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DETERMINATION OF DIRECTIONAL STIFFNESSES OF VEHICLES' TIRES UNDER A STATIC LOAD OPERATION

WYZNACZANIE SZTYWNOŚCI KIERUNKOWYCH OPON POJAZDÓW SAMOCHODOWYCH W WARUNKACH STATYCZNEGO DZIAŁANIA OBCIĄŻENIA*

This paper presents the stand test results on the different tires under static conditions. The tests were conducted on a stand that allows registering the force and deflection of the tested tires. The results were used to determinate directional tire stiffness, by using self-made software identification. In the next stage, tread prints were made, which were used to determine the surface area of the tire contact with the ground, depending on the pressure inside the tire and the load. The results showed correlations of stiffness growth with increasing tire pressure at radial, longitudinal and torsional evaluation, the contrary conclusions were put forward in the evaluation of circumferential stiffness. The tires, which the characteristics were significantly different from the study group, were also presented. The received results can be the input data for dynamic tests.

***Keywords**: stiffness, pneumatic tire, determination, static tests.*

W artykule przedstawiono wyniki badań stanowiskowych różnego rodzaju opon samochodowych w warunkach statycznych. Próby przeprowadzono na stanowisku umożliwiającym rejestrację siły i odkształcenia badanej opony. Uzyskane wyniki posłużyły wyznaczeniu sztywności kierunkowych opon, wykorzystując własne oprogramowanie identyfikujące. W kolejnym etapie wykonano odciski bieżnika, które posłużyły do wyznaczenia pola powierzchni styku opony z podłożem w zależności od ciśnienia wewnątrz opony oraz obciążenia. Wyniki wykazały korelacje dotyczące wzrostu sztywności w miarę zwiększania ciśnienia w oponie przy ocenie promieniowej, wzdłużnej, skrętnej, przeciwne wnioski wysunięto w przypadku oceny obwodowej. Zaprezentowano również opony odstające o charakterystykach znacznie odbiegających od badanej grupy. Otrzymane wyniki mogą być danymi wejściowymi do badań dynamicznych.

***Słowa kluczowe**: sztywność, opona pneumatyczna, wyznaczanie, badania statyczne.*

1. Introduction

Expectations which puts out for car tires are not limited only to give them the longest life. Table 1 shows a number of other requirements for tires.

Table 1. Requirements for car tires

Requirements for car tires			
Material:	**Economic:**	**Performance:**	**Functional:**
retreadability	available production technologies	ability to roughness	proper stress-strain characteristics
resistance to climate conditions	material availability	resistance to aging	ability to aquaplaning
low mass	low price	resistance to wear	static and dynamic balance
ability to recycle		low noise	optimal adhesive properties
		long life	

Also, the requirements for tires are: high abrasion resistance, optimum stiffness characteristics and low rolling resistance [9]. They were presented in Table 1. The behavior of tires largely depends on its interaction with the ground [7] but the impact to their work have also the speed and loads on vehicle axles [15]. The parameters which determine the ability to control the vehicle, in addition to the geometry of the chassis (steering) axle loads and speed, include designated in this paper stiffness and the contact area of the tire tread with the ground. This determines the slip angle in the tire [5].

Basing on the literature analysis, it should be noticed that a small portion of it, deals with the determination of important parameters of tires which are being in exploitation. In the paper [17], methods of determining the radial stiffness were presented, for example: the static method, in which the tests were carried out at different pressures inside the tire, but it was related to brand new tires. On the other hand, in [2, 16] it was focused on influence of the tire stiffness on vehicle vibrations - in result: on the users comfort (also new tires). In [12], there were presented the results of research on the influence of run-flat inserts only on the radial stiffness of the tire, whereas there should also be expected for the assessment of lateral stiffness. The value of stiffness was the main parameter which in [6] was used to formulate the coefficient of longitudinal slip stiffness.

It should be kept in mind that tire manufacturers have tire parameters that can be used for comparative purposes, but do not provide them with the tire. One of the commonly

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl*** available parameters, included in the markings shown on the sidewall of the tire, is load index [10], the values of other parameters are available for an additional charge.

Concerning the above, it was considered purposeful to determine directional stiffness of tires of different construction and purpose, which are in various stages of operating capacity, and as a result would be possible to compare their parameters. Thus designated target characteristic values can be used to model the movement of the vehicle, particularly in simulations of collisions where vehicles involved are not always brand new.

Stiffness characteristics are usually determined in static way by mounting a pneumatic wheel on a stand in the position that allows applying a load and measure the tires deflection.

The overall stiffness of the pneumatic tire C [6] is defined as:

$$C = \frac{\Delta x}{\Delta l} \left[\frac{N}{m}\right] \tag{1}$$

where: Δx – increase of the value of the static reaction (radial, longitudinal, lateral, torsional), Δl – increase of tire deformation

Depending on the direction of the force there are distinguished:

- **Radial stiffness** – it`s calculated as the dependence of the radial tire deflection of the vertical force. Since the radial stiffness largely depends on the size of dynamic load chassis components, so as ride comfort [12]. Due to the fact that the pneumatic tire has a multilayer construction, while the tire is loaded there is a hysteresis effect - the generation of energy loss during the loading and unloading tire [3, 6, 8, 12].
- **Circumferential stiffness** – this is the relationship between the longitudinal movement and the longitudinal force, applied to the wheel axis. Circumferential forces (braking, driving) causes circumferential distortion in tire shell. Circumferential stiffness allows to define the properties of the tire in the longitudinal direction and slip stiffness [6].
- **Lateral stiffness** – the dependence of the lateral displacement of the tire side force, applied to the wheel axle. Lateral stiffness is particularly important with tire overdrive phenomenon during the lateral tire slip [11]. Overdrive phenomenon in tire during the lateral slip is the result of changes in shell tire deformation in the transverse direction [15], in other words, the lateral reaction, resulting in a zone of the tread contact with the ground, is transferred to the wheel rim by means of the elastic element (tire). Due to the lateral stiffness, elastic properties of the tire in the transverse direction can be defined [11].
- **Torsional stiffness** – the relationship between the received angle of rotation of the tire and torque. This parameter significantly affects on the lateral slip angle, which is very important for sports vehicles [4, 14, 16].

Studies on the effect of the pressure in the tires to the size of the surface area of the tire tread contact with the road [13] were already run in the 80's. It has been proved that the pressure variation inside the tire has a significant effect on the change of stress distribution on the surface of the contact with the ground [1].

The research are intended to determine the presence of influence of input signal (tire pressure, load and hence the stress distribution) on the output quantities (tire stiffness, the size of the tire contact area with the ground) without specifying the functional dependence.

2. Tires characteristics

Considering the variety of available car tires, the selection of a representative group of tires was motivated by the general criteria, such as availability, dimensional popularity, type of tread or application.

Research subjects were divided into three groups according to their use, as follows:

- group I (summer tires),
- group II (M+S tires),
- group III (group III (other, which include spare tire, off-road, and after regeneration – poured).

Due to the fact that the tires used in the studies were exploited, coming from the dismantling, and long ago excluded from the production, a certain data were not possible to get.

3. Research methodology

The research consisted of three phases:

- phase I – experimental research (with pressure in tire – 1.2 *bar*; 2.2 *bar*; 3.2 *bar*), the registration of load and deformation,
- phase II – stiffness identification of pneumatic tires,
- phase III – the preparation of the tire tread prints.

Characteristics were made on the machine for static testing of car tires located in Vehicle Laboratory on Faculty of Mechanical Engineering in Bialystok University of Technology (Fig. 1).

4. Registration of the measurements

To receive parameters values (force, deformation) in the course of research there were used: force transducer Dir-1-WT1, displacement transducer Cl100 and laptop computer with NI DAQCard-6024E card. Registration of waveforms was made using the original software created in LabVIEW, where it was visible preview of current parameters, as well it was possible to save the results to a text file. 10 repetitions of each measurement were performed.

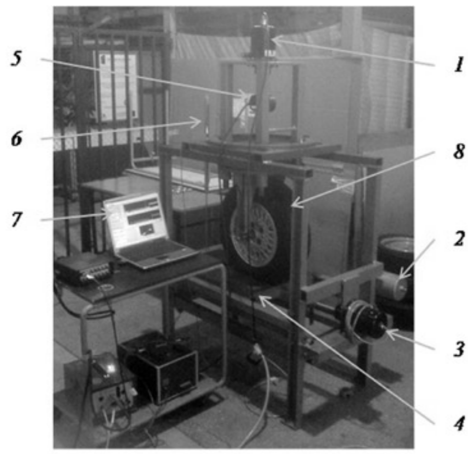

Fig. 1. Research station to determine tire stiffness : 1 - pneumatic cylinder for simulating the radial load , 2 - pneumatic cylinder for simulating torsional load , 3 - pneumatic cylinder for achieving circumferential and lateral load, 4 - sliding plate, 5 - Dir-force transducer 1-WT1, 6 - CL100 displacement transducer, 7 - laptop computer with a NI DAQCard-6024, 8 - tested tire

4.2. Treatment of experimental results

Due to the fact that the electrical equipment (laptop, converters, amplifiers) used in the study were powered by electric site, in addition to the results, disturbances (noise and 'peaks') were also recorded, because test card was very sensitive to the minimum voltage change. These disturbances (from the mains) were the result of, for

Table 2. Brands and models of tires used in research

Nr.	Brand and Model	Size	Load index	Speed index	Year	Internal tire construction		Photo
						Side	Tread	
GROUP I								
1.	Uniroyal Rallye 680	175/65 R13	-	-	-	1 polyester	1 polyester 2 steel	
2.	Kormoran Impulser	155/70 R13	75	T	2000	1 polyester	1 polyester 2 steel	
3.	Debica Passio	135/80 R12	68	T	2004	-	-	
4.	Debica D-164	135/70 R13	68	T	1994	1 rayon	1 rayon 2 steel	
GROUP II								
5.	Marshal Powergrip 749	175/70 R13	82	T	2002	1 polyester	1 polyester 2 steel	
6.	Pirelli Iceplus	155/65 R13	73	Q	2009	1 nylon	1 nylon 2 steel	
7.	Berlin-tyre	155/70 R13	-	-	1995	-	-	
8.	Pirelli P400 Aquamile	155/70 R13	75	T	2006	1 polyester	1 nylon 2 steel 1 polyester	
GROUP III								
9.	USSR ИВ – 167	5,90-13	-	-	-	-	-	
10.	Continental CST 14	105/70 R14	84	M	1991	-	1 rayon 2 steel	
11.	Stomil D-90	165 R13	-	-	-	-	-	

example, other applications requiring switching on and off of electrical equipment near to research station. Interference frequency pointed to the 50 *Hz*, which results from the supply voltage frequency and the amplitude did not exceed 5% of the test range. The transitional recorded peaks did not exceed 20% of the test range. Measurements had static character, so in the registered range of data momentary peaks accounted for 5% of the total results. In order to remove interference, the recorded results were subjected to processing (filtering). Filtration is the process of extinguishing the signal spectrum in the selected frequency ranges. To remove noise from the recorded measurements, Butteworth low-pass filter was used [18].

4.3. Stiffness identification

For identification of the stiffness, the linear regression and the method of least squares were used. Procedures were written in the code of the MATLAB - SIMULINK, Guide addition. FPE_1 (2) index minimalisation was performed numerically using gradientlessness Nelder - Mead simplex method [18], until the desired accuracy of the calculations was achieved, adopted at the $1e^{-6}$ (Fig. 2).

$$FPE_1 = \frac{m+l}{m(m+l)} \sum_{i=1}^{m} \left(F_d - F_m\right)^2 \tag{2}$$

where: m, l –numbers, F_d – measured force values, F_m – model force values

Using the recorded waveforms, force changes in the course of the next iteration the F_m model were sought.

To determine the average error, the FPE_2 index was defined as follows:

$$FPE_2 = \frac{1}{m} \sum_{i=1}^{m} \left(F_d - F_m\right) \tag{3}$$

and the FPE_3 index which is the maximum value of the error

$$FPE_3 = MIN\left(F_d - F_m\right) \tag{4}$$

A qualitative assessment of the experimental F_d and model F_m force fit was based on values of the linear regression coefficient adjusted for degrees of freedom:

$$R^2 = 1 - \frac{m-l}{m-1} \frac{\sum_{i=1}^{m}\left(F_d - F_m\right)^2}{\sum_{i=1}^{m}\left(F_d - \overline{F_m}\right)^2} \tag{5}$$

The dialog box of stiffness identification program (Fig. 3) contains: the function buttons by which the source files are determined, the objective function and procedures (pre-load, pre-deflection) (1), the area that indicates the result of identifying and assumed deflection and pre-load, power – torque coefficient and displacement - the angle (2), the area showing FPE_1, FPE_2, FPE_3 and $R2$ (3), a window showing the course of the force and deflection at the time (4), a window showing the dependence of the strength of the deflection (5) and the search button (6). Block diagram of the program is shown in Figure 2.

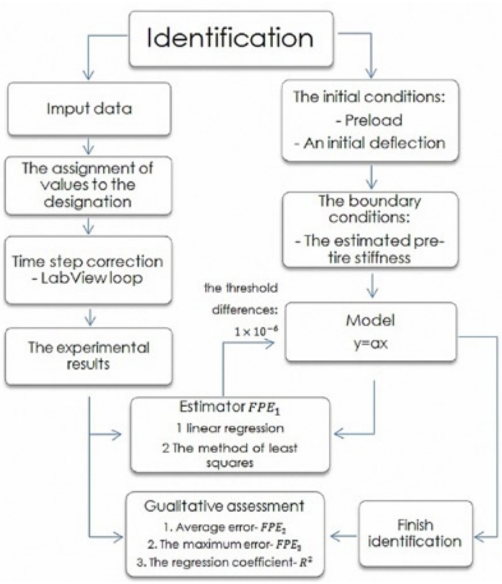

Fig. 2. Block diagram of the stiffness identification program „Tire" in Matlab - Simulink Guide

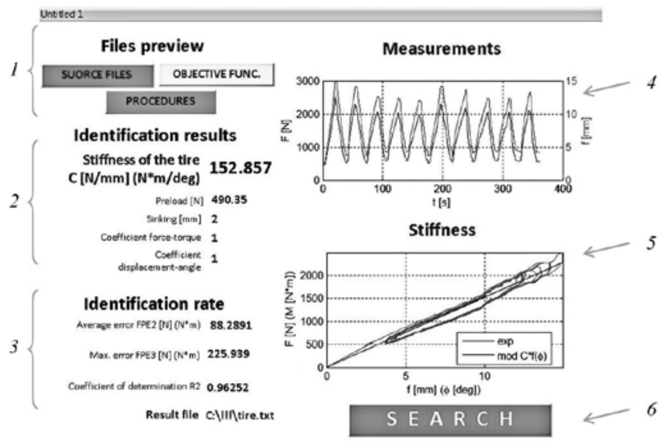

Fig. 3. The dialog box to stiffness identification program (Matlab-Simulink, Guide addition)

4.4. The maximum error of estimate of at the assumed number of repetitions

In order to determine the accuracy of the results (maximum error of estimation) at a predetermined minimum number of measurements, preliminary studies were carried out. The study consisted of a vertical load the tire mounted on the rim and the registration of the deflection as a function of the loading force with the nominal tire pressure of 2,2 *bar*. Tests were repeated 10 times. From the measurements was determined tire radial stiffness, and is shown on a bar graph (Fig. 4). Determined values were adopted for statistical analysis, and to determine the maximum estimation error in the relevant studies.

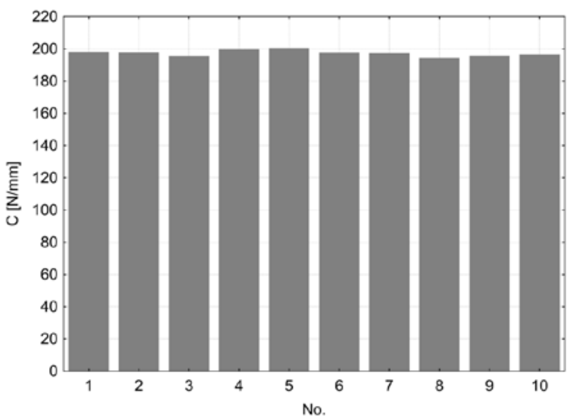

Fig. 4. Bar graph showing the designated values of radial tire stiffness from 10 repetitions

An important parameter that had to be determined in order to calculate the maximum error of estimation is the number of degrees of freedom, comprising as the number of independent observations minus the number of relations that combine these results together. Assuming the number of repetitions of measurements n = 10, and the variance, the maximum error of estimation was specified:

$$D = \sqrt{\frac{S^2 \cdot t_\alpha}{n}} = 0{,}95 \left[\frac{N}{mm}\right] \tag{6}$$

where: S – standard deviation from sample, n – number of repetitions, t_α – statistics.

5. Determination of torsional stiffness

Determination of torsional stiffness consisted on the use of a device that allows the "twist" of the tire. It was implemented by using the swivel plate (Fig. 5) mounted movably on thrust bearings on the table. Torsion angle φ dependence of displacement x is shown in Figure 6.

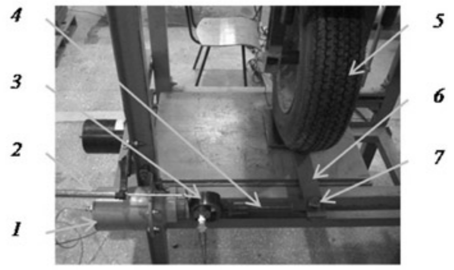

Fig. 5. Mechanism to determine the torsional stiffness: 1 - cylinder, 2 - displacement transducer CL100, 3 - Dir force transducer 1-WT1, 4 - pusher, 5 - tire, 6 - torsion plate, 7 - pin

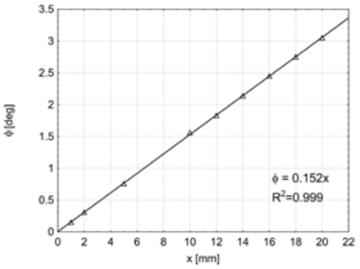

Fig. 6. Dependence of the angle of twist φ [deg] of displacement x [mm]

Knowing arm *r* at which force *F* acts, the torque was determined from the formula:

$$M_s = F \cdot r \ [N \cdot m] \tag{7}$$

where: M_s – torsional torque [*Nm*], *F* – force [*N*], *r* – radius [*m*].

The resulting equation was used in the program for the stiffness identification

Table 3. Coding of test tires

Code	Brand	Model
GROUP I		
L-1	Uniroyal	Ralle 680
L-2	Kormoran	Impulser
L-3	Debica	Passio
L-4	Debica	D-164
GROUP II		
M-1	Marshal	Powergrip 749
M-2	Pirelli	Iceplus
M-3	Berlin-Tyre	-
M-4	Pirelli	P400 Aquamile
GROUP III		
I-1	USSR	ИВ – 167
I-2	Continental	CST14
I-3	Stomil	D-90

6. Results and analysis

The results were presented in the bar charts form, that transparently shows the differences between tires stiffness, depending on the pressure in the tire. In order to improve the readability of graphs, names used car tires were encoded. The summary of results is presented in [10].

6.1. Radial stiffness

When examining radial stiffness of the tested tires (Figure 7), in each case there can be noticed that the higher the pressure in the tire, the stiffness increases. The biggest jump was recorded in the I-1 tire, between the tire pressures 2.2 *bar*–3.2 *bar* (60.3%). The smallest difference in stiffness was observed in the case of a L-3 tire tire pressures between 2.2 *bar*–3.2 *bar* equal to 13.7%.

6.2. Circumferential stiffness

Referring to circumferential stiffness some tire of each group (Fig. 8), it was noted that the value of stiffness, with increasing inflation pressure, decreases.

The L-4 tire's circumferential stiffness, with the lowest pressure (1.2 *bar*), was equal 78.2 *N*/*mm*, while the maximum pressure equal to 3.2 *bar* stiffness amounted only 45.9 *N*/*mm* (difference 70%). In the case of M-4 tires, at the lowest pressure (1.2 *bar*) circumferential stiffness was 133.88 *N*/*mm*, at a nominal pressure of 2,2 *bar* only 84.96 *N*/*mm* (a difference of 57.5%). Further pressure increases caused that the stiffness increased

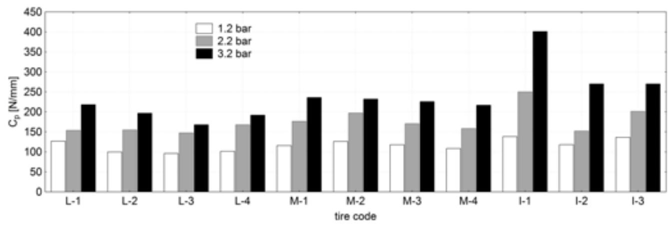

Fig. 7. Tire radial stiffness changes according to the pressure in the tire

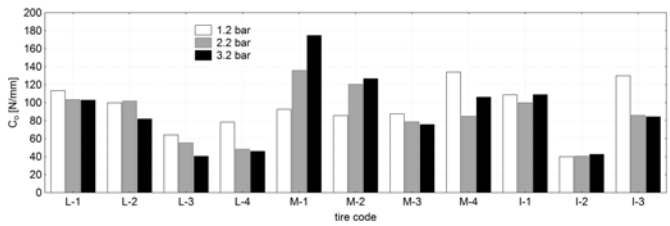

Fig. 8. Changes in the tire of circumferential stiffness depending on the pressure in the tire

of 24.7%. Referring to the circumferential stiffness, tires from group III, it was noted a decrease stiffness with increasing pressure in the I-3 tire. At the lowest pressure (1.2 *bar*) circumferential stiffness was 129.95 *N*/*mm*, at a nominal pressure of 2,2 *bar* – 85.95 *N*/*mm* (a difference of 51.2%). With further pressure increase, it can be noticed a slight decrease in stiffness of 84.25 *N*/*mm*. Changing the tire stiffness as a function of pressure in the tire I-2 is small (about 5%).

6.3. Lateral stiffness

In the case of another determined parameter, which is the lateral stiffness (Fig. 9), as well as the radial stiffness, a bar chart shows an increasing trend in stiffness with the increase of the tire pressure for each tire group, in addition to tire I-2, wherein was noticed stiffness decrease with pressure increase. Also, the tire L-2,where stiffness decreased with increasing inflation pressure from 2.2 *bar* to 3.2 *bar*, up to 18.5%. Relatively high stiffness jump (54.9%) is visible when the pressure is changing from 1.2 *bar* to 2.2 *bar* in the L-2 tire (61.5%) and from 2.2 *bar* pressure to 3.2 *bar* in the L-4 tire (48%).

6.4. Torsional stiffness

In the case of the last designated parameter, which is the torsional stiffness (Fig. 10), as in the case of the circumferential stiffness, there

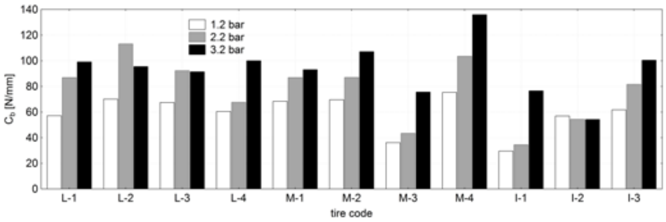

Fig. 9. Lateral stiffness changes of tires depending on the pressure in the tire

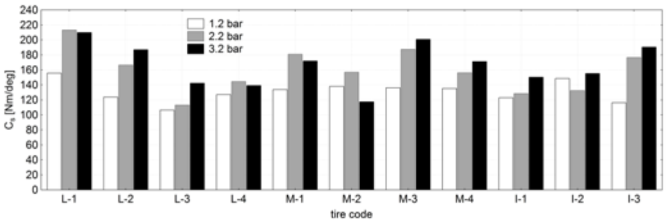

Fig. 10. Changes in torsional stiffness depending on the tire inflation pressure

is a noticeable decrease in stiffness L-1 tire with inflation pressure increase from 2.2 *bar* to 3.2 *bar* of 1.6%.

The relatively large stiffness jump (37%) shows the pressure change from 1.2 *bar* to 2.2 *bar* in the same tire (L-1). A change in pressure from 2.2 *bar* to 3.2 *bar* resulted in as decrease of torsional stiffness in L-4 tire by 3.8%. Another noticeable drop in stiffness with increasing pressure was in the tire M-1 (from 2.2 *bar* to 3.2 *bar* - 5.2%). At I-2 tire pressure change from 1.2 *bar* to 2.2 *bar*, the torsional stiffness decreases by 12%. With further increase of the pressure up to 3.2 *bar*, stiffness increases by 17% compared to the stiffness at a pressure of 2.2 *bar*.

7. Tire deformations

Vertical load applied to the tire in order to determine the radial stiffness, causes deformation of the tire in the manner shown on Figure 11. Force of gravity of the vehicle tire creates a longitudinal force in the area of wheel contact with the ground and results in a deformation shown in Figure 11a. Load tire with braking torque or driving torque results in a longitudinal force in the area of wheel contact with the ground and results in a deformation shown in Figure 11b. However, when determining the lateral stiffness, that is by applying a force in the direction transverse to the longitudinal axis of the tire and recording the movement of the table, there can be seen the deformation of the tire like on the image (Figure 11c).

a) b) c)

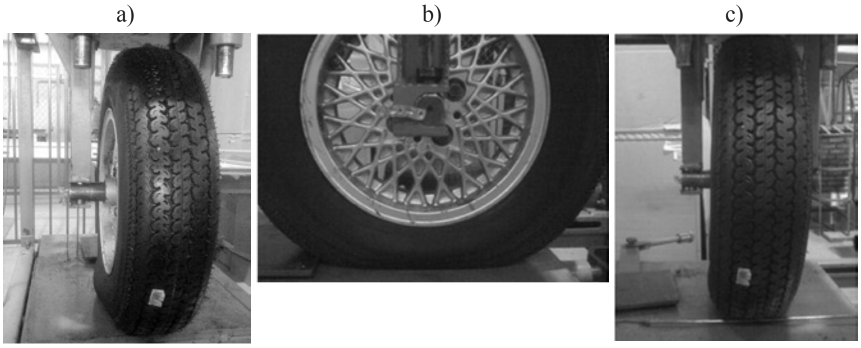

Fig. 11. Deformations of the tire when determining stiffness: a) radial, b) circumferential, c) lateral

8. Tire prints

When turning the wheel on a hard surface, a tire is deformed resulting in the formation of surface contact with the road. All the forces needed for acceleration, braking and steering implementation are carried through the tread with the road contact surface. Changing the tire pressure directly affects the size of the surface area of the tire contact with the road. As it is known, the larger a surface area contact with the ground, the lower the penetration of the tire on soft ground (that is used in off-road vehicles - reducing the pressure during the crossings in difficult terrain). This results in a higher rolling resistance, thus, greater fuel consumption and emitted noise by the tire tread.

In order to see the surface area of the contact patch under various pressures in the tires and different loads, the prints of one of the tire tread have been made (Fig. 12).

On the basis of the results, bar graphs were created, which show dependency of contact area changes as a function of inflation pressure (Fig. 13) and the dependence of the contact area changes as a function of pressure force (Fig. 14).

a) b) c) d)

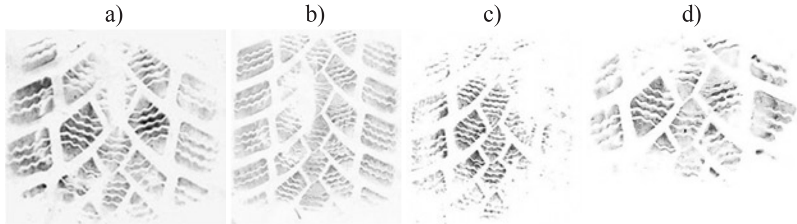

Fig. 12. Uniroyal Rallye 680 tread prints: - tire pressure 2.2 bar - load: a – 1841 N, b – 3682 N; - load 614 N – tire pressure: c – 1.5 bar, d – 3 bar

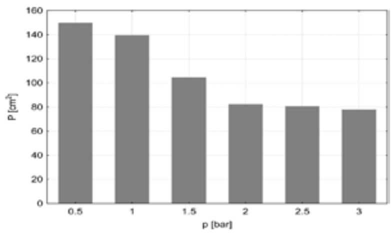

Fig. 13. Changing the area of contact as a function of the pressure in the tire at a constant load.

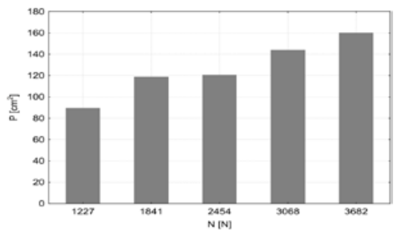

Fig. 14. Changing the contact surface as a function of pressure force at a constant pressure in the tire

With the increase of pressure in the tire, contact area with the ground track decreases. The biggest change in area (about 70%) was observed in the range of pressure from 1 bar to about 2 *bar* (Fig. 13). When increasing tire load at a constant value of 2.2 *bar* nominal pressure in the tire (Fig. 14), the surface area of the tread contact with the road increases. This is most noticeable in the ranges of load changes from 1227 *N* to 1841 *N* (approximately 66%) and from 2454 *N* to 3068 *N* (approximately 80%).

9. Conclusions

The results for the tire stiffness refer to the exploited tire, which are in various stages of utility. The research had comparative character, in which it was tried to find some correlations in static conditions. The research for 11 pieces of various kinds of tires, it was found that the average tire has a radial stiffness of about 180 [*N*/*mm*], circumferential 80 [*N*/*mm*], lateral 65 [*N*/*mm*] and torsional 150 [*Nm*/*deg*]. In the case of radial stiffness, with increasing tire pressure, increased their value, similar to the lateral stiffness. In some cases a declines and torsional stiffness were occured with increasing pressure in the tire.

With the increase in pressure in the tire, contact area with the ground is reduced. The biggest change in area (about 70%) was observed in the range of pressure from 1 bar to about 2 *bar*. When increasing the tire load with constant nominal value pressure in the tire of 2.2 *bar*, the area of contact with the ground increased to 35%.

The parameters are applicable to the modeling of the vehicle motion, which operates mainly for lateral drift angle of tires, which has an effect on vehicle controllability, especially in sports cars [14]. At a later stage it is planned to construct the experimental tire dynamic research stand, particularly to determine the tire side abolition or damping, where provided to base the identification models on the results of previous work.

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