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PROBLEMS OF RELIABILITY MODELLING OF MULTIPLE-PHASED SYSTEMS

PROBLEMY MODELOWANIA NIEZAWODNOŚCI SYSTEMÓW WIELOFAZOWYCH*

This article discusses the basic problems connected with modelling the reliability of multiple-phased systems. Operation and maintenance of such systems is associated with execution of various tasks which together lead to the achievement of the final goal. Such systems include logistic and transport systems. Two types of models are discussed: synthetic models, which cover a system's entire operating life and models in which the particular phases are considered separately. As an example, a "k out of n" system is used with different k parameters for each phase. The advantages and disadvantages of three models are discussed: the "conservative" model which is based on an analysis of the block reliability model, a Markov model with fixed duration of each phase and a Markov model with random phase duration.

Keywords: *reliability, modeling, multiple-phased system.*

W artykule omówiono podstawowe problemy związane z modelowaniem systemów wielofazowych. Eksploatacja takich systemów związana jest z realizacją różnych zadań, które składają się na osiągnięcie celu końcowego. Do takich systemów można zaliczyć systemy logistyczne i systemy transportowe. Omówiono dwa rodzaje modeli: modele syntetyczne ujmujące cały okres eksploatacji systemu i modele, w których poszczególne fazy są rozpatrywane oddzielnie. Wykorzystano przykładowy system o strukturze progowej zmiennej w kolejnych trzech fazach eksploatacji. Przedstawiono zalety i wady korzystania z modelu „konserwatywnego" bazującego na analizie modelu blokowego i modeli Markowa z ustaloną i losowo zmiennych czasem trwania poszczególnych faz.

Słowa kluczowe: *niezawodność, modelowanie, system wielofazowy.*

1. Introduction

There are many systems whose operating life consists of a series of separate time intervals. In each of those intervals, the system carries out different tasks, the results of which contribute to the achievement of the final goal. Such systems are referred to in the literature (e.g., [4]) as phased mission systems (PMS). Examples of such systems, beside transportation and logistic systems, can be found in many areas of application such as the nuclear power industry, aviation, shipbuilding, the telecommunications industry, the construction industry, electronics, and many others (e.g., [8, 9]). Research done currently [10] concerns, among others, a combined rail/water transport system for the transportation of coal in the corridor of the Oder Waterway.

Because the concept of periodic execution of tasks by such complex systems as a transportation or a logistic system is related to a much wider set of systems than suggested by the concept of *mission*, the term multiple-phased systems was proposed in [4].

In multiple-phased systems, the individual phases may be characterized by many different properties [4]:
- a task executed in a given phase may differ from the tasks executed in the remaining phases,
- the requirements regarding performance and reliability may differ among phases,
- during some of the phases, the system may be subjected to a particularly strong influence of the environment, which may cause a considerable increase in failure rate,
- the structure of a system may change over time depending on the functional and reliability requirements formulated for the currently executed phase,
- proper execution of tasks within a given phase may bring other effects for the system than those obtained in other phases.

2. Reliability models of multiple-phased systems

In system reliability modelling, the use of the concept of a multi-phased system allows better approximation of reality on account of the following assumptions:
- a system's operational structure is not constant; it may change between phases depending on the importance / criticality of a given phase,
- the history of failures or repairs of a given component in a given phase affects the behaviour of the system in the following phase. Hence, the state of a component at the beginning of a given phase depends on the state of the component at the end of the previous phase.
- the criteria defining the level to which the requirements related to performance and reliability are met may differ in a given phase from those for the next phase.

These assumptions are used in various ways in the models known from the literature. There are two types of models: synthetic models, which cover a system's entire operating life and models in which the particular phases are considered separately.

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

Synthetic models, in which all phases are represented together (e.g., [1, 2, 5, 14]), have a number of advantages as they make it possible to use similarities among phases to obtain a compact model in which all phases are adequately embedded. The construction of a synthetic model like that may be neither easy nor convenient in the cases where differences outweigh the similarities among the individual phases.

Separate modelling of each phase (e.g., [7, 14]), in turn, enables immediate characterization of the differences among phases with respect to failure rate and structural requirements. Each phase can be solved separately, and the results obtained can be combined with those from other phases to give total results for the system. The main weakness of the separating approach to modelling (which does not occur in synthetic models) is how it deals with the relationships among phases, which have to be taken into account when distributing components among the phases. Such an approach requires clearly specified mapping of the states of a component at the end of a phase relative to the state of the same component at the beginning of a following phase. A task of this sort is conceptually simple, but may be inconvenient and is a potential source of errors for complex systems.

The most difficult decision in the process of modelling regards the way in which the individual phases are combined into a single model and the way in which the reliability characteristics of the entire system are estimated.

An analysis [11] was conducted for an example system described by the following assumptions:

- a system made up of three components (A, B and C),
- during its operating life, the system goes through 3 successive phases (I, II and III),
- the failure rates for the individual components are constant over the time of duration of a given operational phase, but may differ among the individual phases (λ_i^j, i = A, B, C, j = I, II, III),
- the components can be serviced or repaired; the failure rate in the particular phases is constant, but may also change in successive operational phases (μ_i^j, i = A, B, C, j = I, II, III),
- the system's reliability structure is a "k^j-out-of-n" threshold structure; the parameter k is phase-dependent and is $k^I = 1$, $k^{II} = 2$, $k^{III} = 3$, at $n = 3$.

A system has a "k-out-of-n" threshold structure if the system functions if and only if at least k of the n elements function. A block diagram of such a system is shown in figure 1. In the diagram, k elements are connected in a series and may be replaced with any of the (k+1, n) elements (there are models with more formalized redundancy schemes, e.g., [3]).

It is also easy to show that a threshold structure is a generalization of a series and a parallel structure:

- a "1-out-of-n" structure is a parallel structure,
- an "n-out-of-n" structure is a series structure.

The reliability function of a threshold structure system, assuming that the system is made up of identical components ($R_1(t) = R_2(t) = \ldots = R_n(t) = R(t)$), is expressed by formula:

$$R_s(t) = \sum_{i=k}^{n} \frac{n!}{i!(n-i)!} R(t)^i (1 - R(t))^{n-i} \qquad (1)$$

For modelling a system made up of renewable components, the Markov model is most frequently used – appropriate state

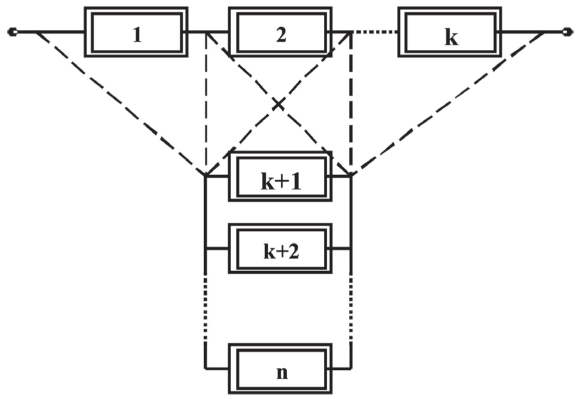

Fig. 1. A block diagram of a threshold structure

graphs for the individual operational phases of an example system are shown in figures 2, 3 and 4. The state of the system has been described using the following notation

$$S = \langle S_A, S_B, S_C \rangle \qquad (2)$$

where: S_i = 0 – failed component, S_i = 1 – working component.

The failed states of the system in the individual phases have been shaded. Emphasis should be given to one of the adopted assumptions [11] saying that no further failures are possible during repair of system components.

2.1. A "conservative" reliability model

The simplest approach to combining phase reliabilities into system reliability is to use a series model of system reliability in which the successive phases of system operation represent structure components. This is possible if the components of the system do not show dynamic changes in properties such as transition errors or incomplete repair [5]. Then, a "conservative" estimate of a system reliability function (a "bottom-up" estimate) is obtained. An example of this kind of structure is shown in figure 5.

The results of calculations of the reliability of such a system, if the components are irreparable, are precise but lead to "bottom-up" estimation of the reliability of the actual system.

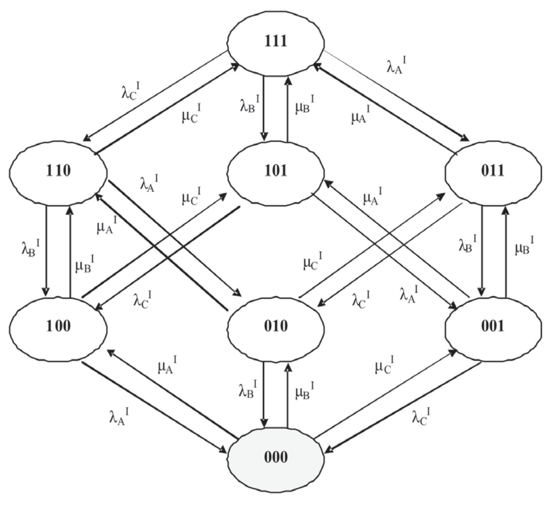

Fig. 2. A Markov model for phase I [11]

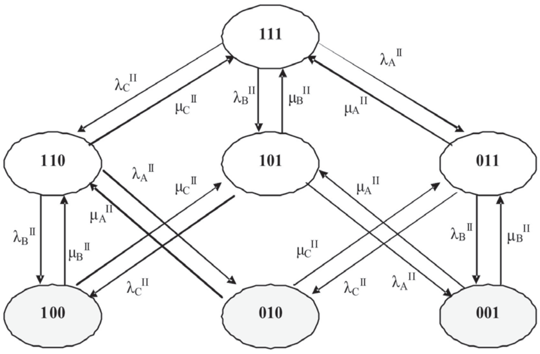

Fig. 3. A Markov model for phase II [11]

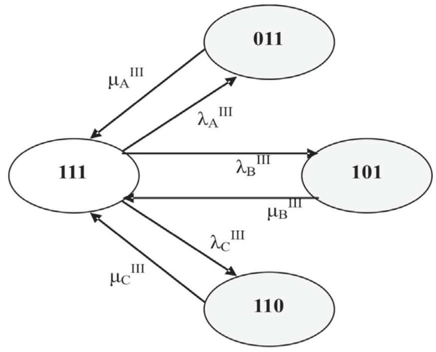

Fig. 4. A Markov model for phase III [11]

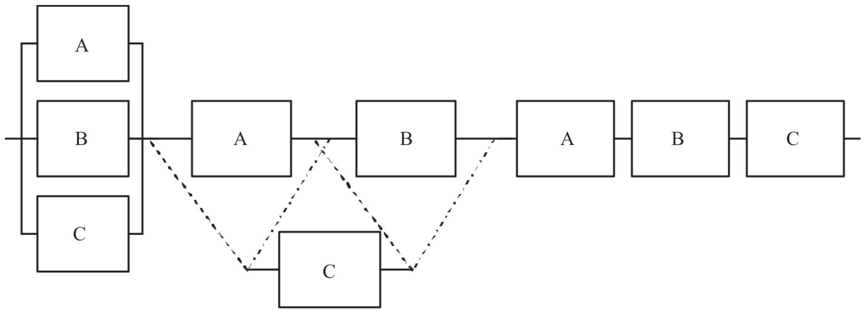

Fig. 5. A structure of system reliability – a series structure of phases [11]

A solution to this problem is to analyze a system using the success path set or the failure cut set methods [6].

The properties of minimal path and minimal cut sets are, among others, as follows [3]:

- the structure of a system can be represented by means of a pseudostructure made up of minimal path sets connected in parallel,
- the structure of a system can be represented by means of a pseudostructure made up of minimal cut sets connected in a series,
- the structure of a minimal path set P_j (j = 1, ..., p) is a series structure,
- the structure of a minimal cut set K_j (j = 1, ..., k) is a parallel structure.

The structure of a system φ can thus be represented by the structures of its minimal paths:

$$\phi(x) = \max_{1\le j\le p}\min_{i\in P_j} x_i \tag{3}$$

which corresponds to the pseudostructure φ_P made up of minimal paths, or by the structures of minimal cuts:

$$\phi(x) = \min_{1\le j\le k}\max_{i\in K_j} x_i \tag{4}$$

which corresponds to the pseudostructure φ_K made up of minimal cuts.

Then, the reliability of the system, which is not worse than the reliability of a system with the pseudostructure φ_K and not better than the reliability of a system with the pseudostructure φ_P can be estimated top-down and bottom-up. The reliabilities of systems with pseudostructures φ_P and φ_K are relatively easy to determine.

$$\max_{1\le j\le p}\left\{P\left(\min_{i\in P_j} x_i = 1\right)\right\} \le R_s \le \min_{1\le j\le k}\left\{P\left(\max_{i\in K_j} x_i = 1\right)\right\} \tag{5}$$

2.2. A Markov model – fixed phase duration

The Markov model may turn out to be an effective tool for reliability analysis of complex systems which show variable behaviour during execution of a task, such as, for example,

- varying transition probabilities between phases or
- a limited number of maintenance kits.

In the proposed modelling strategy, each phase of operation of a multi-phased system is modelled with a separate Markov model. It is assumed that the final reliability state of a system in phase j is the initial state for phase $j + 1$. A process diagram is shown in figure 6.

In phase I, executed over time T_1, working states include, among others, states (111), (101), (110) and (011), and transition to phase II, to analogous working states is possible. States (100), (010) and (001), in turn, will be failed states in phase II, and the final probabilities of being in these states will add up to give the probability of system failure. Obviously, a system may also fail in phase I – state (000). For phase III, only the state in which all components are working (111) is a working state.

Inconveniences of this method of modelling the reliability of multi-phased systems are connected with the difficulties in relating the corresponding states of the system among the individual phases. Other complications arise (e.g., [5]) if a given component is subject to failure in one phase but does not fail in another or when failures in one phase are not diagnosable until the component is used in the subsequent phase.

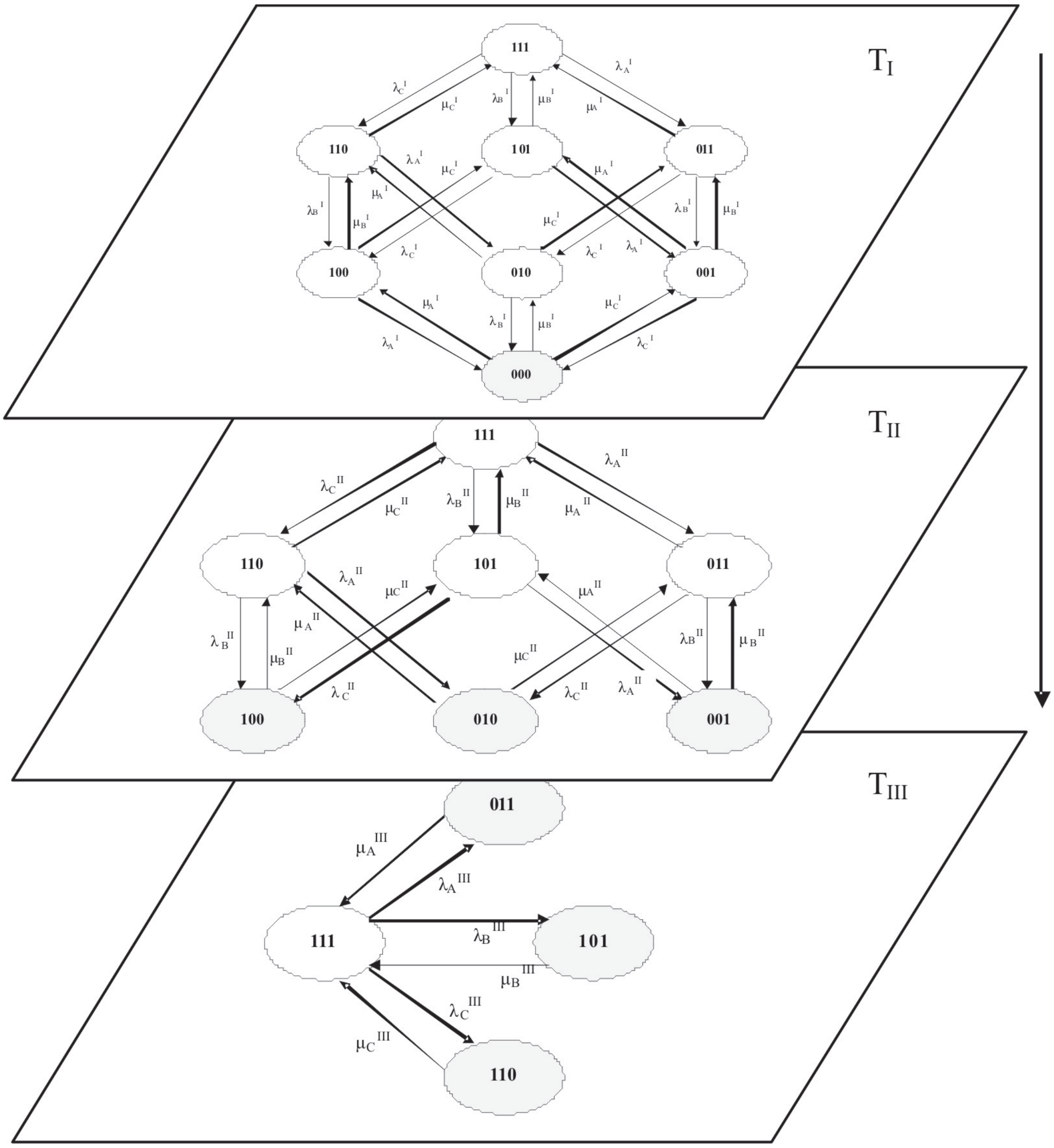

Fig. 6. A diagram of a reliability model with fixed phase duration [11]

A synthetic model of the approach formulated earlier was proposed in [5]. The phase index φ_j was introduced

$$\varphi_j = \begin{cases} 1 & \text{if } (T_{j-1} \le t < T_j) \\ 0 & \text{if } t < T_{j-1},\ t > T_j \end{cases} \quad (6)$$

where: T_j – end-of-phase-*j* time point.

The φ_j index specifies which transition between states belongs to the given phase *j*. The use of this method does not change the state space of a system nor does it require determination of new probabilities for transitions between states. The obtained model is still a Markov model, but is no longer homogeneous – the values of transition probabilities depend on the operating time of a system. An example of such a model is shown in figure 7. To simplify the diagram, only the components' failure rates were taken into account; the notation for repair rates is analogous. For each state number, the number of phase in which this state is a failed state has been given.

2.3. A Markov model – random phase duration

If deterministic duration of the individual phases of system operation cannot be defined and these time periods are random variables, then a non-homogeneous Markov process has to be used for modelling. The approach presented in [14] is based on solution of a single non-homogeneous Markov model in which the concept of state transitions has been generalized to include phase changes.

One merit of this approach is that it can take into account the dependence of phase change on the state of the system and the dependence of failures and repairs on time in the individual phases. There is also no need to correlate the probabilities of being in a state between phases.

The disadvantage of this method is the high level of complexity of the model. The state space can be very large because it represents the sum of the states of all the submodels Because the size of the state space in Markov models is (in the worst

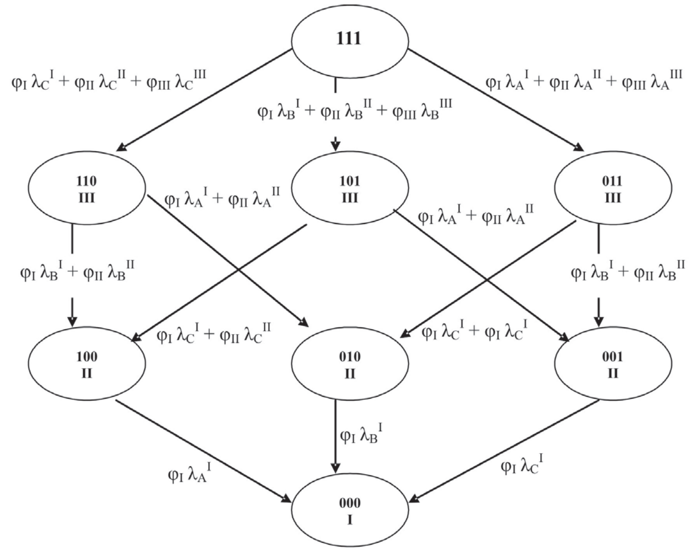

Fig. 7. A synthetic model of a sample multi-phased system [11]

case) an exponential function of the number of components, its growth may be critical for the possibility of modelling.

3. Conclusions

From the models obtained it was possible to estimate the basic measures of system reliability such as, for example, the probability of accurate performance of a logistics task, the probability of the occurrence of an error/failure as a function of task execution time, or the mean time between failures.

The presented approach was used to model the reliability of intermodal transport systems by means of Markov and semi-Markov models (e.g., [12], [13] and [16]). It can be said that the intermodal transport system (from the point of view of reliability modelling) is a standard example of a system with periodic/phased tasks (missions). Transport of loads is divided into phases in which the transportation activities are performed different mean of transport.

Possibilities of reliability and safety modelling of combined transport of coal on Odra River Corridor [10] is shown in table 1.

For example, taking into consideration the whole system, for fixed phase duration of rail and water transport we have obtained the following estimations [10]:

- availability of rail system

$$A(t) = \left[\mu / (\lambda + \mu) \right] + \left[\left(P(t_0)\lambda - (1 - P(t_0)\mu \right) exp(-(\lambda + \mu)t) \right] / (\lambda + \mu) \qquad (7)$$

where: $P(t_0)$ – probability of up state for $t = 0$,

- availability of combined system:

$$A_i(t) = \left[\mu_i / (\lambda_i + \mu_i) \right] + \left[\left(P_i(t_0)\lambda_i - (1 - P_i(t_0)\mu_i \right) exp(-(\lambda_i + \mu_i)t) \right] / (\lambda_i + \mu_i) \qquad (8)$$

$$P_{i+1}(t_0) = A_i(t) \qquad (9)$$

where: i – number of phase

The estimated statistical data are divided into two groups of undesired events of rail transport and waterborne transport (on the first level of the system decomposition). In the case of rail transport these are the events connected with failures and hazards resulting from rail cars operation and maintenance and failures or faults of rail system infrastructure. Whereas for waterborne transport system these are events caused by failures and faults of inland waterway infrastructure and restrictions come from improper water flow and events fixed with failures and maintenance actions of pushers and barges. The difference in failures and hazards structure results from conditions and shares of particular events – see figure 8.

For given results of data analysis, the combined system of coal transportation is more reliable than the rail one, while the total 50 hours mission time is divided on 6 hours rail phase and 46 hours water transport phase – figure 9.

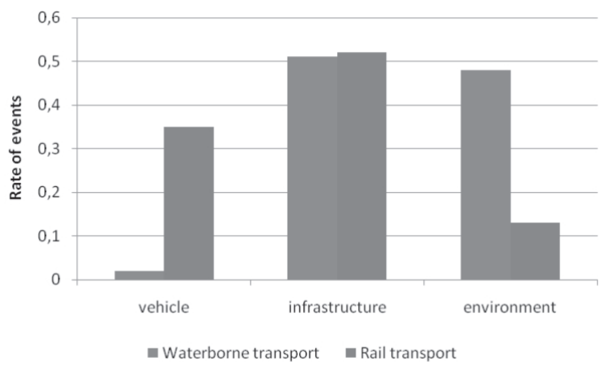

Fig. 8. Rate of undesired events [10]

Table 1. Schema of analysed models of combined transport systems

Transport system	Reliability model		
Rail	Block diagram	Markov	
Rail-water-rail	Block diagram	Markov – fixed phase duration	Markov – random phase duration

One can see that the model correctly shows the effect of change of mean of transport in the 6th hour of the mission and it also shows the increase of the system availability according to higher reliability of water transport. Certainly uncertainly of the assesment of the combined transport availability depends on mentioned uncertainly of input data.

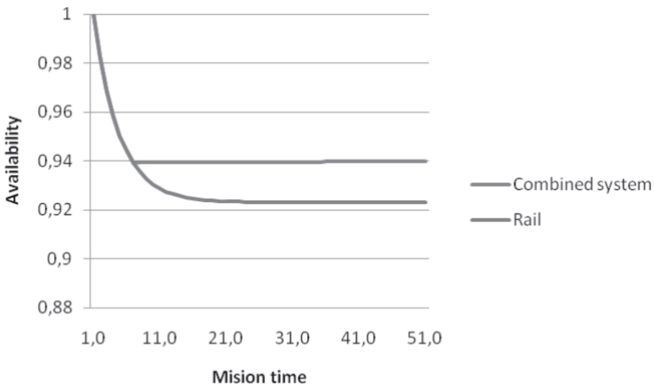

Fig. 9. Results of comparision of rail and combined transportation systems

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