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SYSTEM RELIABILITY DEMONSTRATION WITH EQUIVALENT DATA FROM COMPONENT ACCELERATED TESTING BASED ON RELIABILITY TARGET TRANSFORMATION

STWIERDZANIE NIEZAWODNOŚCI SYSTEMU NA PODSTAWIE RÓWNOWAŻNYCH DANYCH Z PRZYSPIESZONYCH BADAŃ ELEMENTÓW SKŁADOWYCH W OPARCIU O TRANSFORMACJĘ CELU NIEZAWODNOŚCIOWEGO

The reliability demonstration test (RDT) programs in general proceed at various levels, including component, subsystem, and system in the verification and validation phase of the product life cycle. The system reliability demonstration within feasible duration becomes a considerable issue because of the marketplace demands for decreased development time and cost. A method based on reliability target transformation is proposed to accomplish the system reliability demonstration with the data from the RDT of the components. In order to shorten the test time, the RDT plan for component under the accelerated condition is first designed. Then, the reliability target of the system with different lifetimes required by the producer and the consumer is transferred to the target with the same specified mission time, which should meet the time constraint of the system level test. Next, the lower limit confidence of component reliability at the system mission time are estimated and converted to the equivalent binomial component data by the curve fitting method, then they are synthesized to the equivalent binomial system data by the Bayesian method. Finally, the system reliability demonstration is considered. The system classical attribute acceptance sampling plan at the mission time is used to make decisions using the equivalent binomial system data. If the decision cannot be made, the system Bayesian attribute acceptance sampling plan will be designed with the equivalent data as the prior parameters and the complementary system test will be conducted.

Keywords: *reliability demonstration, accelerated testing, equivalent binomial component data; reliability target transformation, Bayesian attribute acceptance sampling plan.*

Ogólnie, oprogramowanie do badań stwierdzających niezawodność (RDT) można stosować na różnych poziomach, w tym na poziomie elementu składowego, podsystemu i systemu, w fazie weryfikacji i walidacji cyklu życia produktu. Stwierdzenie niezawodności systemu w realnym terminie staje się ważkim problemem ze względu na wymogi rynku co do zmniejszenia czasu i kosztów rozwoju. W prezentowanej pracy zaproponowano metodę opartą na transformacji celu niezawodnościowego, wedle której niezawodność systemu stwierdza się na podstawie danych z RDT części składowych. Aby skrócić czas testowania, w pierwszej kolejności tworzy się plan RDT dla części składowej w warunkach przyspieszonych. Następnie cel niezawodnościowy systemu przy różnych czasach pracy wymaganych przez producenta, jak i konsumenta, zostaje przetransponowany na cel o tym samym określonym czasie użytkowania, który powinien spełniać ograniczenie czasowe dla badań na poziomie systemu. Następnie szacuje się dolne granice przedziałów ufności dla niezawodności komponentów w określonym czasie eksploatacji systemu oraz przekształca się je na równoważne dane dwumienne dla części składowych z wykorzystaniem metody dopasowywania krzywych; dalej, są one syntetyzowane do równoważnych dwumiennych danych dotyczących systemu z zastosowaniem metody Bayesa. Pozwala to na stwierdzenie niezawodności systemu. Decyzje podejmuje się na podstawie równoważnych danych dwumiennych dotyczących systemu z wykorzystaniem klasycznego planu wyrywkowej kontroli odbiorczej systemu według zadanych charakterystyk dla określonego czasu użytkowania. Jeżeli decyzja nie może zostać podjęta w ten sposób, konstruuje się bayesowski plan wyrywkowej kontroli odbiorczej systemu wg. zadanych charakterystyk, gdzie dane równoważne stanowią parametry a priori, oraz przeprowadza się uzupełniające badania systemu.

Słowa kluczowe: *stwierdzanie niezawodności;badania przyspieszone; równoważne dane dwumienne dla części składowych; transformacja celu niezawodnościowego; bayesowski planwyrywkowej kontroli odbiorczej systemu wg. zadanych charakterystyk.*

1. Introduction

Before being disposed to the markets, the product is subjected to a number of phases, including product planning, design and development, verification and validation, production. In the planning phase, all the products are required to achieve a reliability target that need to be demonstrated. In the design and development phase, the target of the product (also termed the system) is allocated to its subsystems and components, and according to these targets, the reliability is designed into the system proactively. In the verification and validation phase as well as the production phase, the reliability targets of the components, the subsystems, and the system are demonstrated through the reliability demonstration test (RDT).

Because of the marketplace demands for decreased development time and cost, the RDT is required to be accomplished within feasible test duration. For the components, the accelerated life testing (ALT) can be adopted in RDT to shorten the test time [1, 4-5, 15, 18], which is termed the accelerated life reliability demonstration test (ALRDT). However, for the system reliability demonstration, the accelerated testing may not be applied due to the complexity of the system. The acceptance sampling plan is often employed for system reliability demonstration. And the literature on the acceptance sampling plan can be classified by the classical (commonly used) plan [3, 9, 12] and Bayesian plan [2, 7, 11, 16–17].

Further, the system reliability demonstration method utilizing the component data is generally considered. In the literature, there are two main approaches to demonstrate the system reliability target. One approach is that the system reliability target is demonstrated through the component test based on the derived relationship between the system reliability target and the component test plan. Mazumdar [10] proposed an optimum procedure for component testing with type-I censoring to demonstrate the series system reliability, and Rajgopal and Mazumdar [13] developed a system-based component test plan for a series system with type-II censoring. Yan and Mazumdar [20], and Rajgopal and Mazumdar [14] provided the component-testing procedure for a parallel system with type-II censoring, respectively. The time to failure for the components is assumed an exponentially distributed in the methods above, and the methods can only be applied to the series system or the parallel system, but be not suitable for the complex system that comprises of the components with other lifetime distributions. The other approach is that the system reliability target is demonstrated through designing the system test plan with component data. Li and Cai [7] designed the system attribute acceptance sampling plan through synthesizing the binomial subsystem data by the Bayesian method, which can reduce the sample size for the flight testing. Ten and Xie [17] also proposed a Bayesian reliability demonstration test plan for series-systems with binomial subsystem data, where the approximate lower limit confidence (ALLC) of system reliability was estimated using the binomial subsystem data and was utilized to calculate the system prior distribution parameters. However, the system prior distribution parameters cannot be derived directly utilizing the component data for the case that the components of the system undergo the accelerated testing (AT).

In some applications, the lifetime at the specified reliability is selected as the reliability measure for the target to be demonstrated, where the lifetimes required by the producer and the consumer are different. In this case, the system attribute acceptance sampling test plan at the specified mission time cannot be utilized directly. Additionally, the system attribute test plan is unfeasible when the mission time for the target is long.

In this paper, a system reliability demonstration test methodology with equivalent binomial data from component ALRDT based on reliability target transformation is developed. The ALRDT of the components is first conducted within feasible test duration. With the parameter estimates of the lifetime distribution for the components, the system reliability target with different lifetimes required by the producer and the consumer can be transferred to the target at the same specified mission time with different reliability required by the producer and the consumer, where the mission time meets the time constraint of the system level test. Next, the LLC of component reliability at the specified mission time is estimated and converted to the equivalent binomial component data, and they are synthesized to the equivalent binomial system data. Finally, the system reliability demonstration is considered. The system classical attribute acceptance sampling plan at the mission time is designed to make decisions using the equivalent binomial system data. If the decision cannot be made, the system Bayesian attribute acceptance sampling plan at the mission time will be designed with the equivalent binomial system data as the prior parameters and the complementary system level test for the mission time will be conducted.

2. Assumptions

(1) Life distribution

Assume that the lifetime of the xth component follows the Weibull distribution, then the reliability at t is

$$R_x(t) = \exp\left(-\left(t/\eta_x\right)^{\delta_x}\right) \tag{1}$$

where η_x and δ_x are the scale and shape parameters for the xth component, x=1,2,...,l, and l is the number of components in the system. The time to failure of the components for the system is assumed to be statistically independent.

(2) Accelerated model

The accelerated model often indicate that the η_x is a log linear function of a (possibly transformed) stress S, given as

$$\ln \eta_x = \gamma_{0,x} + \gamma_{1,x} \cdot f(S) \tag{2}$$

where $\gamma_{0,x}$ and $\gamma_{1,x}$ are the parameters of the acelerated model for the xth component and $f(S)$ is the function of the stress S. Additionally, the acceleration factor a_x is equal to $\eta_{0,x}/\eta_{a,x}$, where $\eta_{0,x}$ and $\eta_{a,x}$ are the scale parameters when the stresses are the normal stress S_0 and the accelerated stress S_a, respectively. And the shape parameter δ_x is constant and independent of the stress.

(3) The reliability targets

Let $t_{0,x}(R_x)$ and $t_{1,x}(R_x)$ with the corresponding risks α_x and β_x denote the reliability target of the xth component required by the producer and the consumer, where $t_{0,x}(R_x)$ and $t_{1,x}(R_x)$ are the lifetimes at the specified reliability R_x for x=1,2,...,l. And let $t_{0,s}(R_s)$ and $t_{1,s}(R_s)$ with the corresponding risks α_s and β_s denote the reliability target of the system required by the producer and the consumer, where $t_{0,s}(R_s)$ and $t_{1,s}(R_s)$ are the lifetimes at the specified reliability R_s.

3. Test Methodology

A system reliability demonstration test methodology is developed here. The purpose of the proposed methodology is to demonstrate that the system reliability as well as the components reliability meets the reliability targets required by the producer and the consumer prior to the field deployment. The four generic steps are given as follows, and the flowchart of the test methodology is described in Fig 1.

1) The reliability targets of the components are first to be demonstrated. They are subjected to time-censored ALRDT to accomplish the demonstration within feasible test duration.
2) If all the components pass the test, the system reliability demonstration is considered. As $t_{0,s}(R_s)$ and $t_{1,s}(R_s)$ are different, the system acceptance attribute sampling plan at the mission time cannot be designed. So the reliability target of the system should be transferred. The transferred target is $R_{0,s}(t_{st})$ and $R_{1,s}(t_{st})$, which denote the reliability at the same mission time

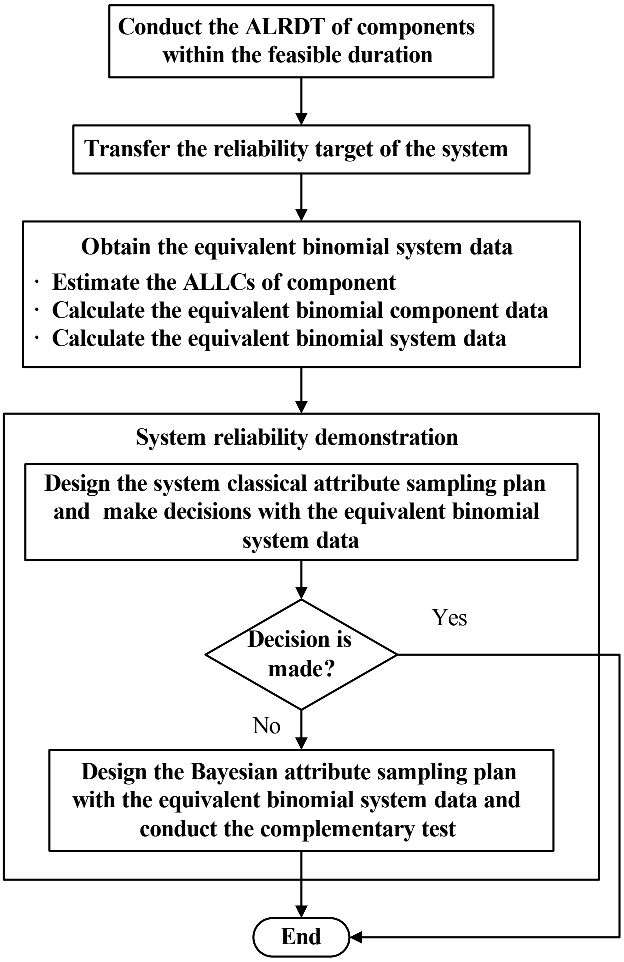

Fig.1 The flowchart of the proposed methodology

t_{st} required by the producer and the consumer, respectively. Note that the specified time t_{st} should meet the constraint of the system test duration.

3) In order to utilize the component data for system reliability demonstration, the LLCs of component at t_{st} under two confidence levels (CLs) are estimated and then converted to the equivalent binomial component data at t_{st} by the curve fitting method. And the system equivalent binomial data can be obtained through synthesizing the equivalent binomial component data by the Bayesian method.
4) According to the system transferred target, the system classical attribute acceptance sampling plan for the mission time t_{st} is designed. And if the decision can be made using the system equivalent binomial data, the system level test is not needed. Otherwise, the system Bayesian attribute acceptance sampling plan is designed, where the system prior distribution parameters are obtained from the equivalent binomial system data. Then, the complementary system level test will be conducted until the time reaches t_{st} and the decision is made in terms of the number of failures occurs during the test.

3.1. Time-censored ALRDT plan for components

For the xth component, assume that the n_x specimens are randomly sampled from a lot and then are tested simultaneously under the specified accelerated stress until the censoring time $t_{a0,x}$, x=1,2,...,l. The producer's risk and the consumer's risk can be expressed through calculating the acceptance probability of operating characteristic (OC) curve when the reliability are set to $R_x\left(t_{0,x}\right)$ and $R_x\left(t_{1,x}\right)$, respectively [1]. Additionally, the cost of time-censored ALRDT for the xth component comprises of two parts:

1) the cost of conducting the test is $C_1 t_{a0,x}$;
2) the cost of samples is $C_2 n_x$.

The optimum plan $D_{cx}=\left(n_x,c_x,t_{a0,x}\right)$ for the xth component can be solved by

$$\underset{D_{cx}}{\mathrm{Min}}[C_1 t_{a0,x}+C_2 n_x] \tag{3}$$

subject to

$$\begin{cases} n_x \le n_{\lim.x} \\ t_{a0,x} \le t_{\lim,x} \\ \alpha_x^{'}=1-\sum\limits_{r=0}^{c_x} C_{n_x}^r \left(1-\exp\left(\dfrac{\ln\left(R_x\left(t_{0,x}\right)\right)}{t_{0,x}^{\delta_x}}\cdot\left(t_{a0,x}\cdot a_x\right)^{\delta_x}\right)\right)^r \left(1-\exp\left(\dfrac{\ln\left(R_x\left(t_{0,x}\right)\right)}{t_{0,x}^{\delta_x}}\cdot\left(t_{a0,x}\cdot a_x\right)^{\delta_x}\right)\right)^{n-r} \le \alpha_x \\ \beta_x^{'}=\sum\limits_{r=0}^{c_x} C_{n_x}^r \left(1-\exp\left(\dfrac{\ln\left(R_x\left(t_{1,x}\right)\right)}{t_{1,x}^{\delta_x}}\cdot\left(t_{a0,x}\cdot a_x\right)^{\delta_x}\right)\right)^r \left(1-\exp\left(\dfrac{\ln\left(R_x\left(t_{1,x}\right)\right)}{t_{1,x}^{\delta_x}}\cdot\left(t_{a0,x}\cdot a_x\right)^{\delta_x}\right)\right)^{n-r} \le \beta_x \end{cases} \tag{4}$$

where c_x is the acceptance number of failures; $\alpha_x^{'}$ and $\beta_x^{'}$ are the actual values of risks; $n_{\lim.x}$ and $t_{\lim,x}$ are the limits of n_x and $t_{a0,x}$, respectively; and x=1,2,...,l. The D_{cx} can be calculated by the exhaustive method when $t_{a0,x}$ is discretized with a fixed step size.

For the xth component, the n_x specimens undergo the AT simultaneously until the time reaches $t_{a0,x}$, x=1,2,...,l. The reliability of the xth component is accepted if $f_x \le c_x$ and is rejected otherwise, where f_x denotes the number of failures at $t_{a0,x}$, x=1,2,...,l. It should be noted that the test plans depend on the design parameters a_x and δ_x for x=1,2,...,l. Pre-estimates of unknown design parameters are needed in previous studies. Such prior pre-estimates may be from past experiences, similar data, testing data in the design and development phase, and the preliminary test.

3.2. System reliability target transformation

After the ALRDT of the components, the $\hat{\eta}_{a,x}$ for x=1,2,...,l can be estimated using the censoring data and then $\hat{\eta}_{0,x}=\hat{\eta}_{a,x}\cdot a_x$. With the $\hat{\eta}_{0,x}$ and δ_x for x=1,2,...,l, the pseudo failure data of the components can be simulated by the Monte-Carlo method. Then, the pseudo failure time of the system can be calculated in terms of the system reliability model. The simulation procedure is repeated for N times, and the N pseudo failure times of the system are approximately fitted to the Weibull distribution. Further, the N pseudo failure times of the system is sorted in ascending order, which is denoted by $t_{ps,k}$, k=1,2,…, N, and the fitting precision is evaluated by the so-called correlative coefficient ρ[6]

$$\rho=\left|1-\sum_{k=1}^{N}\left(F(k)-\hat{F}(k)\right)^2\Big/\sum_{k=1}^{N}\left(F(k)-\overline{F(k)}\right)^2\right|$$

where $F(k)=(k-0.3)/(N+0.4)$, $\hat{F}(k)=1-\exp\left(-\left(t_{ps,k}/\hat{\eta}_s\right)^{\hat{\delta}_s}\right)$, $\overline{F(k)}=\frac{1}{N}\sum_{k=1}^{N}F(k)$, $\hat{\eta}_s$ and $\hat{\delta}_s$ are the scale and shape parameters of the fitted Weibull distribution for the system. The closer ρ approximates 1, the more accurate the fitting precision is.

With the $\hat{\delta}_s$, the target with different lifetimes required by the producer and the consumer can be transferred to a new target at the same mission time t_{st}, given as

$$\begin{cases} R_{0,s}(t_{st})=\exp\left(-\left(t_{st}/\eta_{0,s}\right)^{\hat{\delta}_s}\right)=\exp\left(\ln R_s\cdot\left(\dfrac{t_{st}}{t_{0,s}}\right)^{\hat{\delta}_s}\right) \\ R_{1,s}(t_{st})=\exp\left(-\left(t_{st}/\eta_{1,s}\right)^{\hat{\delta}_s}\right)=\exp\left(\ln R_s\cdot\left(\dfrac{t_{st}}{t_{1,s}}\right)^{\hat{\delta}_s}\right) \end{cases} \tag{5}$$

According to the $R_{0,s}(t_{st})$ and $R_{1,s}(t_{st})$ with α_s and β_s, the system attribute acceptance sampling plan at t_{st} can be designed.

3.3. Equivalent binomial system data

3.3.1. Estimate the LLC of component reliability

After the ALRDT of the components, the ALLC of component reliability at t_{st} can be estimated using the censoring data and the useful degradation information during the test by the Bootstrap method [8, 19].

3.3.2. Calculate the equivalent binomial component data

Using two LLCs at different CLs, the equivalent binomial component data can be obtained by the curve fitting method. The theoretical basis of the method can be seen in [8], and the simplified method is given as follows. For the xth component, let (S_x,F_x) denote the equivalent binomial component data at t_{st} and $R_{L,x}(0.1)$ and $R_{L,x}(0.9)$ denote the LLCs of reliability at t_{st} when CL is 0.1 and 0.9, respectively; $x=1,2,\ldots,l$. Then, (S_x,F_x) can be fully identified by [8]

$$\begin{cases} I_{R_{L,x}(0.1)}(S_x,F_x+1)=1-0.1 \\ I_{R_{L,x}(0.9)}(S_x,F_x+1)=1-0.9 \end{cases} \tag{6}$$

where $I_R(S_x,F_x+1)$ is a Beta distribution function with parameters S_x and F_x, $N_x=S_x+F_x$, and we have

$$I_R(S_x,F_x+1)=\sum_{i=0}^{F_x}C_{N_x}^{F_x}R^{N_x-F_x}(1-R)^{F_x} \tag{7}$$

Note that S_x and F_x are generally decimal.

3.3.4. Calculate the equivalent binomial system data by the Bayesian method

The methods commonly used to synthesize the component reliability include the MML method, the Bayesian method, and Bootstrap method etc. In [8], it is verified that the estimate precision of the Bayesian method with the equivalent binomial component data is higher than other methods. The equivalent binomial system data (S_s,F_s) can be obtained by the Bayesian method as follows

$$\begin{cases} N_s=\dfrac{\mu-\nu}{\nu-\mu^2},\ S_s=N_s\mu,\ F_s=N(1-\mu) \\ \mu=\prod_{x=1}^{l}\mu_x \\ \nu=\prod_{x=1}^{l}\nu_x \end{cases} \tag{8}$$

where N_s, S_s and F_s are the equivalent binomial system data, μ_x and ν_x are first and second moments of reliability for the components, $x=1,2,\ldots,l$, and they are calculated using the equivalent binomial component data by

$$\begin{cases} \mu_x=\dfrac{s_0+S_x}{n_0+N_x} \\ \nu_x=\dfrac{(s_0+S_x)(s_0+S_x+1)}{(n_0+N_x)(n_0+N_x+1)} \end{cases} \tag{9}$$

where $s_0=1/2$, $n_0=1$.

3.4. System reliability demonstration

3.4.1. System reliability demonstration through classical attribute acceptance sampling plan using the equivalent binomial system data

The steps of the decision procedure are given as follows.

Step 1: Let $n_s=[N_s]$, $[\cdot]$ is an integral function, the system classical attribute plan (n_s,c_s) is solved by

$$\begin{cases} n_s=[N_s] \\ \alpha_s'=1-\sum_{r=0}^{c_s}\Pr\{n_s,r\mid R_{0,s}(t_{st})\}\le\alpha_s \\ \beta_s'=\sum_{r=0}^{c_s}\Pr\{n_s,r\mid R_{1,s}(t_{st})\}\le\beta_s \end{cases} \tag{10}$$

where n_s is the sample size of the system and c_s is the acceptance number of failures at t_{st}, $N_s=S_s+F_s$, and $\Pr\{n,r\mid R\}=C_n^r(1-R)^rR^{n-r}$.

1) When (n_s,c_s) can be solved, the reliability of the system is accepted if $c_s\ge F_s$ for all the solutions; the reliability of the system is rejected if $c_s<F_s$ and $(F_s-c_s)>(N_s-n_s)$ for all the solutions; otherwise, the decision cannot be made and the step 2 is considered.
2) When (n_s,c_s) cannot be obtained, the decision cannot be made and the step 2 is considered.

Step 2: Let $c_s < F_s$ and $n_s > N_s$, the system attribute sampling plan (n_s, c_s) is solved by

$$\begin{cases} c_s < F_s \\ n_s > N_s \\ \alpha_s' = 1 - \sum_{r=0}^{c_s} \Pr\{n_s, r \mid R_{0,s}(t_{st})\} \le \alpha_s \\ \beta_s' = \sum_{r=0}^{c_s} \Pr\{n_s, r \mid R_{1,s}(t_{st})\} \le \beta_s \end{cases} \tag{11}$$

The reliability of the system is rejected if the solution can be obtained; otherwise, the decision cannot be made, and the complementary system test will be considered, where the system Bayesian attribute sampling plan is designed.

3.4.2. System Bayesian reliability demonstration test plan

For the system Bayesian attribute acceptance sampling plan, the Beta distribution has been widely used as the prior distribution for binomial sampling as recommended in [17]. The prior distribution of system reliability can be expressed as follows [11].

$$\pi(R) = \frac{R^{p_a - 1}(1-R)^{p_b - 1}}{\beta(p_a, p_b)}, 0 \le R \le 1 \tag{12}$$

where p_a and p_b are the beta distribution parameters, and

$$\beta(p_a, p_b) = \frac{\Gamma(p_a)\Gamma(p_b)}{\Gamma(p_a + p_b)} \tag{13}$$

Then, with the equations (12) and (13), the system posterior distribution is

$$\pi(R \mid D) = \frac{R^{n_s - f_s + p_a - 1}(1-R)^{p_b + f_s - 1}}{\beta(n_s - f_s + p_a, p_b + f_s)} \tag{14}$$

where D denote the (n_s, f_s), f_s is the number of system failures at t_{st}.

For binomial sampling,

$$\Pr\{\text{Accept} \mid R\} = \sum_{r=0}^{c_s} C_{n_s}^{r} (1-R)^{r} R^{n_s - r} \tag{15}$$

$$\Pr\{\text{Reject} \mid R\} = \sum_{r=c_s+1}^{n_s} C_{n_s}^{r} (1-R)^{r} R^{n_s - r} \tag{16}$$

By applying Bayesian theorem, the constrains of the risks can be expressed by

$$\begin{cases} \alpha_s' = \dfrac{\int_{R_{0,s}}^{1} \Pr\{\text{Reject} \mid R\} \cdot \pi(R \mid D) dR}{\int_0^1 \pi(R \mid D) dR} \\ \beta_s' = \dfrac{\int_0^{R_{1,s}} \Pr\{\text{Accept} \mid R\} \cdot \pi(R \mid D) dR}{\int_0^1 \pi(R \mid D) dR} \end{cases} \tag{17}$$

where α'_s and β'_s are the actual values of risks.

With the equivalent binomial system data (S_s, F_s) at t_{st}, the prior parameters p_a and p_b can be obtained, and $p_a = S_s$ and $p_b = F_s$. Then, the test plan (n_s, c_s) can be derived by

$$\underset{n_s, c_s}{\text{Min}}[n_s] \tag{18}$$

subject to

$$\begin{cases} n_s \le n_{\lim,s} \\ \alpha_s' = \dfrac{\int_{R_{0,x}}^{1} \Pr\{\text{Reject} \mid R\} \cdot \pi(R \mid D) dR}{\int_0^1 \pi(R \mid D) dR} \le \alpha_s \\ \beta_s' = \dfrac{\int_0^{R_{1,x}} \Pr\{\text{Accept} \mid R\} \cdot \pi(R \mid D) dR}{\int_0^1 \pi(R \mid D) dR} \le \beta_s \end{cases} \tag{19}$$

where $n_{\lim,s}$ is the limit of n_s.

The n_s systems will be tested simultaneously until the time reaches t_{st}, the reliability of the system is accepted if $f_s \le c_s$ and is rejected otherwise.

4. Illustrative example

Without loss of generality, assume that a series system comprises of three components, and the system reliability model is depicted in Fig. 2.

Fig. 2. System reliability model

The assumed reliability target of the system is shown in Table 1. According to the target, the procedure of system reliability demonstration is given as follows.

Table .1 The reliability targets of the system

Target	$t_{0,s}(R_s)$ (h)	$t_{1,s}(R_s)$ (h)	R_s	α_s	β_s
1	26280	17520	0.78	20%	20%

4.1. Time-censored ALRDT plans for the components

The lifetimes of the components are assumed to follow the Weibull distribution. And the Arrhenis model is employed when the temperature is the accelerating variable. The use temperatures and the accelerated temperatures for three components in ALRDT are shown in Table 2.

Table 2. The use temperatures and the accelerated temperatures for three components

x	Use temperature (K)	Accelerated temperature (K)
1	293	393
2	293	403
3	293	383

The assumed prior values of the accelerated model parameters and the shape parameters as well as the acceleration factors for three components are shown in Table 3.

Table 3. The prior values of model parameters

x	$\gamma_{0,x}$	$\gamma_{1,x}$	δ_x	α_x
1	25.5762	-0.0485	1.9	128
2	24.5620	-0.0431	1.3	114
3	28.1278	-0.0545	1.4	135

According to the reliability targets and the limits of the sample size and the test duration for three components shown in Table 4, the time-censored ALRDT plans are designed with the values of the design parameters α_x and δ_x for x=1,2,3 in Table 2, which are shown in Table 5.

Table 4. The reliability targets and the limits of the sample size and the test duration for three components

x	$t_{0,x}(R_x)$ (h)	$t_{1,x}(R_x)$ (h)	R_x	α_x	β_x	$n_{lim,x}$	$t_{lim,x}$(h)
1	26280	17520	0.92	20%	20%	30	1000
2	26280	17520	0.93	20%	20%	30	1000
3	26280	17520	0.94	20%	20%	30	1000

Table 5. The time-censored ALRDT plans for three components

x	n_x	c_x	$t_{a0,x}$ (h)	α'_x	β'_x
1	30	5	264	16.79%	19.54%
2	30	10	736	19.36%	19.94%
3	30	9	576	18.28%	19.87%

4.2. System reliability target transformation

Assume that the reliability of three components satisfies the requirements of the corresponding targets respectively and the model parameters of three components shown in Table 3 are the estimate values. The pseudo failure times of the components are simulated using the Monte-Carlo method. Then, the pseudo failure time of the system can be obtained according to the system reliability model in Fig. 2. The simulation procedure is conducted for N=100000 times, and the N pseudo failure times of the system are fitted into the Weibull distribution and ρ=0.9999, which indicate that the Weibull distribution is proper to describe the statistical properties of the system lifetime. And $\hat{\delta}_s$=1.61.

Let $t_{st}=8760$ h, then $R_{0,s}(t_{st})$ and $R_{1,s}(t_{st})$ can be calculated by equation (5). The transferred target of the system is shown in Table 6.

Table 6. The transferred reliability target of the system

t_{st} (h)	$R_{0,s}(t_{st})$	$R_{1,s}(t_{st})$	α_s	β_s
8760	0.9585	0.9218	20%	20%

4.3. System reliability demonstration

4.3.1. Calculate the equivalent binomial system data

The assumed LLCs of three components when CL is 0.1 and 0.9 are shown in Table 7.

The equivalent binomial component data are calculated by equations (6) and (7) are shown in Table 8.

Table 7. The assumed LLCs of three components

x	CL=0.1	CL=0.9
1	0.9960	0.9729
2	0.9921	0.9561
3	0.9962	0.9731

Table 8. The equivalent binomial component data

x	S_x	F_x
1	146.5461	1.1101
2	102.7690	1.5278
3	148.9287	1.0713

Then, the equivalent binomial system data (S_s, F_s) are calculated by equations (8) and (9), where $(S_s, F_s)=(121.2495, 5.0488)$.

4.3.2. System reliability demonstration through attribute sampling plan

(1) According to the transferred target in Table 6, the system classical attribute sampling plan is designed and the decision is made with (S_s, F_s) as follows.

Step1: Let $n_s=[N_s]=126$, the solution cannot be solved by equation (10), the decision cannot be made. Then, the step 2 is considered.

Step2: Let $c_s \le [F_s]=5$ and $n_s > N_s$, the solution cannot be solved by equation (11), the decision cannot be made.

(2) Assume that $n_{\lim,s}=10$, the system Bayesian attribute sampling plan is designed by equations (18) and (19) with (S_s, F_s), which are shown in Table 9.

Table 9. The system Bayesian attribute sampling plan

n_s	c_s	α'_s	β'_s
5	3	17.02%	19.75%

Five systems will be arranged to undergo the test until the time reaches t_{st}=8760h. After the complementary test, the reliability of the system will be accepted if the number of system failures is not greater than three and will be rejected otherwise.

4.4. System reliability demonstration for other targets

Another three reliability targets of the system are assumed and shown in Table 10. And suppose that the t_{st} and the (S_s, F_s) are unchanged, the transferred targets of the system are given in Table 11.

Table 10. Another three reliability targets of the system

Target	$t_{0,s}(R_s)$ (h)	$t_{1,s}(R_s)$ (h)	R_s	α_s	β_s
2	26280	17520	0.75	20%	20%
3	17520	8760	0.95	20%	20%
4	17520	8760	0.92	20%	20%

Table 11. The transferred reliability targets of the system

Target	$t_{0,x}(R_x)$ (h)	$R_{0,s}(t_{st})$	$R_{1,s}(t_{st})$	α_s	β_s
2	8760	0.9521	0.9101	20%	20%
3	8760	0.9833	0.95	20%	20%
4	8760	0.9731	0.92	20%	20%

According to the transferred targets, the corresponding system reliability demonstration test plans are designed and the decisions are made.

(1) System reliability demonstration for the target 2

Let $n_s = [N_s] = 126$, the system classical attribute sampling plan is solved and shown in Table 12. As $c_s > F_s$, the reliability of the system is accepted.

Table 12. The system classical attribute sampling plan when n_s=126 for the target 2

n_s	c_s	α'_s	β'_s
126	8	15.11%	19.12%

(2) System reliability demonstration for the target 3

Let $n_s = [N_s] = 126$, the system classical attribute sampling plan is solved and shown in Table 13. As $c_s < F_s$ and $(F_s - c_s) > (N_s - n_s)$, the reliability of the system is rejected.

Table 13. The system classical attribute sampling plan when n_s = 126 for the target 3

n_s	c_s	α'_s	β'_s
126	3	15.98%	12.00%

(3) System reliability demonstration for the target 4

1) Let $n_s = [N_s] = 126$, the system classical attribute sampling plans are solved and shown in Table 14. As $c_s < F_s$ and $(F_s - c_s) < (N_s - n_s)$, the decision cannot be made. Then, the step 2 is considered.

2) Let $c_s = [F_s] = 5$ and $n_s > N_s$, the classical sampling plan can be solved, and one of the solutions is shown in Table 15. So the reliability of the system is rejected.

Table 14. The system classical attribute sampling plan when n_s = 126 for the target 4

Plan	n_s	c_s	α'_s	β'_s
1	126	5	12.61%	5.67%
2	126	6	5.50%	11.50%

Table 15. A system classical attribute sampling plan when c_s = 5 and $n_s > N_s$ for the target 4

n_s	c_s	α'_s	β'_s
127	5	12.95%	5.41%

5. Conclusion

(1) A system reliability demonstration test methodology with the equivalent binomial data from ALRDT of components based on the reliability target transformation is developed in this paper. The feasibility of the method is illustrated through a numerical example.

(2) The data from the ALRDT of components are used to estimate the LLCs of components and converted to the equivalent binomial component data by the curve fitting method. Then, they are synthesized to the equivalent binomial system data by the Bayesian method, which is utilized for system reliability demonstration. If the equivalent binomial system data is sufficient for decision-making, the system classical attribute sampling plan is employed and the system level test is not needed; otherwise, the system Bayesian attribute sampling plan will be designed and the complementary system test at the transformed mission time will be conducted. As the equivalent binomial system data are used to calculate the parameters of prior distribution, the sample size of the system level test is reduced. Additionally, the proposed method is also applicable to the complex systems with various types of component data, where the LLCs of components can be obtained. For the case that the lifetimes of the components follow different distributions, several types of distributions such as Weibull distribution and lognormal distribution can be used to fit the pseudo failure times of system and the one with maximum correlative coefficient ρ is selected to describe the statistic properties of the system. Then, the target of the system is transferred using the similar way shown in equation (5) according to the cdf expression of the selected distribution.

(3) In terms of the model parameter estimates for the components and the system reliability model, the system pseudo failure times can be simulated by the Monte-Carlo method and fitted to the Weibull distribution. Then, with the estimate of the shape parameter for system lifetime distribution, the system reliability target $t_{0,s}(R_s)$ and $t_{1,s}(R_s)$ can be transferred to the target $R_{0,s}(t_{st})$ and $R_{1,s}(t_{st})$ at the same specified mission time t_{st}. As the t_{st} is specified to meet the time constraint of the system level test and the ALRDT of components are employed, the reliability target with long lifetime can be demonstrated. Note that the t_{st} can not be too short; otherwise, $R_{0,s}(t_{st})$ and $R_{1,s}(t_{st})$ will become larger, and so does the sample size of the system level test.

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