

# MODELE OPTIMALIZACJI GRUPOWEJ DLA ZŁOŻONYCH ZADAŃ OBSŁUGOWYCH DOTYCZĄCYCH SYSTEMÓW WIELOSKŁADNIKOWYCH

## GROUP OPTIMIZATION MODELS FOR MULTI-COMPONENT SYSTEM COM- POUND MAINTENANCE TASKS

*W ostatnich latach prowadzi się coraz więcej badań w zakresie optymalizacji eksploatacji systemów wieloskładnikowych, czego wynikiem są licznie proponowane metody optymalizacji oraz modele matematyczne. Jednakże najczęściej bada się proste zadania obsługowe, a rzadko występujące w praktyce zadania złożone, wymagające kilku rodzajów obsługi. W artykule przedstawiono strategię obsługi grupowej służącą optymalizacji przerw na złożone czynności obsługowe w systemach wieloskładnikowych oraz zaproponowano etapy i metody optymalizacji. Przeprowadzono analizę struktury kosztów utrzymania systemu oraz wyznaczono modele kosztów w celu optymalizacji przerw na złożone czynności obsługowe. Wydajność proponowanych modeli zilustrowano przykładem numerycznym.*

**Słowa kluczowe:** złożone czynności obsługowe, optymalizacja grupowa, system wieloskładnikowy, zależność ekonomiczna, przerwa konserwacyjna.

*More and more researches have been made on maintenance optimization of multi-component system in recent years, and a lot of optimization methods and mathematical models have been proposed. However, the maintenance tasks in present researches are mostly simplex, while the compound maintenance tasks integrating several kinds of maintenance types that exist in practice are seldom studied. To optimize the compound maintenance intervals of multi-component system, the group maintenance strategy is introduced in this paper, and the optimization steps and methods are proposed. The maintenance cost structure and composition are analyzed from system point of view, and the cost models to optimize the compound maintenance intervals are established. Finally, a numerical example is presented to illustrate the efficiency of the proposed models.*

**Keywords:** compound maintenance, group optimization, multi-component system, economic dependency, maintenance interval.

### 1. Introduction

With the development of modern devices and equipments, the number of their components is becoming more and more, and the structures and relationships between components are becoming more and more complex, which result in so-called "multi-component system" consisting of multiple dependent components [4]. Different from the single component system or simple system with independent components, interactions between components complicate the maintenance modeling and optimization. However, the interactions also offer the opportunity to group maintenance tasks, reduce maintenance costs, and improve availability further [11].

The present researches on multi-component system maintenance are primarily based on the stochastic, structural or economic dependency between components [11]. This paper exclusively deals with multi-component system with economic dependency. Economic dependency implies the maintenance costs can be saved when several components are jointly maintained instead of separately [12]. Many relevant researches have been done for the maintenance optimization of multi-component system with economic dependency. References [1, 13, 14] adopted the fixed group maintenance strategy, and optimized the intervals of block replacement, minimal repair and preventive replacement. References [3, 5, 6, 10, 15, 16] focused on the optimized group maintenance strategy. Among

them, reference [10] proposed a heuristic approach to group the maintenance tasks of periodic replacement; reference [3] dealt with the joint execution of traditional periodic replacement and functional check considering potential failure; and reference [16] eliminated the maintenance tasks unworthy of grouping with the principle of maximum gradient, which optimized the optimal solution further.

However, the maintenance tasks in above researches are all simplex, such as periodic replacement, functional check, operational check, and so on. In practice, there still exist the compound maintenance tasks. The compound maintenance means the maintenance mode integrating two or more kinds of maintenance types. For example, the maintenance policy for the transmissions of the locomotive is usually under periodic major repair with some times of preventive minor repair [8]. The researches on such type of maintenance mode are little.

Because functional check and periodic replacement are typical in practical maintenance, the maintenance mode of periodic replacement with functional checks is illustrated to study the multi-component system compound maintenance optimization. The mathematical models are established for expected system maintenance cost per unit time, and the intervals of functional check and the inspection times in a periodic replacement span are optimized, which minimize the whole system maintenance cost.

**2. The group optimization strategy of multi-component system compound maintenance tasks**

**2.1. Periodic replacement with functional checks**

The detailed process of periodic replacement with functional checks is as follows: the component is preventively replaced with the interval of  $T_r$ , and between successive replacements the functional checks are implemented with the interval of  $T_n = T_r/k$ , which means there are  $(k-1)$  times of inspections before the replacement (see fig. 1). When carrying on functional checks, if a potential failure is identified, preventive maintenance should be adopted; if not, the component will continue to work until either a failure occurs or the next check. During the replacement period, if a functional failure occurs, the item should be repaired.

Through compound maintenance, the component life can be made full use of, the failure rate can be effectively reduced, and the maintenance cost can be greatly saved in practical maintenance.

**2.2. The group optimization strategy of compound maintenance**

Group maintenance is a maintenance optimization strategy fit for multi-component system. Under this strategy, an occasion for preventive maintenance is determined at a basis maintenance interval, then each components is maintained at an integer multiple of this interval [16]. From the viewpoint of

system availability or cost, the group maintenance is an effective method to optimize multi-component system maintenance tasks, and it is especially suitable when the overhaul or set-up costs are relatively high.

For compound maintenance tasks, the inspection and replacement intervals of the system should be determined first, to which the maintenance time of the components should then be adjusted, thus some maintenance tasks can be carried out simultaneously, and the times of breakdown and set-up costs could be reduced (see fig. 2).

**3. The group optimization models of multi-component system compound maintenance**

**3.1. Modeling notation and assumption**

- The run time of the system is far longer than its maintenance interval;
- The failures of the components occur independently with single failure mode;
- Inspection is perfect in that any potential failure present will be identified at an inspection time;
- The system consists of  $L$  components. The inspection and replacement intervals of component  $i$  are respectively  $T_{ni}$  and  $T_{ri}$  before grouping, and  $T_{Sni}$  and  $T_{Sri}$  after grouping;
- $U_i$ : The time when potential failure of component  $i$  arises, and its *p.d.f* and *c.d.f* are denoted by  $g_i(u)$  and  $G_i(u)$ , respectively;

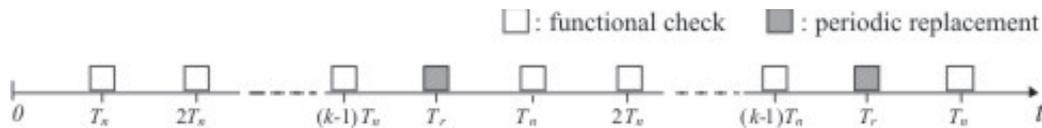


Fig. 1. The sketch map of periodic replacement with functional checks

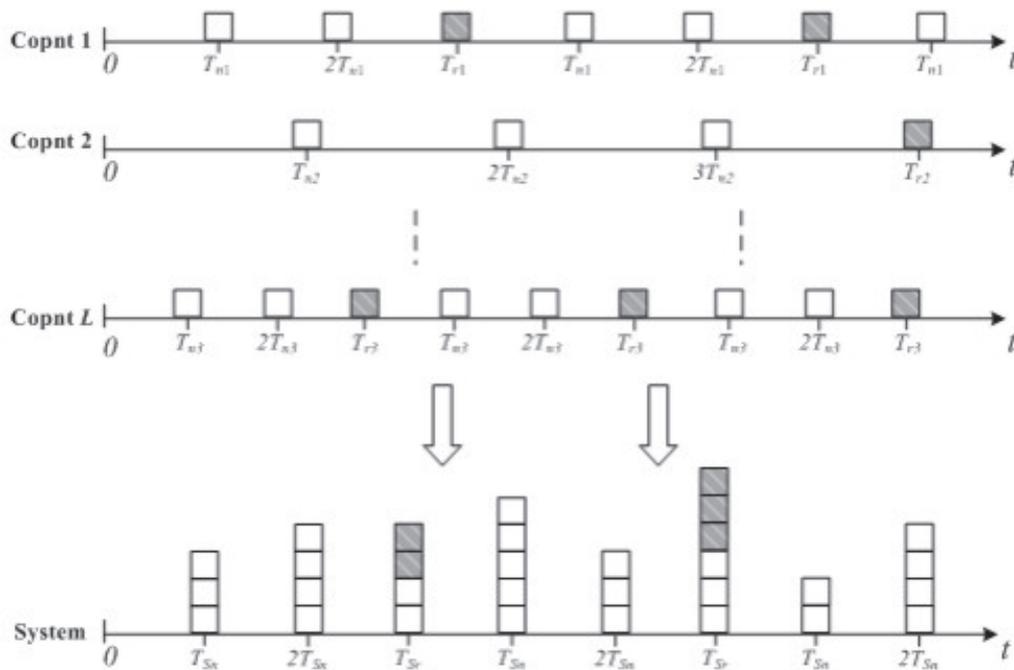


Fig. 2. The group optimization strategy of multi-component system compound maintenance tasks

- $H_i$ : The delay time of component  $i$  from potential failure to functional failure, and its  $p.d.f$  and  $c.d.f$  are denoted by  $f_i(h)$  and  $F_i(h)$ , respectively;
- $C_{ri}$ : the periodic replacement cost of component  $i$ ;
- $C_{ni}$ : the functional check cost of component  $i$ ;
- $C_{pi}$ : the inspection renewal cost of component  $i$ ;
- $C_{fi}$ : the failure renewal and failure loss cost of component  $i$ ;
- $D_{ri}$ : the cost of set-up and system shutdown loss for periodic replacement of component  $i$ ;
- $D_{ni}$ : the cost of set-up and system shutdown loss for functional check of component  $i$ ;
- $D_{ji}$ : the cost of set-up and system shutdown loss for the  $j$ th maintenance tasks package.

**3.2. The compound maintenance interval optimization of single component**

According to the optimization process proposed in 2.3, it starts from the analysis of maintenance cost of single component's periodic replacement with functional check. Considering from the aspect of single component, the component is under the periodic replacement policy in infinite time horizon. From the renewal reward theorem, component  $i$ 's mean cost per unit time can be expressed as:

$$\frac{\text{Total expected cost in one periodic replacement cycle}}{\text{The length of one periodic replacement cycle}}$$

That is  $CR_i(T_{ri}) = \frac{CP_i(T_{ni}, T_{ri}) + C_{ri} + D_{ri}}{T_{ri}}$  (1)

Where  $CP_i(T_{ni}, T_{ri})$  denote component  $i$ 's expected cost during  $[0, T_{ri}]$  with  $T_{ni}$  as its functional check interval. As can be seen, the key of the equation is  $CP_i(T_{ni}, T_{ri})$ .

During every periodic replacement period  $[0, T_{ri}]$ , the component  $i$  is under functional check policy in finite time horizon, and the inspection time is  $k_i = \left\lceil \frac{T_{ri}}{T_{ni}} \right\rceil - 1$  ( $\lceil * \rceil$  means the upper

bound integer of  $*$ ). The cost  $CP_i(T_{ni}, T_{ri})$  is made up of the following three mutually exclusive events:

Event A: Neither inspection renewal nor failure renewal occurs during periodic replacement period, that is, there is no renewal event over  $T_{ri}$ . The cost can be expressed as  $k_i \cdot C_{ni}$ , and may be resulted from the following two cases:

Case 1: there is no potential failure occurring before  $T_{ri}$ , i.e.  $U \geq T_{ri}$ ;

Case 2: a potential failure occurs at  $u$  between the last two checks, and there is no functional failure occurring before replacement, i.e.  $k_i T_{ni} < U_i < T_{ri} \cap U_i + H_i > T_{ri}$ .

Therefore, we have the probability of no renewal event occurring before  $T_{ri}$

$$P_{ni}(T_{ri}) = 1 - \int_0^{T_{ri}} g_i(u) du + \int_{(k_i-1)T_{ni}}^{T_{ri}} g_i(u) [1 - F_i(T_{ri} - u)] du$$
 (2)

Event B: Renewal events occur, and the first renewal is an inspection renewal at the  $l$ th inspection. The cost can be expressed as:  $l \cdot C_{ni} + C_{pi} + CP_i(T_{ni}, T_{ri} - l T_{ni})$ .

To derive the probability of an inspection renewal at time  $lT$ , we note that the condition for a defect occurring in  $(u, u+du)$

$((l-1)T_{ni} < u < lT_{ni})$  and being identified at the  $l$ th inspection is a combination of the following events:

- The defect didn't occur before  $(l-1)T_{ni}$  and was identified at the  $l$ th inspection.
- The delay time of the defect must be longer than  $lT_{ni} - u$ .

The probability of this event is  $g_i(u)du[1 - F_i(lT_{ni} - u)]$ . Integrating all possible  $u$  between  $((l-1)T_{ni}, lT_{ni})$ , we have the probability of a defect being identified at inspection  $lT$  as

$$P_{mi}(lT_{ni}) = \int_{(l-1)T_{ni}}^{lT_{ni}} g_i(u) [1 - F_i(lT_{ni} - u)] du$$
 (3)

Event C: Renewal events occur, and the first renewal is a failure renewal at time  $x$  ( $(j-1)T_{ni} < x < jT_{ni}$ ). The cost can be expressed as:  $(j-1)C_{ni} + C_{fi} + CP_i(T_{ni}, T_{ri} - x)$ .

To derive the  $p.d.f$  of a failure renewal at time  $x$ , we assume that a defect arises at  $(u, u+du)$  ( $(j-1)T_{ni} < u < jT_{ni}$ ), for it is to become a failure in  $(x, x+dx)$ , the delay time  $h$  must satisfies  $x-u < h < x+dx-u$ . So the probability density is:

$$p_{bi}(x) = \int_{(j-1)T_{ni}}^{jT_{ni}} g_i(u) f_i(x-u) du$$
 (4)

Plus the system shutdown and set-up activities, we have the function of expected maintenance cost over  $T_{ri}$  as:

$$CP_i(T_{ni}, T_{ri}) = k_i \cdot C_{ni} \cdot P_{ni}(T_{ri}) + \sum_{l=1}^{k_i} [l \cdot C_{ni} + C_{pi} + CP_i(T_{ni}, T_{ri} - lT_{ni})] \cdot P_{mi}(l \cdot T_{ni}) + \sum_{j=1}^{k_i+1} \int_{(j-1)T_{ni}}^{jT_{ni}} [(j-1)C_{ni} + C_{fi} + CP_i(T_{ni}, T_{ri} - x)] \cdot p_{bi}(x) dx + k_i \cdot D_{ni}$$
 (5)

By the equations (1)-(5), we can get the function of component  $i$ 's mean cost per unit time, then the optimal interval  $T_{ni}$  and  $T_{ri}$  can be obtained.

**3.3. The adjustment of compound maintenance interval**

The maintenance intervals need optimization from the viewpoint of system to obtain  $T_{Sn}$  and  $T_{Sr}$ . The adjustment rules of maintenance intervals for component  $i$  are as follows: the

interval of replacement  $T_{Sri} = \left\lceil \frac{T_{ri}}{T_{Sr}} \right\rceil \cdot T_{Sr} = N_{ri} \cdot T_{Sr}$  ( $T_{Sr} \leq T_{ri}$ ), the

interval of inspection  $T_{Sni} = \left\lfloor \frac{T_{ni}}{T_{Sn}} \right\rfloor \cdot T_{Sn} = N_{ni} \cdot T_{Sn}$  ( $T_{Sn} \leq T_{ni}$ ), the

inspection times in a replacement span  $k_{Si} = \left\lfloor \frac{T_{Sri}}{T_{Sni}} \right\rfloor - 1$ . Note

that,  $k_i$  sometimes may not be an integer (see fig. 3), then the equation (5) for  $CP_i(T_{ni}, T_{ri})$  needs amendment.

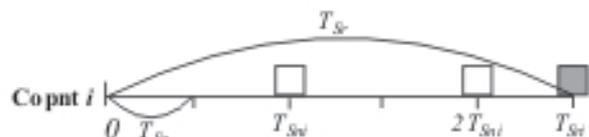


Fig. 3. The case when  $k_i$  is not an integer

$$\begin{aligned}
 CP_i(T_{ni}, T_{ri}) &= k_i \cdot C_{ni} \cdot P_{ni}(T_{ri})S + \\
 &+ \sum_{l=1}^{k_i} [l \cdot C_{ni} + C_{pi} + CP_i(T_{ni}, T_{ri} - lT_{ni})] \cdot P_{ni}(l \cdot T_{ni}) + \\
 &+ \sum_{j=1}^{k_i} \int_{(j-1)T_{ni}}^{jT_{ni}} [(j-1)C_{ni} + C_{fi} + CP_i(T_{ni}, T_{ri} - x)] \cdot p_{bi}(x) dx + (6) \\
 &+ \int_{k_i T_{ni}}^{T_{ri}} [(j-1)C_{ni} + C_{fi} + CP_i(T_{ni}, T_{ri} - x)] \cdot p_{bi}(x) dx + k_i \cdot D_{ni}
 \end{aligned}$$

Integrating equation (5) and (6), we can get

$$CP_i(T_{Sni}, T_{Sri}) = \begin{cases} k_{Si} \cdot C_{ni} \cdot P_{ni}(T_{Sri}) + \sum_{l=1}^{k_{Si}} [l \cdot C_{ni} + C_{pi} + CP_i(T_{Sni}, T_{Sri} - lT_{Sni})] \cdot P_{ni}(l \cdot T_{Sni}) + \\ \sum_{j=1}^{k_{Si}+1} \int_{(j-1)T_{Sni}}^{jT_{Sni}} [(j-1)C_{ni} + C_{fi} + CP_i(T_{Sni}, T_{Sri} - x)] \cdot p_{bi}(x) dx + k_{Si} \cdot D_{ni}, & \frac{T_{Sri}}{T_{Sni}} = N \\ k_{Si} \cdot C_{ni} \cdot P_{ni}(T_{Sri}) + \sum_{l=1}^{k_{Si}} [l \cdot C_{ni} + C_{pi} + CP_i(T_{Sni}, T_{Sri} - lT_{Sni})] \cdot P_{ni}(l \cdot T_{Sni}) + \\ \sum_{j=1}^{k_{Si}} \int_{(j-1)T_{Sni}}^{jT_{Sni}} [(j-1)C_{ni} + C_{fi} + CP_i(T_{Sni}, T_{Sri} - x)] \cdot p_{bi}(x) dx + \\ \int_{k_{Si} T_{Sni}}^{T_{Sri}} [(j-1)C_{ni} + C_{fi} + CP_i(T_{Sni}, T_{Sri} - x)] \cdot p_{bi}(x) dx + k_{Si} \cdot D_{ni}, & \frac{T_{Sri}}{T_{Sni}} \neq N \end{cases} \quad (7)$$

**3.4. The optimization of compound maintenance intervals for multi-component system**

The system maintenance cost consists of two parts: one is the cost for inspection, preventive maintenance and corrective maintenance; the other is the cost for set-up and shutdown loss of system group maintenance [16].

The system maintenance cost in a unit time can be expressed as:

$$\begin{aligned}
 C_{S1}(T_{Sn}, T_{Sr}) &= \sum_{i=1}^L \left[ CR_i(T_{Sri}) - \frac{k_{Si} \cdot D_{ni} + D_{ri}}{T_{Sri}} \right] = \\
 &= \sum_{i=1}^L \left[ \frac{CP_i(T_{Sni}, T_{Sri}) - k_{Si} \cdot D_{ni} + C_{ri}}{T_{Sri}} \right] \quad (8)
 \end{aligned}$$

The system set-up and shutdown cost in a unit time can be expressed as:

$$C_{S2}(T_{Sn}, T_{Sr}) = \frac{\sum_{j=1}^M D_{Sj}}{T_{Sr \max}} \quad (9)$$

Where  $T_{Sr \max} = \max(T_{Sri})$ ,  $M = \frac{T_{Sr \max}}{T_{Sn}}$ ,  $D_{Sj} = \max(D_{nX}, D_{rY})$

$$X = \left\{ x \mid \frac{T_{Snx}}{T_{Sn}} \in N \right\}, Y = \left\{ y \mid \frac{T_{Sry}}{T_{Sr}} \in N \right\}$$

Tab. 1. Maintenance costs and lifetime distribution parameters

<i>i</i>	$C_{ri}$	$D_{ri}$	$C_{pi}$	$C_{fi}$	$C_{ni}$	$D_{ni}$	$m_{ui}$	$l_{ui}$	$m_{hi}$	$l_{hi}$
1	1500	2000	500	6000	100	1000	1	30	1	25
2	1000	3000	400	4000	50	800	3	18	1.6	20
3	1400	2100	800	3000	150	1200	2	20	1	15
4	3800	7000	1900	9000	400	2800	3	19	5	27
5	2900	5500	700	6700	200	1100	2	25	1.5	10

$$\begin{aligned}
 C_S(T_{Sn}, T_{Sr}) &= C_{S1}(T_{Sn}, T_{Sr}) + C_{S2}(T_{Sn}, T_{Sr}) = \\
 &= \sum_{i=1}^L \left[ \frac{CP_i(T_{Sni}, T_{Sri}) - k_{Si} \cdot D_{ni} + C_{ri}}{T_{Sri}} \right] + \frac{\sum_{j=1}^M D_{Sj}}{T_{Sr \max}} \quad (10)
 \end{aligned}$$

If the maintenance activities are not packaged, the system may need shutdown for every task, which would result in higher maintenance cost. Then, the system maintenance cost in a unit time can be expressed as:

$$C_{S0} = \sum_{i=1}^L CR_i(T_{ri}) = \sum_{i=1}^L \frac{CP_i(T_{ni}, T_{ri}) + C_{ri} + D_{ri}}{T_{ri}} \quad (11)$$

By (1)—(9), the equations (10) and (11) can be solved. If the result of (10) could reduce the maintenance cost satisfactorily compared with that of (11), the effectiveness of group maintenance can be validated.

**4. A numerical example**

A simple numerical case is computed here to demonstrate and validate the group optimization for the compound maintenance tasks of multi-component systems. Assuming that a system consists of five components, the initial time and delay time of each component all follow Weibull distribution, the related maintenance costs and life distribution parameters are listed in Table 1.

By the models established in this paper, the intervals and costs before and after group maintenance for the compound maintenance tasks of multi-component system can be optimized with MatLab 7.1. Fig. 4 is the three-dimensional diagram reflecting the changes of system maintenance cost with intervals of replacement and times of inspection in a replacement span.

It can be obtained that when the basis intervals for replacement and inspection are respectively 186 and 31, the whole system maintenance cost can be minimized with the result of 797.5324. If the maintenance tasks are not grouped, the sys-

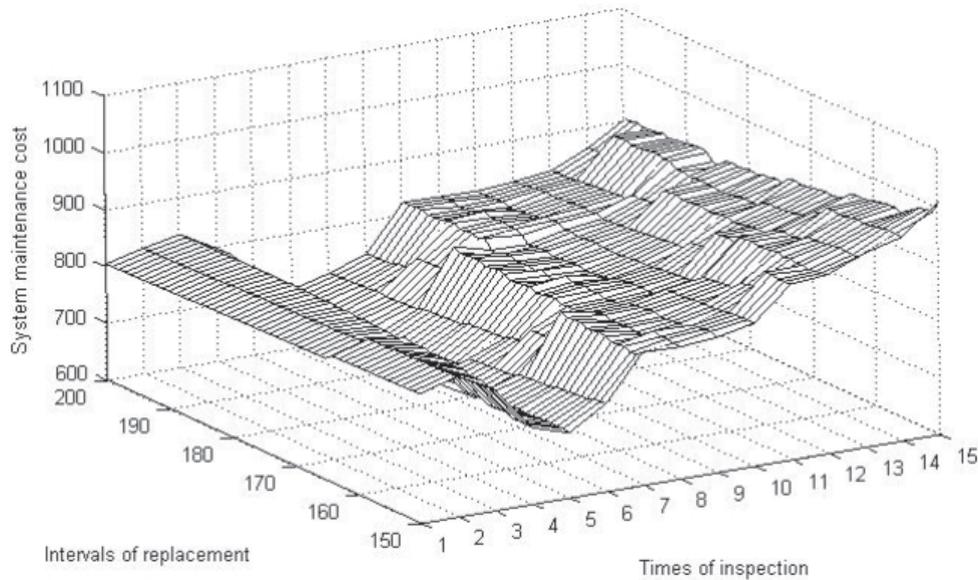


Fig. 4. The three-dimensional diagram of cost optimization for multi-component system

tem maintenance cost would rise to 912.1948. The maintenance intervals and costs before and after group optimization can be seen in Table 2. As is seen, the group optimization of compound maintenance tasks can reduce the system maintenance cost by 12.57% than before.

**5. Conclusions**

Aiming to the requirements of maintenance tasks combination optimization for multi-component system, the group maintenance policy is introduced to optimize the periodic replace-

ment and functional check from the viewpoint of system. The mathematical models for system group maintenance intervals are established, and the effectiveness is validated by a case study. Actually, the model is established only from the aspect of cost; if the failure consequences are evaluated by other factors, such as availability, risk and so on, the corresponding models can also be established in the similar way. The researches on maintenance modeling and optimization of multi-component system, could provide reference for maintenance decision; furthermore, they are of great significance for improving decision scientificity and practical application.

Tab. 2. Maintenance intervals and costs before and after group optimization

Result \ Compone	Before group optimization					After group optimization				
	1	2	3	4	5	1	2	3	4	5
Intervals of inspection	43	32	51	35	26	31	31	62	31	31
Times of inspection	5	6	7	8	6	6	6	6	12	6
Intervals of replacement	215	192	357	280	156	186	186	372	372	186
System maintenance cost	912.1948					797.5324				

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