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SPARE PARTS ALLOCATION OPTIMIZATION IN A MULTI-ECHELON SUPPORT SYSTEM BASED ON MULTI-OBJECTIVE PARTICLE SWARM OPTIMIZATION METHOD

OPTYMALIZACJA ALOKACJI CZĘŚCI ZAMIENNYCH W WIELOSZCZEBLOWYM SYSTEMIE WSPARCIA NA PODSTAWIE METODY WIELOKRYTERIALNEJ OPTYMALIZACJI ROJEM CZĄSTEK

Spare parts allocation optimization in a multi-echelon support system presents a difficult problem which involves non-linear objective function and integer variables to be optimized. In this paper, a multi-objective optimization model was developed, which maximizes support probability and minimizes support costs. In order to solve the optimization problem, an improved multi-objective particle swarm optimization (MOPSO) method was utilized. In this method, techniques of dimensions reduction and rules-based multi-objective optimization were employed, which can improve the efficiency of MOPSO method. A numerical example was given to show the performance of proposed method.

***Keywords**: MOPSO; spare parts; allocation; optimization; support probability.*

Optymalizacja alokacji części zamiennych w wieloszczeblowym systemie wspomagania stanowi trudne zagadnienie, które wymaga optymalizacji nieliniowej funkcji celu oraz zmiennych całkowitych. W niniejszej pracy, opracowano wielokryterialny model optymalizacyjny, który maksymalizuje prawdopodobieństwo wsparcia i minimalizuje jego koszty. W celu rozwiązania problemu optymalizacyjnego, wykorzystano ulepszoną metodę wielokryterialnej optymalizacji rojem cząstek (MOPSO). W metodzie tej wykorzystano techniki redukcji wymiarów oraz wielokryterialnej optymalizacji algorytmowej, które mogą poprawić efektywność metody MOPSO. Zasady proponowanej metody zilustrowano przykładem numerycznym.

***Słowa kluczowe**: MOPSO; części zamienne; alokacja; optymalizacja; prawdopodobieństwo wsparcia.*

1. Introduction

The importance of spare parts management has increased in the past decades. The reason may be the increasing value of spare part inventory investment, and higher requirement of system availability and support level for technically advanced systems. Spare parts allocation is to allocate spare parts over different echelons of inventories to grantee a high efficiency of support system. Therefore, the purpose of spare parts optimal allocation is to achieve the maximal integrated economic benefit and spare part supportability. This presents a NP hard problem and efforts have been made by using methodologies of system decision-making, operational research and engineering economics theory [14, 21]. For example, DuyQuang Nguyen [9], Dubi [7] studied spare parts allocation and optimization questions in preventive maintenance by Monte Carlo simulation method; CAO Huizhi [12] set up a spare parts optimization model by fuzziness theory. Samuel L. Dreyer [19] and Doc Palmer [6] researched on spare parts management and optimization through enterprise resources plan (ERP); Derek T. Dwyer [5] and Ilgin M. Ali [13] studied spare parts stock quantity based on genetic arithmetic, and developed an optimization model; Faisal I. Khan [10] studied on spare parts allocation and optimization based on risk analysis within spares costs restriction. Although the above studies have obtained some productions, most of them took single equipment as the objective and optimized only for single objective. In fact, spare parts allocation should think about different kinds of equipments in a whole and think over more objectives such as spare parts support probability, support costs etc. The spare parts allocation and optimization for multi-objects are more difficult for the complex relations among every objective, which may be conflict or supplement one another.

Particle swarm optimization (PSO) is a kind of evolutionary calculation technology based on colony aptitude theory, which was brought forward by American Doctor Eberhart and Kennedy in 1995. The idea of PSO came from birds' preying behavior. In the method, it was assumed that a group of birds were searching for foods at random, and there was only one piece of food in a certain area. All those birds didn't know where the food was, while they knew the distance between the food and their current positions. Then the optimal strategy of finding food was to get the area where is nearest to the food from every bird's current position. In PSO, each unity was taken as a particle with certain position and speed. The particle's position presents the solution of the question.

By comparing with PSO, MOPSO needs to take into account of a number of objectives and make a choice from a set of feasible solutions. Therefore, the key problem is that how to confirm a proper fitness function, which is used to measure the quality of a solution scheme for multi-objective case. According to fitness function, the solution methods for PSO can be classified as objective polymerization method, Pareto dominated based method and rule-based method. Objective polymerization method [4, 17] takes multi-objective function polymerized by power adding and converts the multi objectives to a single objective. In Pareto dominated based method [1, 3, 16], the best isolated none-inferior solution was endowed with global minimum, which can induct MOPSO method to find a none-inferior solution with a symmetrical distribution. About rule based method [2, 8], not all objectives were considered at one time. According to different circs in optimization process, the fitness function was converted from different objective.

A multi-objective optimization model of spare parts allocation was developed by using an improved MOPSO method, which takes the maximum support probability of spare parts and the minimal support costs as the objective functions. The solution techniques, such as dimensions reduction and rules-based multi-objective optimization, were utilized in order to improve the solving efficiency of MOPSO method.

2. Modeling of spare parts allocation and optimization

2.1. Problem description and model assumptions

Most inventory control problems in the real world involve multiple echelons. For example, in many military logistics areas, support systems are operated in echeloned manner. In the paper, a two-echelon spare support system is considered, where there are a number of maintenance facilities and spare parts inventories in the first echelon, and only one maintenance facility and one warehouse in the second echelon. Each maintenance facility has a spare inventory correspondingly, which stores certain types of spare parts. When a repairable unit in an equipment fails, the failed unit will be replaced if there is a spare in the first echelon of inventory. Otherwise, a spare part is back-ordered from the second echelon of inventory. The local repair facility tries to repair the failed units which are repairable at the facility; otherwise, the units will be sent to the second facility for repair. The units which have been repaired will be stored in local inventories as spare parts. Besides, we have following assumptions.

(1) The demand for each type of spare part is independent, and each unit's failure obeys exponential distribution;
(2) All the replaceable units are significant components, and lack of any one will lead to equipment down;
(3) It is assumed that some failure modes of a unit are repairable, and the repair rate is constant;
(4) Only corrective maintenance is considered and related spare parts demand is assumed to follow Poisson distribution;
(5) The maintenance capability is infinite, and repair of failed units may be conducted once they are replaced from equipment;
(6) Continuing examine strategy (*S*-1, S) is adopted for the first echelon of spare inventory;
(7) The lateral supply in the same maintenance facilities is not considered.

The spare part allocation relation is shown as Figure 1. Where, k denotes the sequence of specific type of equipment, and N_j denotes the equipment number of this type.

2.2. Notations

The notations used to develop the model are listed below:

i – types of spare parts, which can be 1,2,…,I;
I – total number of spare parts types;
J – the number of maintenance facilities in the first echelon;

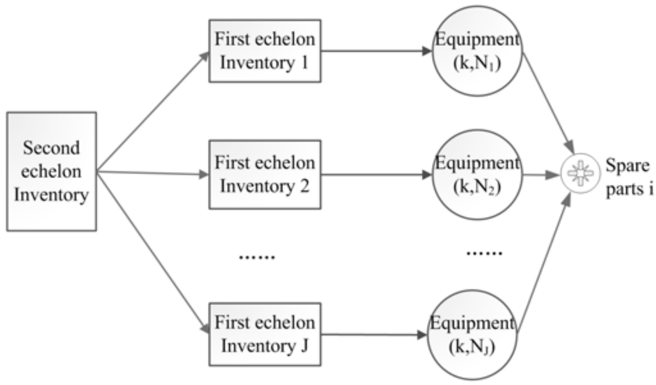

Fig.1. Relation of the spare parts allocation

K – the number of equipment with certain type;
T – equipment working time;
E_{ji} – the demand of spare part i at jth inventory in the first echelon in a period of equipment working time;
E_i – the average demand of spare part i in the first echelon in a period of equipment working time;
$E[B(S_i)]$ – the expected shortage of spare *i* for the first echelon when the stock level in local inventory is S_i;
$E[D(S_i, S_{oi})]$ – the expected shortage of spare i in the second echelon, when stock level in the first echelon is S_i, and the second echelon is $S_{o,i}$;
t_o – the average lead time in the first echelon to backorder spare parts;
t_m – the average lead time in the second echelon inventory to acquire spare parts;
S_{ji} – the stock level of spare part *i* at the *j*th inventory of the first echelon;
S_{oi} – the stock level of spare part *i* at the inventory of the second echelon;
$P(x_{ji})$ – the probability of *i*th spare part demand for jth inventory in the first echelon *j*, and x_{ji} is demand value of spare part *i*;
x'_{ji} – the amount of shortage for spare *i* at the *j*th inventory of the first echelon;
$P_o(x'_{ji})$ – the probability of *i*th spare demand at *j*th inventory of the first echelon;
ξ_i – the shortage probability of spare parts *i* in the second echelon;
η_i – the shortage probability of spare parts *i* in the first echelon;
R_i – the probability of repair for *i*th replaceable unit in the first echelon;
E_i^v – the maximum inventory for spare parts *i* in the first echelon;
E_{oi} – the average demand quantity of spare parts *i* in the second echelon;
E_{oi}^v – the maximum inventory for spare parts *i* in the second echelon;
C_i – the cost of a spare part *i*;

T_{bfi} – the mean-time between failures of spare parts i.

2.3. Development of the model

(1) Demand of spare parts in the first echelon

Because $P(x_{ji})$ denotes probability of spare parts demand for the first echelon j, and the number of demand is x_{ji}. The expected demand of spare part i in the first echelon can be expressed as:

$$E_{ji} = \sum_{x_{ji}=0}^{\infty} x_{ji} P(x_{ji}) \tag{1}$$

(2) Shortage ratio of spare parts in the first echelon

Denote $E[B(S_{ji})]$ as the inventory shortage of spare i for the first echelon. It is the expectation of spare parts demand, which exceeds the stock level, S_{ji}, in the first echelon. Hence, it can be expressed as:

$$E[B(S_{ji})] = \sum_{x_{ji}=S_{ji}+1}^{\infty} (x_{ji} - S_{ji}) P(x_{ji}) \tag{2}$$

The average demand of spare parts in the first echelon can be given by:

$$E_i = \sum_{j=1}^{J} E_{ji} \tag{3}$$

As there are J inventories in the first echelon, the expected shortage of spare i can be obtained through summing the shortage over the J inventories. That is,

$$E[B(S_i)] = \sum_{j=1}^{J} E[B(S_{ji})] \tag{4}$$

From Eqs.(3) and (4), we can have that the shortage ratio of spare i in the first echelon:

$$\eta_i = \frac{E[B(S_i)]}{E_i} \tag{5}$$

(3) The spare parts demand ratio in the second echelon

It is noted that when the demand for spare parts exceeds the stock level of the first echelon of inventory, a spare backorder for the second echelon of inventory will be conducted. Therefore, the probability of ith spare part demand for the second echelon $P_0(x'_{ji})$ can be expressed as:

$$\begin{cases} P_o(x'_{ji}) = P_j(x'_{ji} + S_{ji}) & x'_{ji} > 0 \\ P_o(x'_{ji}) = \sum_{x_{ji}=0}^{S_{ji}} P_j(x_{ji}) & x'_{ji} = 0 \end{cases} \tag{6}$$

The demand probability $P_o(y_i)$ of spare i can be calculated using the $P(x'_{ji})$ dispersed k-fold discrete convolution, that is:

$$P_o(y_i) = \sum_{x'_{1i}+x'_{2i}+\cdots x'_{Ji}=y_i} P_o(x'_{1i}) P_o(x'_{2i}) \cdots P_o(x'_{Ji}) \tag{7}$$

Actually, the right part of the above equation is a calculation of the k-fold discrete convolution, which can be solved by the Conv function in Matlab [11].

In the following, we will show that the demands of spare parts still follow Poisson distribution for the second echelon.

Consider the interval $[0,t]$. Let $N(t)$ denote the number of spare part demands within the interval $[0,t]$ and $X(t)$ denote the number of failures in the interval, and then $X(t)$ can be represented by:

$$X(t) = \sum_{n=1}^{N(t)} W_n(t) \tag{8}$$

Here, $W_n(t)$ is a binary variable defined by:

$$W_n(t) = \begin{cases} 1 & \mathrm{i} \in \theta \\ 0 & otherwise \end{cases} \tag{9}$$

Where θ is the set of failure modes of replaceable units which are repairable. And according to the previous assumption that $P[W_n(t)=1] = 1 - R$.

For any real number u, the characteristic function, $\phi_{X(t)}(u)$ of $X(t)$ can be given by:

$$\phi_{X(t)}(u) = E\{\exp[iuX(t)]\} \tag{10}$$

Where i is an imaginary number, i.e. $i = \sqrt{-1}$, and $E(Y)$ is the expectation of Y.

It can be shown that:

$$E\{\exp[iuX(t)]\} = \sum_{k=0}^{\infty} E\{[\exp(iuX(t))] | N(t) = k\} P[N(t) = k] \tag{11}$$

When k failures have occurred in $(0,t]$, i.e. $N(t) = k$, $X(t)$ is the sum of k independent and identical random variables of $W_n(t)$, then:

$$E\{[\exp(iuX(t))] | N(t) = k\} = (E\{\exp[iuW_n(t)]\})^k \tag{12}$$

As assumed above, $N(t)$ is a NHPP, and then the probability of k demands occurring in the interval $(0,t]$ is given by:

$$P[N(t) = k] = (\int_0^t \lambda(\tau) d\tau)^k \exp[-\int_0^t \lambda(\tau) d\tau] / k! \tag{13}$$

Where $\lambda(\tau)$ is the rate of failure occurrence at time τ.

Substituting equations (12) and (13) into Equation (11), and using equation (10), the characteristic function can be rewritten as:

$$\phi_{X(t)}(u) = \sum_{k=0}^{\infty} (E\{\exp[iuW_n(t)]\})^k (\int_0^t \lambda(\tau)d\tau)^k \exp[-\int_0^t \lambda(\tau)d\tau]/k!$$

$$= \exp\{[E(e^{iuW_n(t,)} - 1)]\int_0^t \lambda(\tau)d\tau\}$$

$$= \exp\{(e^{iu} - 1)P[W_n(t) = 1]\int_0^t \lambda(\tau)d\tau\} \qquad (14)$$

From equations (14) it follows that:

$$\phi_{X(t)}(u) = \exp\{(e^{iu} - 1)(1 - R)\int_0^t \lambda(\tau)d\tau\} \qquad (15)$$

This is a characteristic function of a random variable with a Poisson distribution. This implies that $X(t)$ is a NHPP, and the expected number of failures in $(0,t]$ is given by:

$$\Lambda(0,t) = (1 - R)\int_0^t \lambda(\tau)d\tau \qquad (16)$$

For the case of constant failure rate of replaceable unit, the demand rate of spare in the second echelon of system can be expressed as: $\lambda_{oi} = (1 - R_i)J\lambda_i$.

When the demands of unit i obey Poisson distribution, we have:

$$P_o(y_i) = \sum_{k=0}^{y_i} \frac{(\lambda t)^k}{k!} \mathrm{e}^{-\lambda t} \qquad (17)$$

On the basis of demand probability of spare i in the second echelon, we can obtain the expected demand of spare parts for the second echelon of inventory:

$$E_{oi} = \sum_{y_i=0}^{\infty} y_i P_o(y_i) \qquad (18)$$

The expected shortage of spare i for the second echelon can be expressed as:

$$E[D(S_i, S_{oi})] = \sum_{y_i=S_{oi}+1}^{\infty} (y_i - S_{oi})P_o(y_i) \qquad (19)$$

Then the shortage probability of spare parts i for the second echelon can be denoted as:

$$\xi_i = \frac{E[D(S_{i,}S_{oi})]}{E_{oi}} \qquad (20)$$

(4) The average delay time for spare parts in the first echelon

The supply delay of spare parts in the first echelon can be divided into two cases. Firstly, there is a shortage of spares in first echelon but there are a number of spares in the second echelon. In this case, the delay time can be given by $(1 - \xi_i)\eta_i t_o$. Secondly, there is a shortage of spares both in first echelon and second echelon. In this case, the delay time can be expressed as $\eta_i \xi_i (t_o + t_m)$. Therefore, the average delay time of spare i in the first echelon can be given by:

$$T_{Di} = (1 - \xi_i)\eta_i t_o + \xi_i \eta_i (t_o + t_m) = \eta_i t_o + \xi_i \eta_i t_m \qquad (21)$$

(5) The support probability for spare i

It is assumed that life distribution of component i is known, and the mean time between failures of the component i can be obtained, then the steady-state support probability of spare parts i can be expressed as:

$$P_i = \frac{T_{bf_i}}{T_{bf_i} + T_{Di}} \qquad (22)$$

Through the collation, we can obtain:

$$P_i = \frac{T_{bf_i} E_i E_{oi}}{T_{bf_i} E_i E_{oi} + \sum_{j=1}^{J} \sum_{x_{ji}=S_{ji}+1}^{\infty} (x_{ji} - S_{ji}) p(x_{ji}) \cdot [t_o E_{oi} + t_m \sum_{y_i=S_{oi}+1}^{\infty} (y_i - S_{oi}) p_o(y_i)]}$$

$$= \frac{\sum_{j=1}^{J} \sum_{x_{ji}=1}^{\infty} x_{ji} p(x_{ji}) \cdot \sum_{y_i=0}^{\infty} y_i p_o(y_i) T_{bf_i}}{\sum_{j=1}^{J} \sum_{x_{ji}=1}^{\infty} x_{ji} p(x_{ji}) \cdot \sum_{y_i=0}^{\infty} y_i p_o(y_i) T_{bf_i} + \sum_{j=1}^{J} \sum_{x_{ji}=S_{ji}+1}^{\infty} (x_{ji} - S_{ji}) p(x_{ji}) \cdot [t_o \sum_{y_i=0}^{\infty} y_i p_o(y_i) + t_m \sum_{y_i=S_{0i}+1}^{\infty} (y_i - S_{oi}) p_o(y_i)]} \qquad (23)$$

Therefore, the support probability of all spare parts can be given by:

$$P_S = \prod_{i=1}^{I} P_i \qquad (24)$$

(6) Multi-objective allocation and optimization model

The allocation and optimization model of spare parts can be expressed as:

$$\max \quad P_S = \prod_{i=1}^{N} \frac{T_{bf_i} E_{oi} \sum_{j=1}^{J} E_{ji}}{T_{bf_i} E_{oi} \sum_{j=1}^{J} E_{ji} + (\sum_{j=1}^{J} (E_{ji} - S_{ji})[t_o E_{oi} + t_m (E_{oi} - S_{oi})]} \qquad (25)$$

$$\min \quad C_S = \sum_{i=1}^{N} C_i (\sum_{j=1}^{J} (S_{ji} + S_{oi}) \qquad (26)$$

Subject to

$$0 \le \frac{\sum_{j=1}^{J} E_{ji} - S_{ji}}{\sum_{j=1}^{J} E_{ji}} \le 1, \forall i \ = 1,2,...,N \qquad (27)$$

$$0 \le \frac{E_{oi} - S_{oi}}{E_{oi}} \le 1, \forall i \ = 1,2,...,N \qquad (28)$$

$$0 \le S_{ji} \le E_i^v, S_{ji} \text{is an integer}, \forall i \ = 1,2,...,N \qquad (29)$$

$$0 \le S_{oi} \le E_{oi}^v, S_{oi} \text{is an integer}, \forall i = 0,1,2,...,N \qquad (30)$$

Where, Eqs.(25) and (26) represent the optimization objectives which maximizing support probability of spare parts and minimizing the costs of spare parts.

Constraint (27) means that the shortage rate of spare parts i in the first echelon is greater than or equal to 0, and less than or equal to 1.

Constraint (28) means that the shortage rate of spare parts i in the second echelon is greater than or equal to 0, and less than or equal to 1.

Constraint (29)means that the stock quantity of spare parts i in the first echelon is an integer, which does not exceed the maximum inventory limit.

Constraint (30) means that the stock quantity of spare parts i in the second echelon is an integer, which dose not exceed the maximum inventory limit.

3. Solution of spare parts allocation and optimization models

3.1. Characteristic of the optimization model

It can be seen that spare parts allocation and optimization models are high-dimensional, non-linear, multi-objective optimization models from formula (25) to (30). So, the traditional optimization algorithm is ineffective to these models. Therefore, an improved algorithm is presented in order to solve such an optimization problem, which includes the following characteristic of analysis and processing.

(1) In practice, the supportability between various spare parts is independent among them. Therefore, when support probability of each spare parts getting the maximum, the probability of such spare parts will obtain maximum. Similarly, when the costs of each type of spare parts are minimal, the costs of such spare parts are also minimal. So dimensionality reduction can be used. Namely, optimized for each variety of spare parts, and then we can get the optimization solution of all kind of spare parts.

(2) In the spare parts optimization and allocation models, there are two different objective functions. Decision-makers preferences of these two objectives are different at different stages and with different tasks. Therefore, the rule based method for solving the MOPSO is more reasonable.

(3) In the particle swarm optimization, ω is the most important controllable parameter. For the premature of PSO method and late oscillation phenomenon in the near global optimal solution, the PSO method needs to be improved in order to enhance the quality and efficiency of the solution.

3.2. Particle representation

Using vector-based method, particle representations can be shown in Figure 2. Each particle corresponds to a spare part configuration program.

	First level station 1	First level station 2	...	First level station m	Second level station		
Partical X_1	$X_{1,1}$	$X_{1,2}$	···	$X_{1,m}$	$X_{1,m+1}$	⇨	Scheme 1
Partical X_2	$X_{2,1}$	$X_{2,2}$	···	$X_{2,m}$	$X_{2,m+1}$	⇨	Scheme 2
Partical ...	...	...	...	...	...	⇨	...
Partical Xn	$X_{n,1}$	$X_{n,2}$	···	$X_{n,m}$	$X_{n,m+1}$	⇨	Scheme n
Partical ...	...	...	...	...	...	⇨	...
Partical X_N	$X_{N,1}$	$X_{N,2}$	···	$X_{N,m}$	$X_{N,m+1}$	⇨	Scheme N

Fig.2. The sketch map of particle's denotation

It is noted that $x_n = [x_{n,1},...x_{n,j},...x_{n,m},x_{n,m+1}]$ $n \in \{1,2,...,N\}$, (N presents the number of particles), Particle dimension is m+1; $x_{n,j}$ denotes the allocation number of spares i in the first echelon. $j \in \{1,2,...,m\}$,and m is the number of the first echelon institutions; $x_{n,m+1}$ denotes the allocation number of spares i in the second echelon.

3.3. Fitness calculation

Fitness function is used to evaluate the pros and cons of the individual groups. The function values guide the movement direction and speed of the particle swarm. A rule-based method was employed to determine the fitness function, which compromises the two goals of optimization problem, that is, maximizing the support probability (Goal 1) and minimizing the support cost (Goal 2) for spare parts i. Clearly, when Goal 1 was implemented, we can simply confirm the neighborhood of Goal 2, and then calculate the fitness of each particle.

3.4. Particle update

Particle swarm method takes on a rapid convergence speed, which can easily lead to the loss of population diversity. In order to prevent its fall into local optimal solution, this paper puts forward a dynamic swarm strategy for the selection of the two particle swarms. In this strategy, each particle swarm optimizes a single goal, and the global optimal value of the first particle swarms are used in the rate equation of the second particle swarm. While the global optimal value of the second particle swarm are used to update the first particle swarm. Namely:

$$S_{1\cdot v_{id}}(t+1) = \omega S_{1\cdot v_{id}}(t) + c_1 r_1 [S_{1\cdot p_{id}}(t) - S_{1\cdot x_{id}}(t)] + c_2 r_2 [S_{2\cdot p_{gd}}(t) - S_{1\cdot x_{id}}(t)] \quad (31)$$

$$S_{2\cdot v_{id}}(t+1) = \omega S_{2\cdot v_{id}}(t) + c_1 r_1 [S_{2\cdot p_{id}}(t) - S_{2\cdot x_{id}}(t)] + c_2 r_2 [S_{1\cdot p_{gd}}(t) - S_{2\cdot x_{id}}(t)] \quad (32)$$

$$S_{1.x_{id}}(t+1) = S_{1.x_{id}}(t) + v_{id}(t+1), \qquad 1 \le i \le n,\ 1 \le d \le D \quad (33)$$

$$S_{2.x_{id}}(t+1) = S_{2.x_{id}}(t) + v_{id}(t+1), \qquad 1 \le i \le n,\ 1 \le d \le D \quad (34)$$

$$v_{id}(t) = [S_{1.v_{id}}(t) + S_{2.v_{id}}(t)]/2 \quad (35)$$

3.5. Weight improvement

In particle swarm optimization methods, the inertia factor is the most important controllable parameter, which is used to control the influence degree of the current speed on the updating speed. The larger the parameter value, the more beneficial for a wide range of global search. The smaller the parameter value, the more favorable in the current range of local search. A large number of experiments[15, 18, 20] have shown that the effectiveness of the algorithm is larger when the parameter value is between 0.4 and 1.4. In this article, the inertia factor was selected by the following formula:

$$\omega_g = \omega_{\max} - g\frac{\omega_{\max} - \omega_{\min}}{G_{\max}} \quad (36)$$

In this formula, ω_{max} denotes the maximum inertia weight, ω_{min} denotes the minimum inertia weight, g denotes the iteration steps, and G_{max} represents the maximum number of iteration steps.

3.6. Procedure of solution algorithm

The procedure of MOPSO solution algorithm is shown in Figure 3.

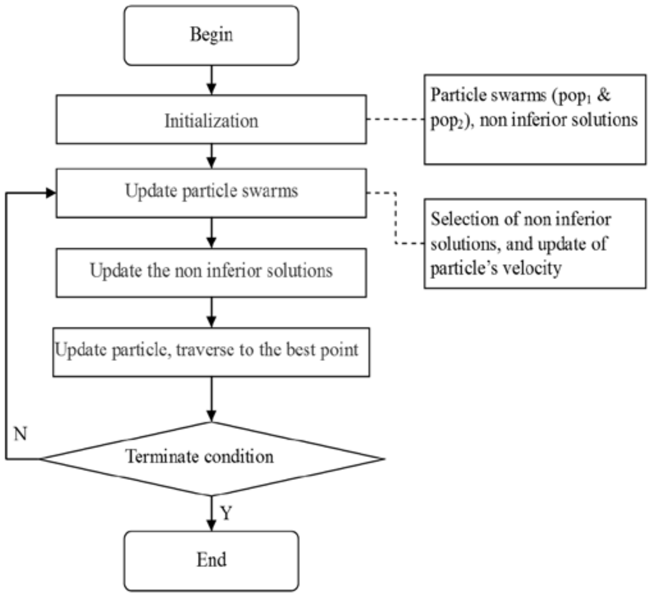

Fig. 3. Procedure of rules based MOPSO solution algorithm

Step 1: Initialization. The non-inferior solution and particle swarm POP_1, POP_2 were initialized randomly. In the mean time, the objectives and constraints of each particle in particle swarm was computed. While, each particle's velocity as well as the best point of each particle have traversed were initialized in the particle group.

Step 2: Update the particle swarms. The particle velocity can be updated by equations (31) and (32), while the position of the particle can be updated according to equations (33) to (35).

Step 3: Update the non-inferior solutions.

Step 4: Update the optimal solution, which was found by each particle.

Step 5: Terminate the condition judgment. If the condition was satisfied, the procedure will be terminated, otherwise will go to step 2.

4. Numerical example

4.1. Description of the problem

It was supposed that in workshops X and Y both there are18 sets of production equipment. Spare parts A and B are two critical spare parts of the equipment. The mean time needed for backordering spares from the second echelon is 480 h, while the mean time for acquiring spares in the second echelon is 720 h. P_{ji}, S_{ji}, S_{oi} and T_{bf} were listed in Table 1. In order to complete annual production tasks (at 1600 h), how to allocate such spare parts which can get the maximum support probability of the spare parts system, while the guaranteeing costs is minimum.

4.2. Problem solving and analysis

The basic data such as E_{ji} E_{oi} and the highest reserves E^v_i, E_{oi}^v in different institutions can be calculated, which are shown in the Table 2.

Table 1. Input data of the example

Workshop	Number of Equipment	Types of Spare parts	P_{ji}	T_{bf}(h)	S_{ji}	S_{oi}
X	18	A	0.8	3500	5	2
		B	0.75	2500	5	3
Y	18	A	0.9	3500	8	2
		B	0.95	2500	8	3

Table 2. Basic data of the numerical example

Sequence number	Spare name	T_{bf}(h)	E_{1i}	E_{2i}	E_{oi}	E^v_i	E_{oi}^v	C_i (RMB YUAN)
1	A	3500	7	8	2	10	3	1200
2	B	2500	9	11	7	12	8	500

In this paper, Matlab was used to program the MOPSO algorithm. The computer configuration is the Intel (R) Core (TM) i5, CPU 2.27 GHz and Memory 2G. The size of the two particle swarms are all 40 particles, and the number of iterations is 100 generations, ω_{max} is 1.2, ω_{min} is 0.5, C_1 is 0.5, and C_2 is 0.5. As a result, the average computation time is 6 sec. The results of the optimization of spare parts are shown in Table 3. The individual fitness values are shown in Figure 4, and the non-inferior solutions distribution in the objective space is shown in Figure 5.

Table 3. Optimization results of the spare parts allocation

Scheme	Allocation for spare A			Allocation for spare B			Support probability P	C_A(RMB YUAN)
	S_{11}	S_{21}	S_{o1}	S_{21}	S_{22}	S_{o2}		
1	0	0	2	0	0	9	0.792	6900
2	0	0	2	0	0	10	0.823	7400
3	0	0	2	0	0	11	0.856	7900
4	0	0	3	0	0	9	0.871	8100
5	0	0	3	0	0	10	0.905	8600
6	0	0	3	0	0	11	0.941	9100
7	0	0	3	1	1	11	0.944	10100
8	1	1	3	0	1	11	0.946	12000
9	0	1	3	2	2	11	0.948	12300
10	1	1	3	2	2	11	0.950	13500
11	2	2	3	1	2	11	0.953	14200
12	2	2	3	2	2	11	0.955	15900

As can be seen from Table 3, when the support cost is ¥6900, the optimal allocation scheme is that 2 of spare part A are allocated in the second echelon and none in other echelons; and stock level of spare parts B is 9 in the second echelon. With this scheme, the spare support probability is 0.792. With the increasing support costs, the spare support probability is also increased. Decision-makers can select the suitable spare parts allocation scheme from the different optimization schemes according to the needs and actual conditions, which can meet the demands of the support probability and limit of spare parts costs.

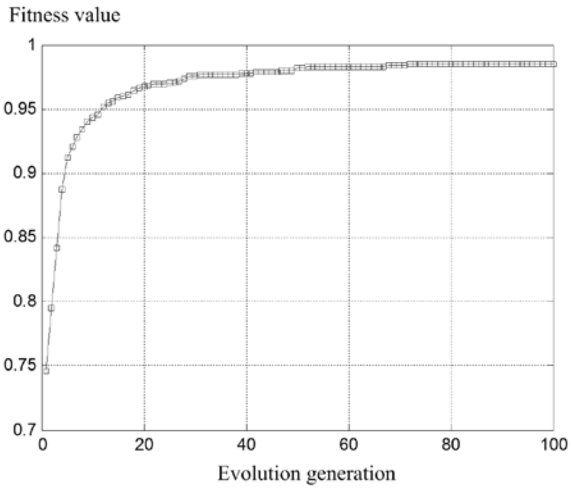

Fig. 4. The changing of best fitness in MOPSO for spare parts

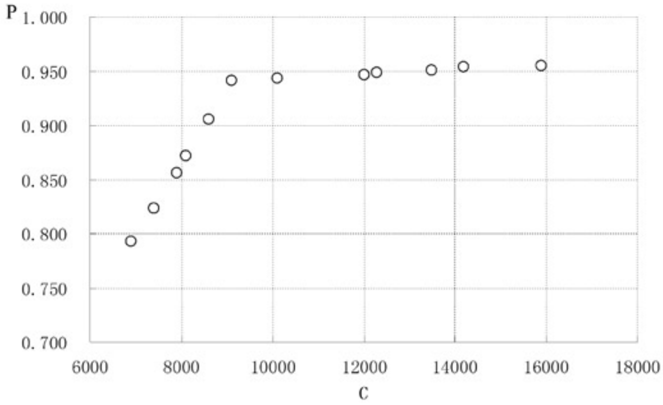

Fig. 5. The distribution of Pareto solutions in the objective space for spare parts

Figure 4 shows the relation between optimal individual fitness and evolution generation. It is seen that the best individual fitness value basically turns to stabilization when the evolution generation is beyond 50.

The distribution of the non-inferior solutions in the objective space is shown in Figure 5. It can be seen that with the increasing of support costs with in a certain range, the support probability of spare is also rising. However, when the support probability of spare reached 0.95, even with increasing the number of spare parts support costs, the changes in the probability is very small. Therefore, decision-makers can determine a suitable solution by the tradeoff between the two goals of the spare parts support probability and support costs. As a result, we can select a allocation scheme of spare parts from non-inferior solutions.

In addition, the collections of non-inferior solutions in the target space are corresponding to the 12 different schemes in Table 3 respectively. Policy-makers can make a tradeoff from the two goals of the probability of spare supportability and support costs to determine a suitable solution from the non-inferior solutions for the spare parts allocation. For example, if the spare support probability is not less than 0.90, and with a minimum cost, one can select the target space of the fifth non-inferior solutions (P=0.905, C=8600RMB¥). In this scheme, 3 of spare part A are allocated in the second echelon and none in other echelons; and stock level of spare part B is 10 in the second echelon.

5. Conclusions

This paper develops an improved MOPSO method of spare parts allocation and optimization, which takes into account of multiple objectives such as support probability of spare parts and support costs, and focuses on the whole system of maintenance support. Compared with other methods, such as GA, and ANN, this method is more efficient to solve the non-linear, multi-objective and high-dimensional problems of the allocation and optimization models. At last, a numerical example has verified the feasibility and effectiveness of the models and algorithm for the spare parts. In this paper, we have assumed that the failure rate of a replaceable unit is constant, and there is no lateral supply between the inventories at the same echelon. However, it can be extended to consider lateral supply and other distributions of unit failures. This topic will be studied in our following research.

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