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A METHOD OF EVALUATING FATIGUE LIFE OF SOME SELECTED STRUCTURAL COMPONENTS AT A GIVEN SPECTRUM OF LOADS – AN OUTLINE

The paper has been intended to introduce a method of evaluating the damage hazard and fatigue life of a structural component of an aircraft for: a given spectrum of loading the component; the Paris formula with the range of values of the material constant – differentiated.
Fatigue crack growth while the aircraft is operated shows random nature. To describe the dynamics of the crack propagation as a random process, a difference equation was used to arrive at a partial differential equation of the Fokker-Planck type. Having solved this equation enables the density function of growth in the crack length to be found. With the density function of the crack length and the boundary value thereof, probability of exceeding the boundary condition has been determined. The function obtained in this way enables estimations of fatigue life of a structural component.

***Keywords**: damage hazard, fatigue life, density function, stress intensity factor, probability, cyclic loading.*

1. Introduction

In many papers one could find a method to evaluate fatigue life of a structural component for some selected notations of the Paris formula and a simplified fatigue loading pattern.

An attempt has been made in this paper to present a method of evaluating fatigue life of a structural component of an aircraft, assuming that:

- fatigue loading of the component is determined with some spectrum of loads, set up using a pattern of loading in the course of aircraft operation,
- the crack growth process, approached in a deterministic way, has been described with the Paris formula in the following form:

$$\frac{da}{dN_Z} = C \quad M_k^m \quad E[(\sigma_{max})^m]\pi^{\frac{m}{2}} \quad a^{\frac{m}{2}} \tag{1}$$

where: C, m – material constants, a – crack length, N_z – the number of fatigue cycles, M_k – coefficient of the finiteness of the component's dimensions and position of the crack $E[(\sigma_{max})^m]$ – the expected value determined on the spectrum of loads, found in the following way:

$$E[(\sigma_{max})^m] = P_1(\sigma_1^{max})^m + P_2[(\sigma_2^{max})^m + \ldots + P_L[(\sigma_L^{max})^m \tag{2}$$

$$\sigma_i^{max} = \frac{\sigma_i^{max} + \sigma_i^{min}}{2} + \sigma_i^a \qquad i=1,2,\ldots,L \tag{3}$$

σ_i^{max} – maximum value of cyclic loading within the i-th value interval (discrete loading value), σ_i^{min} – minimum value of cyclic loading within the i-th value interval, σ_i^a – amplitude of cyclic loading within the i-th value interval, P_i – frequency of threshold values of loading, determined with the following dependence:

$$P_i = \frac{n_i}{N_c} \qquad N_c = \sum_{i=1}^{L} n_i \tag{4}$$

n – the number of repetitions of specific threshold values of loading, using a single aircraft flight (standard flight), N_c – the total number of cycles in the course of aircraft standard flight.

Relationship (1) can be expressed against time, i.e. – in more detail – against flying time of the aircraft. Hence, the following assumption is made:

$$N_z = \lambda t \tag{5}$$

where: λ – intensity of the occurrence of cycles of fatigue loading of the structural component, t – flying time of the aircraft.

In our case, $\lambda = \frac{1}{\Delta t}$, where Δt – loading-cycle duration. A provisional formula to determine Δt could be accepted in the following form:

$$\Delta t = \frac{T}{N_c} \tag{6}$$

where: T – time of a standard aircraft flight.

Using assumptions and notifications made earlier, one can set to describing – in probabilistic terms – the dynamics of crack growth in the component.

In [1], the following difference equation has been used:

$$U_{a,t+\Delta t} = P_1 U_{a-\Delta a_1,t} + P_2 U_{a-\Delta a_2,t} + \ldots + P_L U_{a-\Delta a_L,t} \tag{7}$$

where: $U_{a,t}$ – probability that for the flying time equal to "t", the crack length was "a", Δa_i – crack length increment in time interval "Δt" for the stress σ_i^{max} $(i = 1,2,\ldots,L)$.

The following differential equation of the Fokker-Planck type has been obtained from equation (7) in [3]:

$$\frac{\partial u(a,t)}{\partial t} = -\alpha(a)\frac{\partial u(a,t)}{\partial a} + \frac{1}{2}\beta(a)\frac{\partial^2 u(a,t)}{\partial a^2} \tag{8}$$

where: $u(a,t)$ – a crack-length density function that depends on the flying time of the aircraft, $\alpha(a)$ – coefficient that determines average crack-length increment per time unit, defined with the following dependence:

$$\alpha(a) = \lambda \sum_{i=1}^{L} P_i \Delta \quad a_i \tag{9}$$

$\beta(a)$ – square of the crack-length increment as referred to time unit, determined with the following dependence:

$$\beta(a) = \lambda \sum_{i=1}^{L} P_i(\Delta \quad a_i)^2 \tag{10}$$

Δa_i – crack-length increment determined with the following dependence:

$$\Delta \quad a_i = C_m(\sigma_i^{max})^m a^{\frac{m}{2}} \tag{11}$$

$$C_m = C \quad M_k^m \quad \pi^{\frac{m}{2}} \tag{12}$$

2. How to find a crack-length density function for m ≠ 2

With the accepted notification applied, eq (1) could be presented in the following form:

$$\frac{da}{dt} = \lambda \quad C \quad M_k^m E[(\sigma_{max})^m]\pi^{\frac{m}{2}} a^{\frac{m}{2}} \tag{13}$$

The following dependence is a solution of eq (13):

$$a = (a_o^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda \quad C \quad M_k^m E[(\sigma_{max})^m]\pi^{\frac{m}{2}} t)^{\frac{2}{2-m}} \tag{14}$$

With eq (14) applied, coefficients α(a) and β(a) could be developed to take then the following forms:

$$\alpha(t) = \lambda \quad C \quad M_k^m \pi^{\frac{m}{2}} E[(\sigma^{max})^m] \cdot$$
$$\cdot [a_o^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda \quad C \quad M_k^m E[(\sigma_{max})^m]\pi^{\frac{m}{2}} t]^{\frac{m}{2-m}} \tag{15}$$

$$\beta(t) = \lambda \quad C^2 \quad M_k^{2m}\pi^m E[(\sigma^{max})^{2m}] \cdot$$
$$\cdot \{[a_o^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda \quad C \quad M_k^m E[(\sigma_{max})^m]\pi^{\frac{m}{2}}]^{\frac{2}{2-m}}\}^m \tag{16}$$

Taking eqs (15) and (16) into account, eq (8) takes the form:

$$\frac{u(a,t)}{\partial t} = - \quad \alpha(t)\frac{\partial u(a,t)}{\partial a} + \frac{1}{2}\beta(t)\frac{\partial^2 u(a,t)}{\partial a^2} \tag{17}$$

Solution of eq (17) is the requested crack-length function that depends on the flying time:

$$u(a,t) = \frac{1}{\sqrt{2\pi \quad A(t)}} e^{-\frac{(a-B(t))^2}{2A(t)}} \tag{18}$$

where: $B(t)$ – an average value of crack length for the flying time equal to "t", whereas $A(t)$ – a variance of crack length for the flying time equal to "t".

Computational formulae take the following forms:

$$B(t) = \int_0^t \alpha(z)dz \tag{19}$$

$$A(t) = \int_0^t \beta(z)dz \tag{20}$$

Having calculated the integrals, we arrive at:

$$B(t) = [a_o^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda \quad C \quad M_k^m \pi^{\frac{m}{2}} E[(\sigma^{max})^m]t]^{\frac{2}{2-m}} - a_o \tag{21}$$

$$A(t) = \frac{2}{2+m} C \quad M_k^m \quad \pi^{\frac{m}{2}} \cdot \frac{E[(\sigma^{max})^{2m}]}{E[(\sigma^{max})^m]} \cdot$$
$$\cdot \quad [(a_o^{\frac{2-m}{2}} + \frac{2-m}{2} \lambda \quad C \quad M_k^m \quad \pi^{\frac{m}{2}} E[(\sigma^{max})^m]t)^{\frac{2+m}{2-m}} - a_o^{\frac{2+m}{2}}] \tag{22}$$

Notations:

$$\_\_ = CM_k^m \pi^{\frac{m}{2}} E[(\sigma^{max})^m] \tag{23}$$

$$\omega = \frac{E[(\sigma^{max})^{2m}]}{(E[(\sigma_{max})^m])^2} \tag{24}$$

With account taken of (23) and (24), eqs (21) and (22) take the following forms:

$$B(t) = [a_o^{\frac{2-m}{2}} + \frac{2-m}{2} \quad \_\lambda t]^{\frac{2}{2-m}} - a_o \tag{25}$$

$$A(t) = \frac{2}{2+m} \_\omega \quad [(a_o^{\frac{2-m}{2}} + \frac{2-m}{2}\_\lambda t)^{\frac{2+m}{2-m}} - a_o^{\frac{2+m}{2}}] \tag{26}$$

3. How to find a crack-length density function for m = 2

When the material constant takes value equal to two ($m = 2$), the crack-length density function takes the following form [3]:

$$u(a,t) = \frac{1}{\sqrt{2\pi \quad \hat{A}(t)}} e^{-\frac{(a-\hat{B}(t))^2}{2\hat{A}(t)}} \tag{27}$$

where:

$$\hat{B}(t) = a_o(e^{\lambda \bar{C} t} - 1) \tag{28}$$

$$\hat{A}(t) = \frac{1}{2} a_o^2 \bar{C} \bar{\omega}(e^{2\bar{C}\lambda t} - 1) \tag{29}$$

$$\bar{C} = CM_k^2 \pi \quad E \quad [(\sigma^{max})^2] \tag{30}$$

$$\bar{\omega} = \frac{E[(\sigma^{max})^4]}{(E[(\sigma_{max})^2])^2} \tag{31}$$

4. How to determine the hazard of a catastrophic damage to a structural component of an aircraft, from the point of view of fatigue and fatigue life

To determine some critical value of the crack length, the stress intensity factor in the following form could be used:

$$K = M_k \sigma \sqrt{\pi \quad a} \tag{32}$$

where: M_k – correlation factor that comprises geometric characteristics of the finiteness of dimensions of the component and the crack shape, σ – the loading that affects the component.

The stress intensity factor, determined with dependence (32), becomes a quantity of a critical value K_c when the crack length and the stress take critical values a_{kr} and σ_{kr}, respectively. Then it is called 'resistance of the material to cracking':

$$K_c = M_k \sigma_{kr} \sqrt{\pi \quad a_{kr}} \tag{33}$$

With eq (33) applied, the critical value of the crack length can be defined:

$$a_{kr} = \frac{K_c^2}{M_k^2 \sigma_{kr}^2 \quad \pi} \tag{34}$$

Having exceeded the critical value of the crack length usually leads to a catastrophic damage to the component.

If the factor of safety is introduced, one can find the admissible value of the crack.

The computational formula takes then the following form:

$$a_d = \frac{K_c^2}{k \quad M_k^2 \sigma_{kr}^2 \quad \pi} \tag{35}$$

where: k – factor of safety, σ_{kr} – maximum value of service stress affecting the aircraft component.

With the crack-length density function (18) and formula (34), one can determine a given dependence to estimate the hazard of a catastrophic crack of the component for the flying time equal to t:

$$Q(t) = \int_{a_{kr}}^{\infty} u(a,t)da \tag{36}$$

The hazard of damaging the component will be determined in the following way, with the factor of safety taken into account:

$$\bar{Q}(t) = \int_{a_d}^{\infty} u(a,t)da \tag{37}$$

Applying one of dependences, i.e. (36) or (37), one could estimate fatigue life of the component for the assumed level of damage hazard.

The computational formula will be then as follows:

$$\bar{Q}(t)_{dop} = \int_{a_d}^{\infty} \frac{1}{\sqrt{2\pi \ A(t)}} e^{-\frac{(a-B(t))^2}{2A(t)}} da \tag{38}$$

To normalise the crack-length density function, the following dependence could be used:

$$Z_t = \frac{a - B(t)}{\sqrt{A(t)}} \tag{39}$$

The computational formula (38) after normalisation takes the following form:

$$\bar{Q}(t)_{dop} = \frac{1}{\sqrt{2\pi}} \int_{\frac{a_d - B(t)}{\sqrt{A(t)}}}^{\infty} e^{-\frac{1}{2} z_t^2} dz \tag{40}$$

The right side of dependence (40) can be found using tables of normal distribution.

When the (required) $\bar{Q}(t)^*_{dop}$ is determined, some value of time should be found – such as to make the left side of eq (40) equal the right one.

The value of "t" found in this way will be the searched for "life" for the assumed level of $\bar{Q}(t)_{dop}$.

Eq (40) could be converted into a form convenient enough to use tables of normal distribution.

With the following notifications introduced:

$$\frac{a_d - B(t)}{\sqrt{A(t)}} = \gamma(t) \tag{41}$$

$$\Phi(t) = \int_0^t \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} z^2} dz \tag{42}$$

the computational formula (40) can be reduced to take the following form:

$$\bar{Q}(t)_{dop} = \frac{1}{2} - \Phi(\gamma(t)) \quad \text{for } \gamma(t) > 0 \tag{43}$$

$$\bar{Q}(t)_{dop} = \frac{1}{2} + \Phi(\gamma(t)) \quad \text{for } \gamma(t) < 0 \tag{44}$$

While computing either the damage hazard or fatigue life of the structural component, one should take value of the m coefficient into account and select a suitable function.

5. Final remarks

Presented density function of fatigue crack length and function enabled to calculate damage hazard and fatigue life of selected structural components are very important in research about reliability and durability of aviation technology.

The main advantage of the presented method is: fact that the method takes into consideration random value of stress and can be use for material with "m" coefficient not equal two. Authors are going to carry out further verification the method with use real maintenance data.

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