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MODELING THE RELIABILITY AND EFFICIENCY OF FLEXIBLE SYNCHRONOUS PRODUCTION LINE

MODELOWANIE NIEZAWODNOŚCI I WYDAJNOŚCI SYNCHRONICZNEJ ELASTYCZNEJ LINII PRODUKCYJNEJ*

The paper introduces a mathematical model of operation of a flexible synchronous production line (FSPL) of multifunctional CNC machines that includes one redundant multifunctional CNC machine which can take over the functions of every FSPL machine. The graph of FSPL state, relations and equations used to calculate reliability and productivity are shown. Maple, the software used for reliability and productivity calculations and modelling, as well as the mathematical results are presented.

***Keywords**: model, reliability, efficiency, synchronous flexible production line, redundant technological cell.*

Przedstawiono model matematyczny funkcjonowania synchronicznej elastycznej linii produkcyjnej (SELP) z obrabiarek wielozadaniowych CNC, w skład której wchodzi maszyna technologiczna rezerwowa. Maszyna technologiczna rezerwowa może przejmować funkcje każdej z obrabiarek SELP. Przedstawiono graf stanu SELP, zależności, równania do obliczania niezawodności i wydajności SELP. Opracowano program obliczeń niezawodności i wydajności (Maple) oraz zaprezentowano wyniki modelowania i optymalizacji ilości obrabiarek.

***Słowa kluczowe:** model, niezawodność, wydajność, synchroniczna elastyczna linia produkcyjna, maszyna technologiczna rezerwowa.*

1. Introduction

Multi-role CNC machines are mainly designed for processing frame type parts which have many holes with different diameters and precision (class 5 to 11), on which resistance points are based, and additional tools are connected to the frame and to mounting connections (to attach the part using screws or pegs and to facilitate processing, establishing datums, and assembly). The dimensions of the main hole diameters vary within a wide range (from 16 to 50 mm) and depend on the type of part [1, 2, 5, 7]. The work [8] introduces specification of processing and classification of holes for the system of automated design of technological processes. Modern market conditions require production characterised by quick start and quick change of the assortment of produced parts. CNC machine tools and Flexible Production Systems (FPS), combing the high flexibility of traditional equipment and the high efficiency of machine tools, are the most effective equipment for multi-nomenclature production [1, 2, 3, 10, 11].

2. Methodology of modeling FSPL reliability and efficiency

Every multi-role CNC machine tool can be considered as a complex system. If the system contains "n" number of serial connected elements, damage of any of them leads to the failure of the whole system and can be described by graph – fig. 1.

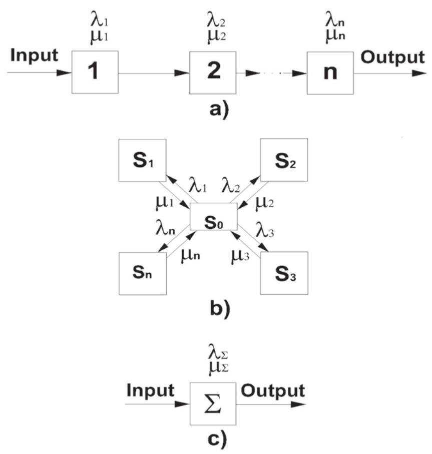

Fig.1. The conditions graph of the multirole CNC machine tool: a) system elements from 1 to n; b) elements conditions; c) machine as the sum of all elements

States on the graph:
S_0 – all n elements of the system are operating,
S_1 – the first element failed and the system is non-operational,
S_2 – the second element failed and the system is out of order;
. . ,
S_n – nth element failed and the system is not working.
Indications on the graph:

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

$\lambda_i, (i = 1, \overline{n})$ – the intensity of failure stream of 1 to n elements.
$\mu_i, (i = 1, \overline{n})$ – the intensity of restoration stream of working ability of 1 to n elements.

Because of the fact that after failure of any element the rest of the elements cannot function properly until the time of restoration of its work ability, it is considered that only one element can fail at a time. All failure and restoration streams are considered as simple.

The system of equations for the qualification of final probabilities is presented below:

$$\begin{aligned} &P_0 \sum_{i=1}^{n} \lambda_i = \sum_{i=1}^{n} P_i \mu_i; \\ &P_1\mu_1 = P_0\lambda_1; \\ &P_2\mu_2 = P_0\lambda_2; \\ &\ldots\ldots \\ &P_i\mu_i = P_0\lambda_i; \\ &\ldots\ldots \\ &P_n\mu_n = P_0\lambda_n. \end{aligned} \qquad (1)$$

The standardization condition:

$$\sum_{j=0}^{n} P_j = 1 \qquad (2)$$

After change of the first equation of system (1) to the standardization condition (2) and solutions, every probability $P_i, (i = 1, \overline{n})$ is expressed by P_0:

$$P_i = P_0 \frac{\lambda_i}{\mu_i} \qquad (3)$$

The set of numbers i is marked as I ($i \in I$). Let us introduce the j, belonging to this set: $j \in I$. With regard of new letters, after the substitution of 3 to the standardization condition 2 the following formula is received:

$$P_0 = \frac{1}{1 + \sum_{j=1}^{n} \frac{\lambda_j}{\mu_j}} \qquad (4)$$

After substitution of (4) to (3):

$$P_i = \frac{\lambda_i}{(1 + \sum_{j=1}^{n} \frac{\lambda_j}{\mu_j})\mu_i} = \frac{\rho_i}{1 + \sum_{j=1}^{n} \rho_j} \qquad (5)$$

where: $\rho_i = \frac{\lambda_i}{\mu_i}, \rho_j = \frac{\lambda_j}{\mu_j}$.

The output system (fig.1) is replaced with the simple two-state element: working and non-work (in the damage condition; non-operational). The diagram of such an element or new system is introduced in fig. 2.

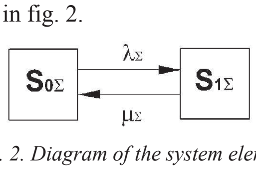

Fig. 2. Diagram of the system element

where λ_Σ is definied as:

$$\lambda_\Sigma = \sum_{i=1}^{n} \lambda_i \qquad (6)$$

The value μ_Σ is defined from dependence:

$$\mu_\Sigma = \frac{P_0}{1 - P_0} \lambda_\Sigma \qquad (7)$$

After substitution of (4) to (7) the following formula is received:

$$\mu_\Sigma = \frac{\lambda_\Sigma}{\sum_{j=1}^{n} \rho_j} \qquad (8)$$

Obtained dependencies allow defining the total intensity of failure stream and total intensity of stream of restoring it to work for the system presented in figure 2, therefore allow modeling of efficiency of system work.

3. The mathematical model of functioning of FSPL with redundant technological cell

At present we use the structure of the flexible synchronous production line (FSPL) from the multi-role CNC machine tools with a redundant technological cell (RTC) which can replace any multi-role machine – technological cell (TC) [4, 6]. Figure 3 introduces the structure of such a flexible system (FSPL). The redundant technological cell (RTC) can replace only one damaged machine (TC), so whole system (FSPL) stops working after failure of two machines (TC).

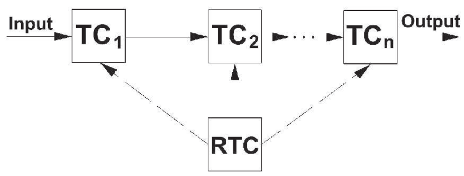

Fig.3. The FSPL structure

The graph of states (FSPL), including RTC, is introduced on fig. 4. States on the graph:
S_0- all multirole machines (TC) are operating; S_1- 1st TC_1 does not operate; S_2- 2nd TC_2 does not operate; … , S_n- n - e TC_n does not operate; $S_{1,1}$ - second TC_2 failure while TC_1 does not operate; . . $S_{1,2}$ - third TC_3 failure while TC_1 does not operate; …; $S_{1,n-1}$ - the n – e TC_n failure while TC_1 does not operate; $S_{2,1}$ - first TC_1 failure while TC_2 does not operate; $S_{2,2}$ - third TC_3 failure while TC_2 does not operate; …; $S_{2,n-1}$ the n - e TC_n failure while TC_2 does not operate; $S_{3,1}$ - first TC_1 failure while TC_3 does not operate; $S_{3,2}$ - second TC_2 failure while TC_3 does not operate; $S_{3,3}$ - (not shown on graph) the TC_4 failure while TC_3 does not operate; ... ; $S_{3,n-1}$ - the n–e TC_n failure while TC_3 does not operate; $S_{i,1}$ - first TC_1 failure while TC_i does not operate; $S_{i,2}$ - second TC_2 failure while TC_i does not operate; … ; $S_{i,n-1}$ - failure n-e TC_n while TC_i does not operate; $S_{n,1}$ - first TC_1 failure while TC_n does not operate; $S_{n,2}$ - second TC_2 failure while TC_n does not operate; … ; $S_{n,n-1}$ - n–1 - e TC_n failure while TC_n does not operate; States S_0, S_1, S_2, S_3, S_4,..., S_n - able to work and remaining for an emergency. Graph clarification: $\lambda_i (i = 1, \overline{n}); \mu_i (i = 1, \overline{n})$ - intensity of the failure stream and restoring the working ability of technological devices TC_i $i = 1, \overline{n})$.

The number of states is considerable (eg. for n = 10 the number of states is $N = n^2 + 1 = 101$, which makes model construction and analysis difficult). That is the approach based on the increase of states is proposed.

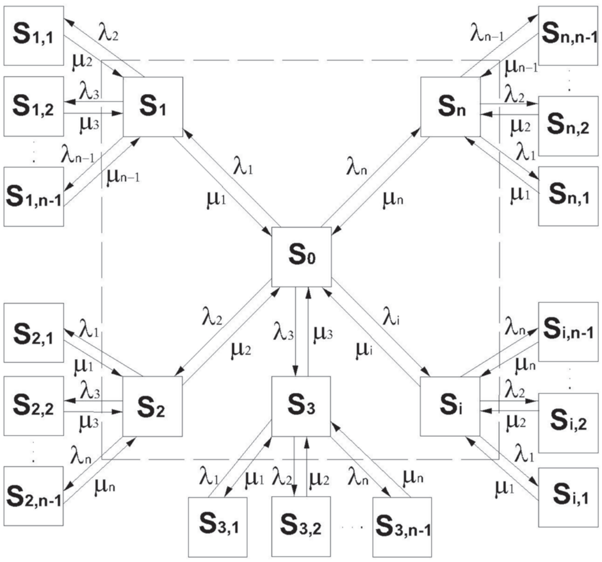

Fig. 4. Graph of FSPL conditions, including one reserved RTC place

We isolate the following subsets in E set (power N):

$$E_1 = \{S_1, S_{1,1}, S_{1,2}, ..., S_{1,n-1}\};\ E_2 = \{S_2, S_{2,1}, S_{2,2}, ..., S_{2,n-1}\};$$

$$E_3 = \{S_3, S_{3,1}, S_{3,2}, ..., S_{3,n-1}\};\ \ldots;\ E_i = \{S_i, S_{i,1}, S_{i,2}, ..., S_{i,n-1}\};$$

$$\ldots;\ E_n = \{S_n, S_{n,1}, S_{n,2}, ..., S_{n,n-1}\}$$

We will qualify the probability of system existence in these subsets. In this case we will consider a diagram of equivalent enlarged system shown in figure 5.

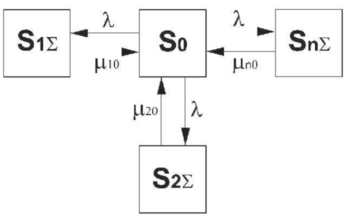

Fig. 5. Diagram of conditions equivalent of enlarged system

States on the diagram (fig. 5):

S_0 - all multirole CNC machine tools are operating; $S_{1\Sigma}$- the system is in one of states of the E_1 subset; $S_{2\Sigma}$- the system is in one of the states of E_2 subset; … ; $S_{n\Sigma}$ - the system is in one of states of the E_n subset. On the graph: $\lambda_i, (i = \overline{1,n})$ - is the intensity of the failure streams $UT_i, (i = \overline{1,n})$; $\mu_{i0}, (i = \overline{1,n})$ - the intensity of the stream restoring the system working ability from subsets $E_i, (i = \overline{1,n})$.

The task consists in defining $\mu_{i0}, (i = \overline{1,n})$. If all diagram (Fig. 4) conditions probabilities are known, then $\mu_{i0}, (i = \overline{1,n})$ can be determined from the dependence:

$$\mu_{i0} = \frac{P_i}{P_i + \sum_{j=1(j\neq i)}^{n-1} P_{ij}} \mu_i \quad (9)$$

where P_{ij} - the states probability of $S_{ij} \in E_i$, the rate before μ_i in (1), equal $\frac{P_i}{P_i + \sum_{j=1(j\neq i)}^{n-1} P_{ij}}$, then conditional probability, that it is include in the subset of states E_i, the system is in the state S_i.

We will mark the component of E set as $S_k, (k = \overline{1,N})$, ($S_k \in E$). Dividing the numerator and the nominative (1) by the probability of system being in the E_i $P_{iz} = P\{S_k \in E_i\} = P_i + \sum_{j=1(j\neq i)}^{n-1} P_{ij}$ subset, we will receive:

$$\mu_{i0} = \frac{P_i}{P_{i\Sigma}} \mu_i = P_{iy}\mu_i \quad (10)$$

where P_{iy} - the conditional probability of system being in the S_i state.

We will qualify the conditional probability of the elements of E_i subsets:

$$P_{iy} = P\{S_k = S_i / S_k \in E_i\}; P_{ijy} = P\{S_k = S_i / S_k \in E_i\}$$

They are equal:

$$P_{iy} = \frac{P_i}{P_{i\Sigma}} \quad (11)$$

$$P_{ijy} = \frac{P_{ij}}{P_{i\Sigma}} = \frac{P_{ij}}{P_i + \sum_{j=1(j\neq i)}^{n-1} P_{ij}} \quad (12)$$

To determine the probabilities P_{iy} and $P_{ijy}, (i = \overline{1,n}; j = \overline{1n-1})$ we should consider subsets $E_i, (i = \overline{1,n})$ as independent subsets. For comfort, the set of numbers j is marked as J, ($j \in J$). Let us introduce numbers m, also belonging to this subset ($m \in J$).

With regard of the new numbers of dependence to determine P_{iy} and P_{ijy} we get:

$$P_{iy} = \frac{1}{1 + \sum_{j=1(j\neq i)}^{n-1} \rho_j} \quad (13)$$

$$P_{ijy} = \frac{\rho_j}{1 + \sum_{m=1(m\neq i)}^{n-1} \rho_m} \quad (14)$$

where $\rho_j = \frac{\lambda_j}{\mu_j}, \rho_m = \frac{\lambda_m}{\mu_m}$ - the imported intensities of streams.

Substituting (13) in (10) we will receive:

$$\mu_{i0} = (1 + \sum_{j=1(j\neq i)}^{n-1} \rho_j)^{-1} \mu_i \quad (15)$$

All intensities in the diagram (Fig.5) are known, and the probability of states $P_0, P_{1\Sigma}, ... P_{i\Sigma}, ..., P_{n\Sigma}$, is defined according to well-known dependences [9]:

$$P_0 = \left[1 + \sum_{i=1}^{n} \rho_i (1 + \sum_{j=1(j\neq i)}^{n-1} \rho_j)\right]^{-1} \quad (16)$$

$$P_{i\Sigma} = \left[1 + \sum_{i=1}^{n} \rho_i (1 + \sum_{j=1(j\neq i)}^{n-1} \rho_j)\right]^{-1} \rho_i (1 + \sum_{j=1(j\neq i)}^{n-1} \rho_j) \quad (17)$$

After calculations according to relations (16) and (17), the graph probability conditions, introduced in Fig. 5, can determine the probability of states $S_i (i = \overline{1,n})$ and $S_{ij} (i = \overline{1,n}; j = \overline{1,n-1})$ of the diagram, introduced in figure 3. According to (11), (12) and (13), (14):

$$P_i = P_{iy} P_{i\sum} = (1 + \sum_{j=1(j\neq i)}^{n-1} \rho_j)^{-1} P_{i\sum} \tag{18}$$

$$P_{ij} = P_{ijy} P_{i\sum} = \frac{\rho_j}{1 + \sum_{m=1(m\neq i)}^{n-1} \rho_m} P_{i\sum} \tag{19}$$

After substituting (17) in (18) and (19) :

$$P_i = \left[1 + \sum_{i=1}^{n} \rho_i (1 + \sum_{j=1(j\neq i)}^{n-1} \rho_j) \right]^{-1} \rho_i \tag{20}$$

$$P_{ij} = \left[1 + \sum_{i=1}^{n} \rho_i (1 + \sum_{j=1(j\neq i)}^{n-1} \rho_j) \right]^{-1} \rho_i \rho_j \tag{21}$$

The whole initial structure of the flexible synchronous line (FSPL) pf multi-role CNC machine tools, including reserve working place (RTC), is replaced through one simplest equivalent element for which the intensities of the failures streams λ_Σ and the restoration of efficiency μ_Σ are known. An element with two states is considered as the simplest: the standby and the working state. A diagram of conditions of such an element is shown in Fig.6.

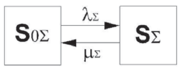

Fig. 6. Graph of FSPL conditions, referred to the simplest element

States in the diagram (fig. 6): $S_{0\Sigma}$ - able to the work; S_Σ - broken (unable to work). We will introduce two new subsets of states for the diagram in figure 4: U - able to work , encircled with dashed line, and V- incapable of working:

$$U = \{S_0, S_1, ..., S_i, ..., S_n\}$$

$$V = \{S_{1,1}, ..., S_{1,j}, ..., S_{1,n-1}, ..., S_{i,1}, ..., S_{ij}, ..., S_{i,n-1}, ..., S_{n,1}, ..., S_{n,j}, ..., S_{n,n-1}\}$$

The subset U answers state $S_{0\Sigma}$ introduced in figure 6, and the subset V - state S_Σ.

The probability of the system being in states $S_{0\Sigma}$ and S_Σ is equal to:

$$P_{0\sum} = P_0 + \sum_{i=1}^{n} P_i \tag{22}$$

$$P_{\sum} = 1 - P_{0\sum} = \sum_{i=1}^{n} \sum_{j=1(j\neq i)}^{n-1} P_{ij} \tag{23}$$

Intensities λ_Σ and μ_Σ for graph introduced on fig. 6 are equal to:

$$\lambda_{\sum} = \sum_{i=1}^{n} (\frac{P_i}{P_{0\sum}} \sum_{j=1(j\neq i)}^{n-1} \lambda_j) = \sum_{i=1}^{n} (\frac{P_i}{P_0 + \sum_{i=1}^{n} P_i} \sum_{j=1(j\neq i)}^{n-1} \lambda_j) \tag{24}$$

$$\mu_{\sum} = \sum_{i=1}^{n} \sum_{j=1(j\neq i)}^{n-1} \frac{P_{ij}}{P_{\sum}} \mu_j = \sum_{i=1}^{n} \sum_{j=1(j\neq i)}^{n-1} \frac{P_{ij}}{\sum_{j=1}^{n} \sum_{j=1(j\neq i)}^{n-1} P_{ij}} \mu_j \tag{25}$$

The developed model for determining the reliability and efficiency of FSPL allows replacing any technological machine in line with redundant technological machine.

4. The software for defining efficiency of FSPL

The program for defining the parameters of functioning of synchronous FSPL was written in the mathematical software for analytic calculations – Maple. This environment is a powerful computer tool, able the solve complex mathematical tasks. It contains tools related to many mathematical fields (algebra, discreet mathematics, differential and integral mathematics, numerical and different methods) and also allows graphical representation, and connection to external modules and programming tools.

The components of the program:
- block pattern of input data,
- block of calculation of required parameters functions of the synchronous line with (without) the reserve place,
- block of formatting results of the experiment and output for these results,

Input data to the execution of research:
- maximum number of cells in the line - N,
- intensity of the stream of damage λ_i and restoring the working ability μ_i of every unit ($i = \overline{1,N}$),
- average time of service for every production individual cell t_i $i = \overline{1,N}$,
- step of calculations Δn (total number equal to the difference between the values of two of the current number of cells in line n of neighbouring cycles).

The block of calculations comprised the following operations:
- defining intensities of streams $\rho_i = \frac{\lambda_i}{\mu_i}$, $i = \overline{1,N}$,
- qualification of the intensity μ_{i0} according to dependence (15),
- calculation of the probability P_0 according to dependence (16),
- calculation of the probability $P_{i\Sigma}$, P_i, P_{ij} according to dependences (17), (18), (21), respectively,
- qualification of the rate of readiness of the line $K_g = P_{0\Sigma}$ according to dependence (22),
- calculation of the efficiency of the line: $Q = \frac{K_g}{t_{max}}$, where t_{max} – maximal time among average times of service for every production cell t_i ($i = \overline{1,N}$,
- defining the parameters of functioning of the synchronous line not including the reserve place:
  - rate of readiness of the line $K_g^{'} = \frac{1}{1+\Sigma\rho_i}$,
  - efficiency of the line $Q^{'} = \frac{1}{t_{max}} K_g^{'}$.
- calculation of current values
  - increase of the coefficient of readiness of the line
    - as absolute value $\Delta K_g = K_g - K_g^{'}$
    - in percentages $\delta K_g = \frac{\Delta K_g}{K_{g\max}, K_{g\max}^{'}} 100\%$

- increase of the efficiency of the line
  - as absolute value $\Delta Q = Q - Q^{'}$
  - in percentages $\delta Q = \frac{\Delta Q}{Q_{\max}, Q^{'}_{\max}} 100\%$

These calculation are taken cyclically until the condition $n = N$ is not met. After that the programme works out the results of the experiment (increase of efficiency) and presents the results on the screen as a matrix and a chart.

5. Results of calculations of reliability and efficiency parameters of FSPL

Above presented software for mathematical calculations (Maple) was used to determine the increase of efficiency of FSPL. Calculations were conducted for sets of different input data of FSPL containing 10 machine tools.

The algorithm of calculating parameters of FSPL that contains multipurpose CNC machine tools was worked out. The following parameters were defined as input parameters for modeling in all considered cases:

- intensity of failure stream ***λ***,
- intensity of restoring to work stream ***μ***,
- average service time of machine tool ***t***.

Calculations of increase of line efficiency were conducted for maximum number of TC equal to 10:

Case 1:

Calculations were conducted for constant parameters of reliability and service:

$$\lambda = 0{,}2\ [h^{-1}],\ \mu = 5\ [h^{-1}],\ t = 0{,}1\ [h]$$

Results of the increase of efficiency calculations conducted in Maple are presented as matrixes and graph in figure 7.

The increase of efficiency ΔQ is almost linear in whole range of numers of machines tools (1 to 10) in FSPL.

Case 2:

Calculations were conducted for constant parameters of reliability and service:

$$\lambda = 0{,}25\ [h^{-1}],\ \mu = 4\ [h^{-1}],\ t = 0{,}1\ [h].$$

Results of the increase of efficiency calculations conducted in Maple are presented as matrixes and graph in figure 8.

Significant and almost linear increase of efficiency of FSPL ΔQ was observed when increasing number of machine tools (up to 8), after further increasing number of machine tools the efficiency decreases.

Case 3:

Calculations were conducted for constant parameters of reliability and service:

$$\lambda = 0{,}3\ [h^{-1}],\ \mu = 3\ [h^{-1}],\ t = 0{,}1\ [h].$$

MATRIX OF RESULTS	
NUMBER OF MACHINE TOOLS IN LINE [pcs.]	EFFICIENCY OF LINE [pcs./h]
1	3.84615384615385
2	7.13305898491084
3	9.94897959183673
4	12.3662306777646
5	14.4444444444444
6	16.2330905306972
7	17.7734375000000
8	19.1000918273646
9	20.2422145328720
10	21.2244897959184

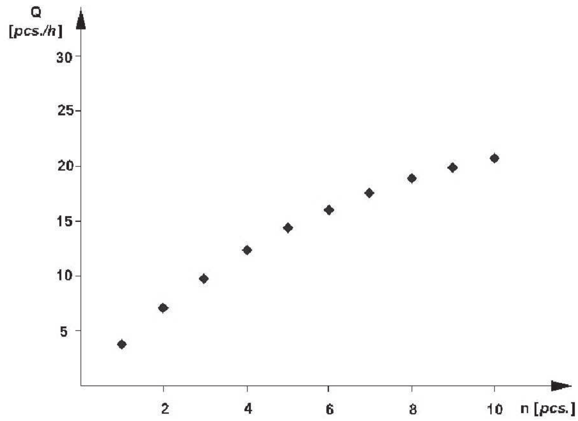

Fig. 7. Dependence of quantity of machine tools on efficiency of line

MATRIX OF RESULTS	
NUMBER OF MACHINE TOOLS IN LINE [pcs.]	EFFICIENCY OF LINE [pcs./h]
1	5.88235294117647
2	10.4938271604938
3	14.1274238227147
4	17.0000000000000
5	19.2743764172336
6	21.0743801652893
7	22.4952741020794
8	23.6111111111111
9	24.4800000000000
10	25.1479289940828

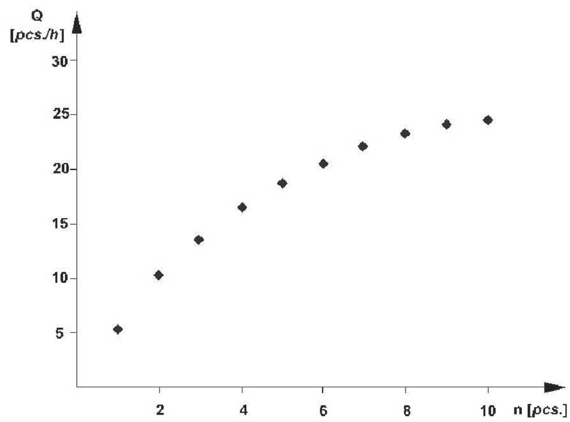

Fig. 8. Dependence of quantity of machine tools on efficiency of line

Results of the increase of efficiency calculations conducted in Maple are presented as matrixes and graph in figure 9.

MATRIX OF RESULTS	
NUMBER OF MACHINE TOOLS IN LINE [pcs.]	EFFICIENCY OF LINE [pcs./h]
1	9.09090909090909
2	15.2777777777778
3	19.5266272189349
4	22.4489795918367
5	24.4444444444444
6	25.7812500000000
7	26.6435986159170
8	27.1604938271605
9	27.4238227146814
10	27.5000000000000

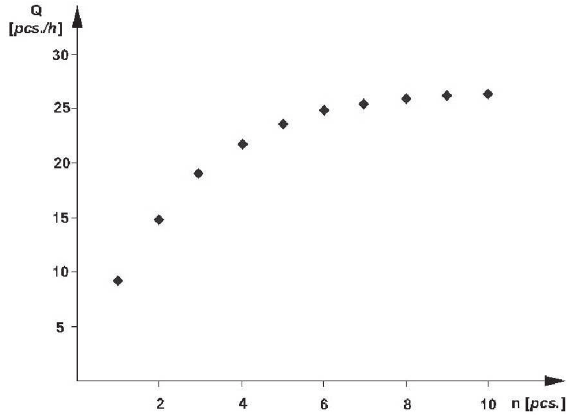

Fig. 9. Dependence of quantity of machine tools on efficiency of line

The efficiency increases steeply right up to the maximum, then decreases together with increasing the number of TM in the line and the diagram can practically reach zero. It is obvious because by significant increasing the number of TM in FSPL, one redundant machine tool can not manage to replace the fixed number of TM, well the productivity of FSPL with redundant machine tool is equal the productivity of FSPL without redundant machine tool.

Significant and almost linear increase of efficiency of FSPL ΔQ was observed when increasing number of machine tools (up to 5), after further increasing number of machine tools the efficiency considerably decreases.

6. Defining the optimal number number of machines in FSPL containing redundant machine tool

The process of running the flexible synchronous production line which consists of consecutively connected technological machines with one redundant technological machine was examined (Fig. 3).

The line consists of technological machines of one type in number of n (TM_1, …, TM_n) and one redundant technological machine which is able to replace every single machine of TM system.

The model of functioning the FSPL is presented in [4] but the whole line's structure is changed, according to reliability parameters, into the simple equivalent element with two states (working and emergency), defining its functioning indexes:

1) the intensity of streams of failure and restoration of working ability;
2) expected value of the production unit service time;
3) availability factor;
4) efficiency ratio taking into consideration reliability parameters.

By creating the model it was assumed that all streams which carry the system from one state to another are simple and service times are disposed exponentially. However the intensity of failure stream λ_p, restoration stream of working ability μ_i and service times t_i are different for every TM_i.

The given model differs from the one in references [6] where those factors were equal.

The aim of the model was to define the productivity gain that is the difference of productivity of FSPL with redundant machine tool and productivity without it:

$$\Delta Q = Q - Q'$$

The analysis of the results obtained by applying the model shows, that by increasing the number of machine tools in the line the diagram of the productivity gain is similar to figure 10.

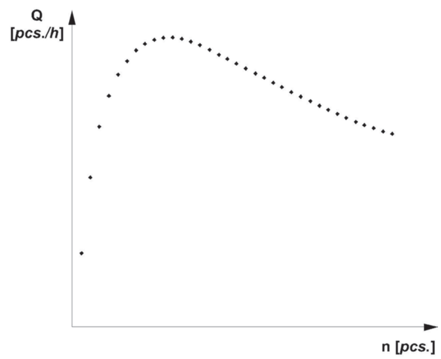

Fig. 10. The theoretical graph of dependence of quantity of the TM's in line on the efficiency

- That optimization of the task where it is necessary to define the number of TM in FSPL with redundant machine tool will allow obtaining the maximum productivity gain. This is the task of integer programming without reservation (taking into consideration that number of TM is even). The solution doesn't require any specially developed algorithm, it is better to choose the quick choice process which consists of repeatable procedure in cycles, in each step j of the mathematical model [6] for the current number of technological machines TM_j it defines the availability factor of FSPL with redundant machine tool and without it and also its productivity (Q_j and Q'_j accordingly). The productivity gain Q_j is defined. If $\Delta Q_j > \Delta Q_{j-1}$, then the current number of TM in the line - n_j is assumed as the optimum point $n_{\text{ïïö}}$. Otherwise the cycle can be stopped.

The conditioning of unimodular target function (there is the only extreme that is the global extreme) is confirmed by findings of research with different parameters. In that procedure the quantity j is changed from 1 to $n_{ñïò}$ and at the beginning of the cycle $n_{ñïò}=1$. The program of searching for optimal number of technological machines is realized in the mathematical software Maple 9. The intensity of failure and restoration stream of working ability and also the service times for every single machine tool are assumed to be equal. The presented below research is connected with defining the influence of reliability parameters on optimal number of machine tools in the line by fixed service times [7].

1. Reliability parameters were analyzed depending on the intensity of failure of every single machine tool λ_i = 0,25…0,35 h^{-1} with constants μ_i = 3 h^{-1} and t_i = 0,05 h (Fig. 11).

Maximum productivity gain and optimal number of machine tools are:

- case 1: ΔQ_{max} = 27,90 pcs /h, n_{opt} = 9 machine tools,
- case 2: ΔQ_{max} = 27,50 pcs /h, n_{opt} = 10 machine tools,
- case 3: ΔQ_{max} = 27,08 pcs /h, n_{opt} = 12 machine tools.

2. Reliability parameters were analyzed depending on the intensity restoration stream of working ability of every single machine tool μ_i = 3, 4, 5 h^{-1}, with constant λ_i = 3 h^{-1} and t_i = 0,1 h (Fig. 12).

Maximum productivity gain and optimal number of machine tools are:

- case 1: ΔQ_{max} = 27,10 pcs /h, n_{opt} = 10 machine tools,
- case 2: ΔQ_{max} = 26,50 pcs /h, n_{opt} = 12 machine tools,
- case 3: ΔQ_{max} = 26,10 pcs /h, n_{opt} = 14 machine tools.

The analysis of obtained results proved that all graphs do have one extremum. The efficiency gain in all cases initially increases when increasing number of TMs in FSPL, reaches maximum and decreases after that.

7. Conclusions

A methodology for modelling CNC machine tools and FSPL is presented. A mathematical model of machine tools and FSPL as a structure of elementary technological cells has been developed. The obtained results relate to machining in flexible systems for Markov chains. All calculations were conducted for universal CNC machining centre KORRADI VH 1000, included in the production line for machining engine blocks. Simulations were performed for FSPL that incorporates a vertical machining centre CINCINNATI SABRE 1000 and a vertical machining centre CINCINNATI ARROW 1000.

After simulation following results were obtained:

- maximal efficiency gain ΔQ_{max} = 27,90 pcs./h optimal number n_{opt} = 9 machine tools, for following parametes of intensity of failure stream of λ_i = 0,25 … 0,35 h^{-1} and constant μ_i = 3 h^{-1} and t_i = 0,05 h.
- maximal efficiency gain ΔQ_{max} = 27,50 pcs./h optimal number n_{opt} = 10 machine tools, for following parametes of intensity of restoring to work stream of λ_i = 3 … 5 h^{-1} and constant μ_i = 3 h^{-1} and t_i = 0,1 h.

The presented values of intensity of streams of each machine tool damage λ_i , restoration to work μ_i and t_i were obtained in industry conditions. The obtained results of simulations of gain in productivity and the optimum numbers of machine tools indicate that with deterioration in reliability parameters there is a decrease in the optimum number of machine tools, but for a specific number of machine tools the gain in productivity is higher than for a line with analogous parameters of maintenance and better indices of reliability.

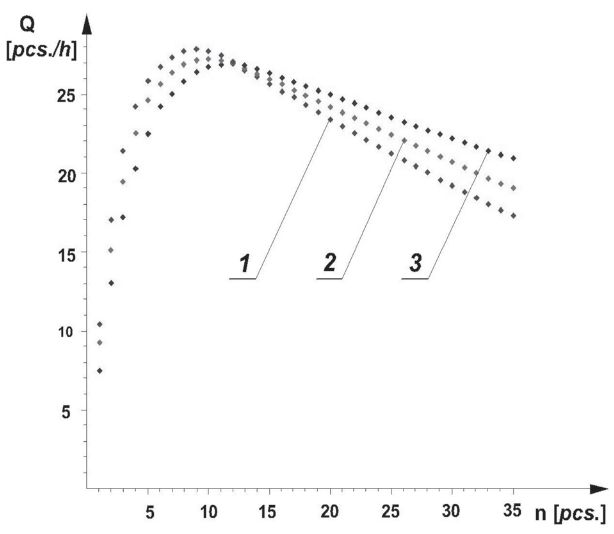

Fig.11. The graph of efficiency vs. number of TM's by following parameters: 1) λ_i = 0,35 h^{-1}, μ_i = 3 h^{-1}, t_i = 0,05 h, 2) λ_i = 0,30 h^{-1}, μ_i = 3 h^{-1}, t_i = 0,05 h,3) λ_i = 0,25 h^{-1}, μ_i = 3 h^{-1}, t_i = 0,05 h.

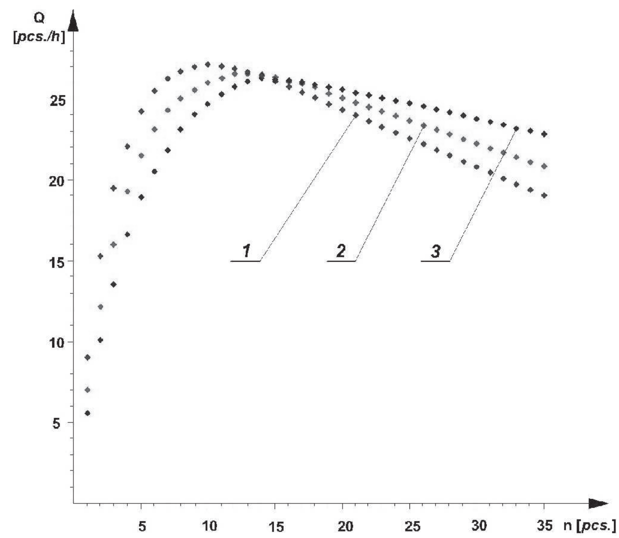

Fig.12. The graph of efficiency vs. number of TM's by following parameters:1) μ_i = 5 h^{-1}, λ_i = 3 h^{-1}, t_i = 0,1 h, 2) μ_i = 4 h^{-1}, λ_i = 3 h^{-1}, t_i = 0,1 h,3) μ_i = 3 h^{-1}, λ_i = 3 h^{-1}, t_i = 0,1 h

8. References

1. Bai Y, Jia X, Cheng Z. Group optimization models for multi-component system compound maintenance tasks. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2011; 1(49): 42–47.
2. Burduk A. Próba adaptacji szeregowych struktur niezawodnościowych do analizy i oceny ryzyka systemów produkcyjnych. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2010; 3(47): 85–96.
3. Farooq S, O'Birien C. Risk calculations in the manufacturing technology selection process. Journal of Manufacturing Technology Management 2010; vol. 21, 1: 28–49.
4. Filipowicz O, Mazurek L, Taranenko V, Świć A. Model matematyczny funkcjonowania elastycznej linii produkcyjnej. Pomiary. Automatyka. Robotyka 2007; 2: 1–11.
5. Halas W, Taranenko V, Swic A, Taranenko G. Investigation of influence of grinding regimes on surface tension state. Lecture Notes In Artificial Intelligence, Vol. 5027. Berlin, Heidelberg: Springer – Verlag 2008: 749–756.
6. Mazurek L, Filipowicz O, Taranenko W, Świć A. Model procesu przezbrajania wielozadaniowych obrabiarek NC w elastycznym systemie produkcyjnym. Przegląd Mechaniczny 2007; 5/S: 104–106.
7. Mazurek L, Świć A, Filipowicz O, Taranenko W. Zwiększenie efektywności pracy obrabiarek wielozadaniowych w elastycznych systemach produkcyjnych. Lublin: Politechnika Lubelska, 2010.
8. Mazurek L, Swic A, Taranenko V. Holes processing and classification in automated technological process projecting system. Acta Mechanica Slovaca**,** Journal published by Faculty of Mechanical Engineering, the Technical University in Kosice, Kosice 2006; 2-A, 325 – 330.
9. Świć A, Taranenko V. Projektowanie technologicznych systemów produkcyjnych. Lublin: Wydawnictwo Politechniki Lubelskiej, 2003.
10. Taranenko G, Taranenko W, Świć A, Szabelski J. Modelowanie układów dynamicznych obróbki skrawaniem wałów o małej sztywności. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2010; 4 (48), s. 4–15.
11. Wang Z, Kang R, Xie L. Dynamic reliability modeling of systems with common cause failure under random load. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2009; 3(43): 47–54.

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