wear = the previous Diameters (after) – the current Diameters (before);
- Total Wear = Re-profiling Amount + Natural Wear;
- Re-profiling Amount % = Re-profiling Amount / Total Wear;
- Natural Wear % = Natural Wear/ Total Wear;
- WearRate_Reprofiling = Re-profiling Amount / Absolute kilometres;
- WearRate_Natural = Natural Wear / Absolute kilometres;
- WearRate_Total = Total Wear / Absolute kilometres;
- Average of the total wear rate = the average of WearRate_Total.

In addition, by comparing the interval of the re-profiling date, we can simply divide each re-profiling episode into seasons (for instance, the summer and warmer times, the winter and cooler times).

In Table 7, we list the statistics for the WearRate_total of all the wheels for the two locomotives. The mean wear rates are 0.516 mm/1000km for locomotive 1 and 0.480 mm/1000km for locomotive 2; in other words, locomotive 1 has a 75% higher wear rate. Wheel-sets 1, 2 and 5 have 11.6 % higher wear rate than wheel-sets 3, 4 and 6.

By comparing the above parameters of the wheels installed in different positions on the locomotives, we can reach the following additional conclusions:

- the average wear rate of the wheels on locomotive 1 is greater than for locomotive 2;
- the natural wear is about 10% ~ 25 % of the total wear; the re-profiling is about 75 %~ 90% of the total;
- the natural wear in winter time is slower than in summer;
- the re-profiling rate in winter is larger than in summer;
- the wheels installed on the second wheel-set in the second bogie have an abnormal higher wear rate compared to the wheels installed in the same bogie but on the other wheel-set; this requires more attention;
- The wheels installed in the same bogie perform similarly.

7. Conclusions

In this paper, the Weibull frailty model is used to analyse the wheels' degradation. The gamma shared frailties $\boldsymbol{\omega}_i$ are used to explore the influence of unobserved covariates within the same locomotive. By introducing covariate $\mathbf{x}_i$'s linear function $\mathbf{x}_i'\boldsymbol{\beta}$, we can take into account the influence of the bogie in which a wheel is installed. The proposed framework can deal with small and incomplete datasets; it can also simultaneously consider the influence of various covariates. The MCMC technique is used to integrate high-dimensional probability distributions to make inferences and predictions about model parameters. Finally, we compare the statistics on re-profiling work orders, the performance of re-profiling parameters (denoted as ΔRd, ΔSd, ΔRr, ΔRx), and wear rates.

The results show the following for the two locomotives: 1) with the specified installation position and operating conditions, the Weibull frailty model is a useful tool to determine wheel reliability by considering an exponential degradation path; 2) rolling contact fatigue (RCF) is the main type of re-profiling work order; 3) the re-profiling parameters can be applied to monitor both the wear rate and the re-profiling loss; 4) the total wear of the wheels can be determined by investigating natural wear and/or loss of wheel diameter through re-profiling loss, but these are different in different locomotives and under different operating conditions; 5) the bogie in which a wheel is installed is a key factor in assessing the wheel's reliability.

Finally, the approach discussed in this paper can be applied to cargo train wheels or to other technical problems (e.g. other industries, other components).

We suggest the following additional research:

- The covariates considered here are limited to the positions of the locomotive wheels; more covariates must be considered. For example, the braking forces and the curving forces should also be considered.
- We have chosen vague prior distributions for the case study. Other prior distributions, including both informative and non-informative prior distributions, should be studied.
- One of our research focuses in the future is to analyse the relationship between re-profiling interval and material removal on the lathe, together with other influencing factors such as the flange wear, wheel diameter, position on the bogie, and other possible covariates mentioned above.
- In subsequent research, we plan to use our results to optimise maintenance strategies and the related LCC (Life Cycle Cost) problem considering maintenance costs, particularly with respect to different maintenance inspection levels and inspection periods (long, medium and short term).

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Appendix A

Table A.1 work order's categories

Code	Description	Code	Description	Code	Description
1	High flange	6	Out-of-round wheel	11	Measurements on the wheel, Miniprof
2	Thin flange	7	Dimension difference in between wheels in bogie	12	Other defect, pressure defect
3	RCF	8	Vibrations	13	Empty, no code
4	Unbalanced wheel	9	Thick flanges	14	Plant to be re-profiled
5	QR measurements	10	Cracks	15	Double flanges

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