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ANALIZA DOSTĘPNOŚCI KONSTELACJI SATELITÓW

AVAILABILITY ANALYSIS OF SATELLITE CONSTELLATION

Artykuł w pierwszej kolejności analizuje pojęcie dostępności konstelacji satelitów. Następnie proponuje metodę analizy dostępności konstelacji opartą na modelu łańcuchów Markowa, analizie przestojów, oraz MTBF i MTTR konstelacji satelitów. Metodę zilustrowano przykładem.

Słowa kluczowe: *Konstelacje satelitów; analiza dostępności; analiza przestojów.*

This paper firstly analyzes the concept of availability of satellite constellation. Then the method of constellation availability analysis is proposed based on Markov chain model, outage analysis, as well as MTBF and MTTR of satellite constellations. Finally, an illustrative example is presented to demonstrate the method.

Keywords: *satellite constellations; availability analysis; outage analysis.*

1. Introduction

A satellite constellation consists of many satellites located in different orbits. The constellation can provide services which single satellites can not deliver, like global navigation, multi-coverage and continuous coverage [9].

Four specifications are important for evaluation of a satellite constellation system, including accuracy, availability, continuity and integrity. The required availability during mission lifetime for a specific service is currently specified for missions where a "steady-state" nominal service is planned, and for which a percentage of the mission time can be specified as an availability performance measure [8]. However, this concept of availability is so simple that lots of important information can not be captured in its expression, like the outage rate and the outage duration of the constellation. The availability of satellite constellation has two different meanings, one is associated with the reliability [5] of the constellation systems' performance, and the other is associated with the failure rates and restoration rates of the constellation.

The concept of availability of satellite constellation is firstly analyzed using MTBF and MTTR of the constellation system in this paper. Then the method of availability analysis of the satellite constellations is proposed on the basis of Markov chain model, outage analysis, as well as MTBF and MTTR of satellite constellations. At the end of the paper, an illustrative example is presented to illustrate the availability analysis method.

2. Concept of Availability of Constellation

"Availability" is a key factor for evaluation of the performance of satellite constellation. However, the definition of availability and its targets have evolved over years. In the early days of the global positioning system (GPS), availability of a minimum of 4 visible satellites or maximum positioning dilution of precision (PDOP) of 6 of an ideal constellation (with 18, 21, or 24 satellites) was adequate [4]. Later, availability of modified requirement was applied to degraded constellations with outages of one or more satellites. Today, the measure of availability has become more complex involving satellite's Markov state probability with different satellite failure and restoration models. For navigation, availability of sustained accuracy, not just accuracy itself, is the dominant requirement. The system performance requirement has also evolved from a simple number of visible satellites to dilution of precision (where satellites' errors are assumed to be identical) to more complex horizontal and vertical user navigation accuracy with satellites' errors changing with time as the satellites' position changes.

As mentioned before, the constellation availability is associated with the failure rates and restoration rates of the satellite constellation. Here, the concept of maintenance of satellite constellation is of great importance. As we know, satellites in the constellation may fail during their operation period. Spare satellite strategy is adopted in constellation design to deal with this problem. These measures can be regarded as maintenance of the constellation.

Under these considerations, the constellation availability may be regarded as the probability that the slots in the satellite constellation will be occupied by satellites transmitting a trackable and healthy SIS (signal in space) [3].

3. Availavility analysis of constellations

3.1. Expected availability

Availability analysis and/or simulations shall be performed in order to assess the availability of the constellation system. The results are used to:

- optimize the system with respect to design, operations and maintenance,
- verify conformance to availability requirements, and
- provide inputs to estimate the overall cost of operating the system.

The availability analysis (predictions/assessments) shall be carried out at the system level using system reliability and maintainability models as well as the data from the outage analysis.

The expected availability of satellite constellation can be expressed as:

$$A_k = \sum_n P_{k,n} \cdot \alpha_{k,n} \tag{1}$$

where: A_k denotes the expected availability of the kth scheme, $P_{k,n}$ denotes the probability of the state that n satellites are working properly, $\alpha_{k,n}$ denotes the probability that the n satellites are meeting the requirement of navigation performance.

3.2. Availability analysis based on Markov chain and outage analysis

In formula (1), $P_{k,n}$ can be calculated through a Markov chain model [6]and the input data such as MTBF and MTTR of the constellation.

Markov chain is a popular tool to deal with evaluation of constellation state probabilities [1]. Here we choose 5 IGSO satellites to illustrate the model, as shown in Fig.1. This model is a Markov chain with a maximum of 5 satellites operating and a minimum of zero operating.

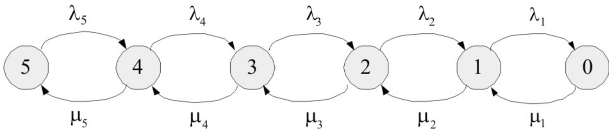

Fig. 1. Markov chain model for 5 igso satellites

Each state is associated with four types of outages.

According to IEC Multilingual Dictionary (2001 edition), outage is defined as the state of an item being unable to perform its required function [2].

The relations between the various values such as MTBF MTTR, and MTTF, which characterize the reliability, maintainability and availability of equipment, are shown in Fig.2.

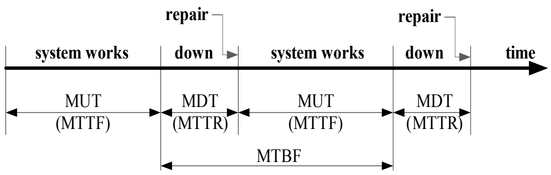

Fig. 2. Relations between the various quantities that characterize reliability, mantainability and availability of constellation

The outage analysis shall be performed in order to supply input data for availability analysis. The outage analysis output includes a list of all potential outages, their causes, probabilities of occurrence and duration. Instead of outage probabilities, failure rates associated with outages can be provided. Furthermore, the means of outage detection and the recovery methods shall be identified in the analysis.

Satellites outages are divided into scheduled and unscheduled outages, which are then each further divided into short term and long-term. Thus, there are four categories of satellite outages: short-term scheduled (STS), long-term scheduled (LTS), short-term unscheduled (STU), and long-term unscheduled (LTU). Scheduled outages, both short- and long-term, are under the control of the Control Segment and can be deferred for a specified time duration and rescheduled as needed. STS outages are for routine maintenance actions such as clock ion pumping, station keeping maneuvers, and switching between redundant subsystems. LTS outages are commonly referred to as End of Life (EOL) events. STU outages result from sudden failures that cannot be predicted or scheduled, but they are problems that can typically be fixed on orbit by switching to a redundant subsystem on board the satellites. LTU outages also result from sudden failures, but the nature of the failure is such that the satellite cannot be fixed on orbit and must be replaced [7].

The four failure mechanisms can be combined into a single aggregate Markov chain, in which the failure and restoration probabilities are:

$$\lambda_i = i(\lambda_S + \lambda_U + \lambda_P + P_E\lambda_E) \tag{2}$$

$$\mu_i = \frac{\lambda_S + \lambda_U + \lambda_P + P_E\lambda_E}{\frac{\lambda_S}{\mu_S} + \frac{\lambda_U}{\mu_U} + \frac{\lambda_P}{\mu_P} + \frac{P_E\lambda_E}{\mu_E}} \tag{3}$$

where: λ_i is the probability that any failure will occur from the state of i satellites, μ_i is the probability that any restoration will occur returning the system to the state of i satellites,

- λ_S is the probability of a STS failure,
- λ_U is the probability of an STU failure,
- λ_P is the probability of a LTU failure,
- λ_E is the probability of a LTS failure on a weak satellite,
- μ_S is the probability of a restoration of a STS failure,
- μ_U is the probability of restoration of an STU failure,
- μ_P is the probability of restoration of a LTU failure,

and μ_E is the probability of the restoration of a LTS failure. When there is no surplus satellite, P_E defines the percentage of weak satellites in the constellation. When there are surplus satellites, P_E is the probability of a surplus satellite not being launched before an LTS failure in the baseline system.

3.3. Performance analysis of the constellations

In formula (1), $\alpha_{k,n}$ denotes probabilities of these n satellites meeting the requirement of navigation performance. To calculate these parameters, PDOP of different satellite constellation must be considered. PDOP of navigation constellation is one of the important specifications in the phase of general system design. Effects of different distributions of faulty satellites on PDOP availability of navigation constellation can be studied by simulation tools like STK or GSSF.

4. An illustrative example

4.1. Availability analysis of regional navigation systems

Continuous and stable coverage in the specified regions are required for the regional navigation constellations, whereas in the unspecified regions the coverage is not required. Three orbits are available in the regional navigation constellations, i.e. GEO, IGSO, and MEO. We focus on the regional navigation constellations having GEO and IGSO satellites. The availability of three schemes can be calculated by the methods proposed in section III. These three schemes are:

1) 5 GEO+3 IGSO
2) 5 GEO+4 IGSO
3) 5 GEO+5 IGSO

The outage data of the regional navigation constellation are given in Table1. By virtue of the Markov chain model and the data in Table 1 the state probabilities of the constellation can be calculated, as shown in Table 2.

$$P_5 = (1 + \sum_{j=0}^{4}(\prod_{k=5}^{5-j}\frac{\lambda_k}{\mu_k}))^{-1}$$

$$= (1 + (\frac{\lambda_5}{\mu_5} + \frac{\lambda_5}{\mu_5}\cdot\frac{\lambda_4}{\mu_4} + \frac{\lambda_5}{\mu_5}\cdot\frac{\lambda_4}{\mu_4}\cdot\frac{\lambda_3}{\mu_3} + \frac{\lambda_5}{\mu_5}\cdot\frac{\lambda_4}{\mu_4}\cdot\frac{\lambda_3}{\mu_3}\cdot\frac{\lambda_2}{\mu_2} + \frac{\lambda_5}{\mu_5}\cdot\frac{\lambda_4}{\mu_4}\cdot\frac{\lambda_3}{\mu_3}\cdot\frac{\lambda_2}{\mu_2}\cdot\frac{\lambda_1}{\mu_1}))^{-1}$$

$$= (1 + (\frac{0.015}{0.125} + \frac{0.015\times0.012}{0.125^2} + \frac{0.015\times0.012\times0.009}{0.125^3} + \frac{0.015\times0.012\times0.009\times0.006}{0.125^4}$$

$$+ \frac{0.015\times0.012\times0.009\times0.006\times0.003}{0.125^5}))^{-1} = 0.8831$$

Tab. 1. Outage data of the regional navigation constellation

Outage Type	MTBF(hrs)	MTTR(hrs)
STS	360	7.5
STU	4380	21
LTS	70080	4380
LTU	420000	5100

$$P_4 = (\prod_{k=5}^{5} \frac{\lambda_k}{\mu_k}) \cdot P_5 = \frac{\lambda_5}{\mu_5} \cdot P_5 = \frac{0.015}{0.125} \times 0.8831 = 0.1060$$

$$P_3 = 0.0102, P_2 = 0.0007, P_1 = 0, P_0 = 0$$

4.2. Comparision of the schemes

Table 3 shows the availabilities of different schemes. From the table, we can see the availability of scheme 3 is the highest, but not much higher than scheme 2. If the costs of the satellites are considered, maybe scheme 2 is the best one.

Tab. 2. The state probabilities of Igso satellites

Numbers of the normal satellites	State probability	Cumulative probability
5	0.8831	0.8831
4	0.1060	0.9891
3	0.0102	0.9993
2	0.0007	1
1	0	1
0	0	1

5. Conclusion

The availability of satellite constellations can be evaluated based on the maintainability of satellite constellations. By means of Markov chain model, PDOP analysis, as well as data from the outage analysis, the availability of constellations can be analyzed quantitatively. This availability analysis method is illustrated through the example of the availability computing of the regional navigation systems, and these results may effectively support the relevant decision-making.

Tab. 3. Availabilities of different schemes

Operation Schemes			State Probabilities $P_{k,n}$	Probabilities of different configurations meeting the performance requirement $\alpha_{k,n}$	Constellation availability A_k
1	**5 GEO+3IGSO** (The baseline configuration meeting the performance requirement)	No IGSO satellite fails (totally 1 kind of configuration)	$P_{1,3}$=0.9301	$\alpha_{1,3} = 1$	A_1=0.9301
2	**5 GEO+4IGSO**	Only one IGSO satellite fails,(totally C_4^3=4 kinds of configurations)	$P_{2,3}$=0.0755	$\alpha_{2,3} = 0.75$	A_2=0.9689
		No IGSO satellite fails (totally 1 kind of configuration)	$P_{2,4}$=0.9123	$\alpha_{2,4} = 1$	
3	**5 GEO+5IGSO**	Two IGSO satellites fail, (totally C_5^3=20 kinds of configurations)	$P_{3,3}$=0.0102	$\alpha_{3,3} = 0.65$	A_3=0.9745
		Only one IGSO satellite fails,(totally C_5^4=5 kinds of configurations)	$P_{3,4}$=0.1060	$\alpha_{3,4} = 0.8$	
		No IGSO satellite fails totally 1 kind of configuration)	$P_{3,5}$=0.8831	$\alpha_{3,5} = 1$	

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