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EFFICIENCY ANALYSIS OF AN AUTOPARAMETRIC PENDULUM VIBRATION ABSORBER

ANALIZA EFEKTYWNOŚCI AUTOPARAMETRYCZNEGO WAHADŁOWEGO TŁUMIKA DRGAŃ*

This paper presents results of a study of a dynamic response of an autoparametric system consisting of the oscillator with an attached pendulum vibration absorber. The harmonic balance method is applied to get the autoparametic resonance conditions. The analytical full vibration absorption condition has been determined and verified by numerical simulations. Additionally, the influence of oscillator and pendulum damping on dynamics and the vibration absorption effect is presented.

***Keywords**: oscillations, pendulum, absorption, damping, resonance.*

W pracy przedstawiono analizę dynamiki autoparametrycznego układu składającego się z oscylatora wraz z dołączonym eliminatorem drgań w postaci wahadła. W celu uzyskania obszarów rezonansu autoparametrycznego zastosowano metodę bilansu harmonicznych. Wyznaczono analitycznie,a następnie zweryfikowano numerycznie warunek pełnej eliminacji drgań. Dodatkowo, przedstawiono wpływ tłumienia oscylatora i wahadła na zjawisko eliminacji drgań oraz dynamikę układu.

***Słowa kluczowe**: drgania,wahadło,eliminacja, tłumienie, rezonans.*

1. Introduction

The problem of undesired vibration reduction has been known since many years ago and becomes more attractive nowadays. The dynamic vibration absorbers (DVA) are special devices, consisting of masses suspended on springs and dampers. In the classical theory of DVA, the primary structure is modelled as a spring mass system. However, other dynamic vibration absorption models also have high interest in research and engineering application. In particular, the pendulum type systems can play an important role in many fields such as machinery, transportation and civil engineering. But, dynamic behaviour of a pendulum absorber is significantly more complex than it is supposed by the widely used additional simple dynamical dampers.

The autoparametric vibration pendulum absorber (AVPA) is designed to absorb energy from the primary system (main mass). This absorption effect is efficient only in the limited band of vibration frequencies of the main system [1]. Unlike the classical absorber, the use of the pendulum absorber does not result in excitation of vibrations with considerable amplitudes at other frequencies. This is due to the rigid regime of excitation of vibrations of the pendulum only near its internal resonance frequency for a resonance excitation frequency ratio of 1/2, [2, 6, 8].

Many papers dealing with various types of dynamic dampers and related topics have been published during the last decades. Some pendulum type absorbers have been applied for vibration protection systems on tower-pipes, chimneys, civil structures (buildings and bridges) affected by wind or seismic vibration, etc. [5]. The collection of many vibration absorbers and their practical applications are presented by Sun [7].

This paper deals with a pendulum absorber connected to a damped oscillator system. In this type of structures different motions are possible: periodic, quasi-periodic, chaotic or the pendulum may rotate [9]. Especially transition to rotation and chaos can lead to unexpected increase of amplitude and eventually to destruction of the structure. If the pendulum plays a role of a dynamical absorber, this kind of motion is unwanted. The first possible intuitive solution is to increase the system damping. This study is to estimate how the system damping influences the absorption efficiency of AVPA. In addition, obtained results allow preparing the control algorithm based on change in the system damping.

2. Model of AVPA

Let us consider a pendulum vibration absorber attached to a damped oscillator. The oscillator is forced by harmonic force $F(t)$ with amplitude q and frequency ϑ near the principal parametric resonance. The suspension of the primary system consists of a linear spring with stiffness reduced in dimensionless form to one and a viscous damping function $\alpha_1 X'$. Damping of the pendulum is described by linear function $\alpha_2\varphi'$.

The differential dimensional equations of motion of the two degrees-of-freedom autoparametric system (Fig. 1) are derived by the second kind of Lagrange equations and they are shown in papers [9]. The equations of motion are express in the dimensionless form:

$$X''+\alpha_1 X'+X+\mu\lambda(\varphi''\sin\varphi+\varphi'^2\cos\varphi)=q\cos\vartheta\tau, \quad (1)$$

$$\varphi''+\alpha_2\varphi'+\lambda\left(X''+1\right)\sin\varphi=0. \quad (2)$$

The second equation represents the pendulum and the first one is the excited system (the oscillator). The μ and λ represent pendu-

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

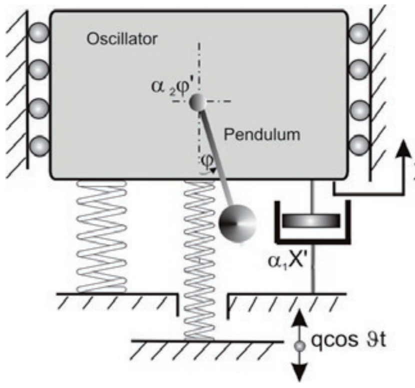

Fig. 1. Scheme of an autoparametric pendulum vibration absorber.

lum parameters. These parameters are responsible for the internal couplings of the pendulum absorber and the oscillator, also. Detailed information and definition of dimensionless parameters: α_1, α_2, μ, λ and q are presented in [10].

3. Parametric analysis of damping

3.1. Harmonic Balance Method

The harmonic balance method (HBM) is used to find an approximate solution for the system applied near the principal internal resonance condition. Thus, in the first approximation, the solutions are assumed as:

$$x(\tau)=A(\tau)\cos\left[\vartheta\tau+\phi_1\right],\varphi(\tau)=B(\tau)\cos\left[(\vartheta/2)\tau+\phi_2\right], \quad (3)$$

where $A(\tau)=A$, $B(\tau)=B$ and φ_1 and φ_2 are amplitudes and phase angles of the oscillator and the pendulum, respectively. Introducing eq. (3) and expand nonlinear terms (*sinφ* and *cosφ*) in Taylor series (up to the third order), for steady states following algebraic equations are obtained:

$$\left(1-\vartheta^2\right)A-\mu\lambda\left(\vartheta/2\right)^2B^2\cos\left(2\phi_2-\phi_1\right)=q\cos\phi_{1,} \quad (4)$$

$$-\vartheta\alpha_1A+\mu\lambda\left(\vartheta/2\right)^2B^2\sin\left(2\phi_2-\phi_1\right)=q\sin\phi_1, \quad (5)$$

$$\left(\vartheta/2\right)^2-\lambda+\left(\lambda/8\right)B^2+A\left(\lambda\vartheta^2/2\right)\cos\left(2\phi_2-\phi_1\right)=0, \quad (6)$$

$$\alpha_2\left(\vartheta/2\right)+A\left(\lambda\vartheta^2/2\right)\sin\left(2\phi_2-\phi_1\right)=0. \quad (7)$$

After some mathematical manipulations we get the phase angles:

$$\mathrm{t}g\phi_1=\frac{4\vartheta\left(4A^2\alpha_1+B^2\alpha_2\mu\right)}{16A^2\left(\vartheta^2-1\right)-B^2\mu\left(\left(B^2-8\right)\lambda+2\vartheta^2\right)},tg\left[2\phi_2-\phi_1\right]=\frac{\alpha_2\left(\vartheta/2\right)}{\left(\vartheta/2\right)^2-\lambda+\left(\lambda/8\right)B^2}, \quad (8)$$

and two equivalent equations for amplitudes of the oscillator:

$$A^2=\frac{B^4}{16\vartheta^4}+\frac{B^2}{16\lambda^2\vartheta^4}\left(4\lambda\vartheta^2-16\lambda^2\right)+\frac{\left(64\lambda^2-32\vartheta^2\lambda+4\vartheta^4+16\vartheta^2\alpha_2^2\right)}{16\lambda^2\vartheta^4}, \quad (9)$$

$$A^2=B^4\frac{\left(2\lambda\mu\left(\vartheta^2-1\right)-\lambda^2\mu^2\vartheta^4\right)}{16\left(\vartheta^4+\left(\alpha_1^2-2\right)\vartheta^2+1\right)}+B^2\frac{\left(16\lambda\mu\left(1-\vartheta^2\right)+4\mu\vartheta^2\left(\vartheta^2-1-2\alpha_1\alpha_2\right)\right)}{16\left(\vartheta^4+\left(\alpha_1^2-2\right)\vartheta^2+1\right)}+\frac{q}{\left(\vartheta^4+\vartheta^2(\alpha_1^2-2)+1\right)}. \quad (10)$$

Equating them, finally, we get the resonance curve which describes the pendulum oscillations in the steady state:

$$B^4\left[\frac{-\lambda^2\mu^2\vartheta^4+2\lambda\mu\left(-1+\vartheta^2\right)}{16\left(1+\left(-2+\alpha_1^2\right)\vartheta^2+\vartheta^4\right)}-\frac{1}{16\vartheta^4}\right]+B^2\left[\frac{16\lambda\mu\left(1-\vartheta^2\right)+4\mu\vartheta^2\left(\vartheta^2-1-2\alpha_1\alpha_2\right)}{16\left(1+\left(-2+\alpha_1^2\right)\vartheta^2+\vartheta^4\right)}+\frac{16\lambda^2-4\lambda\vartheta^2}{16\lambda^2\vartheta^4}\right]+\left[\frac{q^2}{1+\vartheta^2\left(-2+\alpha_1^2\right)+\vartheta^4}-\frac{64\lambda^2-32\lambda\vartheta^2+4\left(4\alpha_2^2\vartheta^2+\vartheta^4\right)}{16\lambda^2\vartheta^4}\right]=0. \quad (11)$$

Detailed derivation of HBM and solution stability analysis is shown in the paper [10]. It should be noticed, that HBM gives reliable results near main parametric resonance and for weakly nonlinear system, only.

3.2. Full absorption condition

If, we assume that $A=0$ (oscillator doesn't vibrate) in equations, the algebraic equations yield:

$$-\mu\lambda\left(\vartheta/2\right)^2B^2\cos\left(2\phi_2-\phi_1\right)=q\cos\phi_{1,} \quad (12)$$

$$\mu\lambda\left(\vartheta/2\right)^2B^2\sin\left(2\phi_2-\phi_1\right)=q\sin\phi_1, \quad (13)$$

$$\left(\vartheta/2\right)^2-\lambda+\left(\lambda/8\right)B^2=0, \quad (14)$$

$$\alpha_2\left(\vartheta/2\right)=0. \quad (15)$$

Based on equation (15), we can conclude, that the full elimination of oscillator's motions condition is possible if damping of the pendulum equals $\alpha_2=0$, or if the system does not vibrate (i.e. trivial solutions $A=0$, $B=0$ and $\vartheta=0$). Then amplitude of pendulum motion can be calculated from eqs. (12)–(13) and eq. (14):

$$B_1^2=\frac{q}{\mu\lambda\left(\vartheta/2\right)^2},\quad B_2^2=\frac{8\lambda-8\left(\vartheta/2\right)^2}{\lambda}. \quad (16)$$

Comparing the amplitudes in equation (16), the two frequency excitation for full absorption effect are obtained:

$$\vartheta_1=\sqrt{2\lambda+\frac{\sqrt{2}\sqrt{2\lambda^2\mu^2-\mu q}}{\mu}},\vartheta_2=\sqrt{2\lambda-\frac{\sqrt{2}\sqrt{2\lambda^2\mu^2-\mu q}}{\mu}}. \quad (17)$$

However, ϑ_2 is located beyond the main parametric resonance. Therefore, ϑ_1 denotes true amplitude for full absorption condition.

4. Absorption effect

4.1. Analysis of full vibration absorption effect (FVAE)

First, we analyse the FVAE of oscillator's motions. For data taken from [10]: α_1=0.1, α_2=0, μ=15.2, λ=0.25, q=0.05, the analytical full absorption frequency, calculated from eqs. (17), is equal to ϑ_1=0.997, and amplitudes of the pendulum B_1=0.23. The analytical resonance curves (eqs. (9)–(11)) for full absorption vibration effect are presented in Figs. 2. Close ϑ=1, the dynamical elimination of oscillator's vibration caused by the pendulum swinging is clearly visible. The analytical resonance curves are in a very good accordance with numerical verification [10].

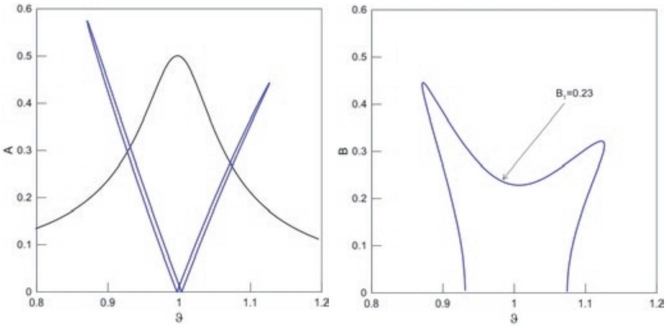

Fig. 2. Analytical resonance curves for full absorption effect.

Figure 3 shows numerical verification of FVAE. The amplitude of the oscillator tends to zero (Fig. 3a), while pendulum execute periodic swinging about amplitude equal φ=0.23, which agrees with analytical results. Additionally, we can conclude, that frequency ratio between oscillator and pendulum equals two. The initial conditions of system were set: φ=0.1, φ'=0, x=0 and x'=0.

(a) (b)

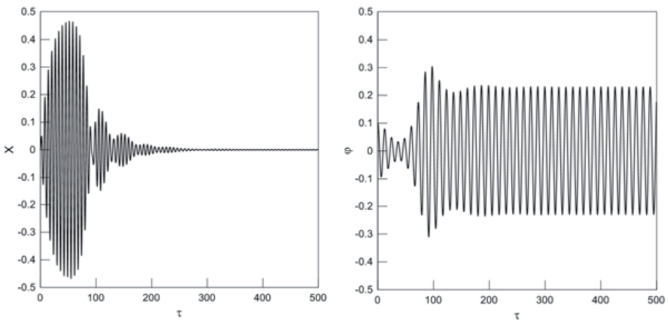

Fig. 3. Numerical verification of full absorption condition for ϑ=0.997, time history of oscillator (a) and pendulum (b).

If the pendulum does not vibrate (i. e. B=0), then it plays just the role of an additional mass of the oscillator. The value of its amplitude can be estimated by the classical relationship for excited linear oscillator:

$$A = \frac{q}{\sqrt{1+\left(\alpha_1^2-2\right)\vartheta^2+\vartheta^4}}. \quad (18)$$

This formula is identical to that obtained from eqs. (4)–(7), if we put B=0. In our example this amplitude, for ϑ=0.997 equals A=0.5 which is consistent with result in Fig. 2a.

4.2. Influence damping on absorption effect

In practice, FVAE is difficult to obtain because of existing friction related to damping in pivot of the pendulum. In this section we analyse the influence of system damping on the absorption effect. In Figs. 4, the influence of oscillator's damping on the oscillator (Fig. 4a) and the pendulum (Fig. 4b) behaviour is shown. Interesting, that increase of oscillator's damping does not eliminate dynamic absorption region, but only reduces it (Fig. 4a). This is very important from dynamic elimination vibrations point of view. This suggests to use this parameter to control the system behaviour.

However, the increase in pendulum's damping causes reduction of the pendulum amplitude (Fig. 5b), but absorption effect completely disappears (Fig. 5a). This denotes, pendulum's damping may impair the efficiently of AVPA.

(a) (b)

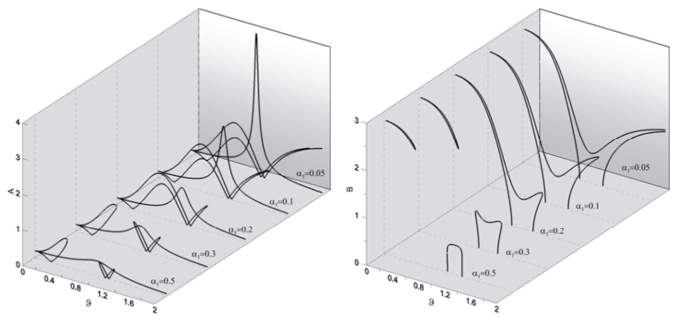

Fig. 4. Influence oscillator's damping on absorption effect (a) and pendulum swings (b) for α2=0.002, μ=6, λ=0.3, q=0.2.

(a) (b)

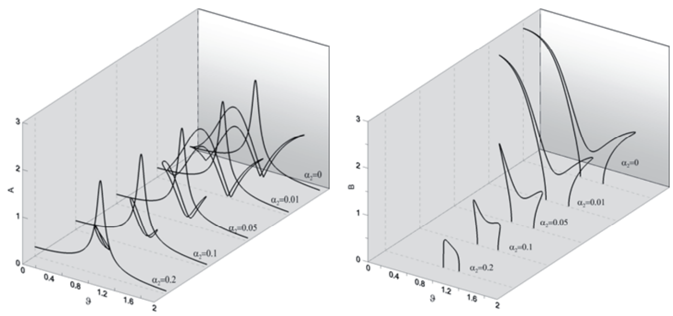

Fig. 5. Influence pendulum's damping on absorption effect (a) and pendulum swings (b) for α1=0.1, μ=6, λ=0.3, q=0.2.

The obtained results show that the best absorption effect exists for small values of system's damping and absorption region is located near the main parametric resonance. Therefore, the dynamic pendulum damper should be properly designed to take system's damping parameters into consideration. The numerical and experimental verification of these results can be found in [3, 4].

5. Conclusions and final remarks

The vibration absorption effect by application of an autoparametric coupled pendulum is investigated in this paper. In the system, the motions of the pendulum and the oscillator are coupled therefore vibration absorption depends on dynamics of both subsystems. Near the autoparametric resonance region, the most effective absorption region is located. Analytical and numerical studies have shown that full absorption effect is possible if the damping of the pendulum is near to zero. The absorber can be highly efficient for correctly tuned subsystems.

The damping analysis shows, that the increase of pendulum's damping can reduce or eliminate the absorption region, while the increase of oscillator' damping only reduces the absorption. Therefore, the control method of AVPA by oscillator damping as a control parameter looks promising.

A smart suspension consisting of SMA spring together with MR damper leading to active dynamic vibration absorber will be prepared in the future.

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