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USE OF VIBROACOUSTICAL SIGNAL IN DETECTING EARLY STAGES OF FAILURES

An understanding of vibroacoustic signal is required for the robust and more effective detection of early stage failure. In the paper the possibility and method of time varying vibration decomposition are discussed. It is shown that analysing the coupling between the structure's components changes from linear to nonlinear or to other kind of nonlinearity together with intermodulation phenomena can be used as measure in structural health monitoring. In addition on an analytical connection is investigated between the tracking method and the physics of the kinematic contact process based on the idea of higher-order spectra analysis, bispectral analysis specially.

Keywords: *Vibroacoustic diagnostics, modulation phenomena, bispectrum, time-frequency representation.*

1. Introduction

Half a century ago, when solving the issue of dealing with effects of failures in machines and devices as well as the task of lubrication of important kinematic pairs, attention was drawn to the fact of relevant organization of operation and maintenance units and introduction of new operational methods. However the need for cost reduction as well as increasing mechanization and complexity of devices and the associated growth of loss in a situation of a failure pointed to the importance of maintenance and repair activities. As a result, procedures were introduced in the years 1950-1970 which defined the scope and the time of scheduled maintenance-and-repair inspections.

The next stage of development, associated with 1970's, was the inclusion of elements of prediction in the existing, essentially preventive methods. This direction of development was to a great extent stimulated by the introduction of "Just – in – Time" manufacturing system which put a lot of focus on minimizing the warehouse stock. The present stage of development, which has continued developing since the beginning of 1980's, is determined by high automation of equipment and development of technologies on the one hand and putting stress on relevant security of operated systems and the need for reducing the threat to the environment on the other. Hence the enormous interest in the procedures of identification and forecasting of emergency situations as well as growth of credibility of reliability assessment related to monitored machines and devices.

This results from the fact that a system which has been designed while accounting for the possibilities of technical condition monitoring (CM) enables us to avoid the loss associated with forced downtimes, reduces the cost of unnecessary storage of spare parts as well as the costs associated with occurrence of unforeseen failures. Thus production cost is reduced by avoiding the need to pay the crew for readiness to work during downtimes.

In such circumstances it is natural to adopt Condition Based Maintenance (CBM), which means introduction of a system of evaluating the technical condition based on the collected data related to the parameters of a machine's operation and the parameters of residual processes as well as performance of preventive maintenance based on the forecasted damage (failure) occurrence, which we could term as "just – in – time" maintenance. The right selection and implementation of a diagnostic system and the relevant training of the diagnostic team who, based on the examination of trends of defect occurrence and development, would able to determine the time to failure and assess the actual, longer operating time are the factors conditioning the ability to accomplish the goals assumed for such an operation process.

One of the basic elements of such a strategy is the use of diagnostic systems for monitoring of technical condition of systems or accordingly process monitoring systems in order to detect and identify the defect development phase and prevent the occurrence of dangerous disturbance of functioning of critical elements and units of a system. This often calls for the need of solving the issue of diagnostic inspection optimization according to the criteria defined by RBI (Risk Based Inspection) and RBM (Risk Based Maintenance) methods. In accordance with the propositions found the publications on the topic [3,16] the basic tasks of advanced security monitoring include: defining the scope of monitoring and the method of limiting the scope and presenting the information related to emergency conditions and values; selection of methods and means enabling monitoring and on-line inference in a manner enabling early detection of growing disturbance and extraction of features characteristic for developing defects from general signals pointing to operating anomalies; controlling the defects and taking corrective actions by the operator so as to minimize, and in particular avoid, the occurrence of undesirable events which are accompanied by extensive consequences; development of a method of forecasting future events based on the current observations and registered, lasting changes of parameters that have been detected during measurements' analyses as well as examination of the results found in the database. The last item is particularly important when monitoring the condition of elements and units subject to degradation and fatigue-related wear in whose case the identification of early stages of defect development may prevent the occurrence of the catastrophic phase of a defect and consequently destruction of the entire system.

Taking this into account, the possibilities of applying vibroacoustics in technical systems, including the issues of vibroacoustic diagnosis of units and elements, inspire increasing interest among engineers and technicians involved in the organization and planning of [machine] operations in companies.

Early detection of defects and determination of their causes occupies a special place among the vibroacoustic diagnosis methods. Let us note that the process of defect formation can lead both to intensification of non-linear phenomena as well as to occurrence of non-stationary effects even if in the early stages the intensity of defects is small while the growth of the level of vibration and noise is negligible in contrast with emergency situations. Here let us only note that the emergence of defects and the low-energy phases of their development are most often accompanied by local disturbance of the signal's run, which may result in tangible changes of the signal's frequency structure that are additionally variable in time. Such a situation inclines one to formulate the diagnosis of origin of defects while relying on the diagnostic information carried by non-stationary disturbance and non-linear effects.

Let us note that the analysis of low-energy pulse-type disturbance, causing broadband response with small amplitude, calls for accounting for not only the information on the changes of the signal's power but also on its phase, which points to the need for reaching beyond the information contained in the second-order process. This is so because even though the correlation function provides a sufficient description of the Gauss process with the mean value equal zero, still in the case of non-Gauss distributions of probability it is accordingly the correlation function of the power spectrum that provides partial information on the process.

For example the information on emergence of a defect can be contained in the low-energy components of the signal that are carried across the structure of a machine from the measurement source as a result of modulation of a relevant carrier function. Thus when examining signals attention is particularly devoted to the analysis of amplitude-and-phase modulation of a signal and occurrence of non-linear and non-stationary effects.

The actual signal can contain both, the component generated by the diagnosed kinematic pair as well as by the components transmitted over structure of the examined object which are generated by other kinematic nodes. This brings us to the necessity of solving the issue of relevant separation of diagnostically-useful information. It is connected with the issue of developing a relevant procedure which could be a part of the algorithm for diagnosing the low-energy phases of defect development

As has been demonstrated by to-date research [10], significant diagnostic information is contained the higher order moments, which underscores the significance of non-linear phenomena in the detection of defect development. Such possibilities are not offered by the analysis of power spectrum which relies on the assumption of mutual independence of respective frequency components, which is a consequence of adopting the linearity and the superposition in the applied methods of power spectrum determination. In majority of cases the development of low-energy defects cannot be adequately presented by means of linear models. An example confirming this situation is the phenomenon of transformation of a harmonic function in which the additional frequency components are coupled. Similarly, the transition through a non-linear phenomenon with a square component of a signal consisting of two harmonics with various frequencies and various initial phases will lead to the emergence among the additional components, at the output points of system, also of components which maintain the same relationships between the original frequencies and phases as those found in the input signal. This phenomenon is called in publications [8] as quadratic-phase coupling.

The above presented introduction points to the need of broader reference to the methods of signal analysis that enable detection of phase relationships between the harmonics and the modulation, inter-modulation and mutual modulation effects which enable examination of resultant multi-dimensional signals.

Treating the diagnosed object as an operator which transforms the parameters of a technical condition of ***X"*** into parameters of vibroacoustic signal ***Z***, we will obtain the solution of the problem of general diagnosis of state in the form of a inverse task:

$$\boldsymbol{Z} = \boldsymbol{AX} \Rightarrow \boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{Z} \qquad (1)$$

where A – system matrix.

Solution of equation (1) for the general case is difficult. Thus the problem of general diagnosis of object's state is in its essence reduced to selection of such subset of diagnostic parameters so that the changes of the values of the technical state parameters are accompanied by changes of one diagnostic parameter (symptom). In such case the quantitative relationships can be established on the basis of analysis of the models of defect symptom generation.

With such approach, the subset of ***X'***, the elements of which are states that ensure the realization of the object's functional tasks, is disregarded.

Assuming that transition from one state to another within the subset of ***X'*** is caused by appearance of defects which do not have direct influence upon decrease of the object's functional value, we are faced with a diagnostic problem that is new in terms of its characteristics.

In this case we are interested in making of a diagnosis of state and forecast of transition into a given state from the subset of ***X"***. In order to carry out this type of diagnosis on the basis of the character and intensity of the changes connected with the degree of object's readiness for work, it is necessary to analyze the relationships between the degradation and wear and tear processes that take place in the functionally essential (kinematic) pairs, and the related changes of the vibroacoustic processes properties. This implies a necessity of defining of such a set of diagnostic parameters whose elements will be sensitive to slight changes of technical parameters and which can be used for location and identification of origins and development of low energy stages of defects. Let us note that similarly as in the case of general diagnosis, also in this case the fundamental problem is the structure and the analysis of the relationships and cause and effect relations, and creation of an adequate set of diagnostic parameters. Let us consider this problem in more detail.

2. Diagnostic model in detection of low-energy defects

While attempting to develop a model oriented on such defects one should on the one hand consider the issue of examining the signal's parameters from the point of view of their sensitivity of to low-energy changes of the signal and, on the other, the issue of quantification of energetic disturbances occurring in the case of defect initiation.

Let us assume that the degree of damage D is the dissipated variable that covers the changes of the structure's condition due wear and tear:

$$dE_d(\Theta, D_0) = \frac{\partial E_d(\Theta, D_0)}{\partial D} dD + \frac{\partial E_d(\Theta, D_0)}{\partial \Theta} d\Theta \qquad (2)$$

where: $dE_d = \frac{df(D,\Theta,\gamma(\Theta))}{d\Theta}$, $\gamma(\Theta)$ - the parameter describing how big a part of the dissipated energy dE_d is responsible for structural changes, Θ - operating time.

Bearing in mind the possibility of diagnosis of the origin and the development of low-energy phases of defect formation, when the extent of the original defect can be different in each case, let us analyze this issue more precisely.

To examine this problem let us recall here the two-parameter isothermal energy dissipation model proposed by Najar [8] where:

$$dE_{d_s} = dE_d - dE_{dq} = Tds = \sigma_\Theta dD \qquad (3)$$

where: dE_{d_q} - energy transformed into heat, dE_{d_s} - energy responsible for internal structural changes, T - temperature, ds - growth of entropy.

The expression (3) shows that the growth of the dissipated variable D is attributable to the dE_{d_s} part of energy, which is the dissipated part of dE_d energy, that causes the growth of entropy *ds*.

The role of the multiplier determining the relation between the increments of dissipated variable and the entropy is played by the dissipation stress σ_Θ.

The assumption of *T*=constans results in independence of dissipation-related loss $dE_{d_s} = dE_{\Theta_s}$, thus following integration the expression (3) takes the following form:

$$E_{d_s} = T\Delta s \qquad (4)$$

The derivative of defect development energy related to *D*, when $E_f(D_0) \le \frac{1}{2}E\varepsilon^2$, means the boundary value of deformation energy and takes the following form:

$$\frac{dE_{d_s}}{dD} = \frac{E_f(D_0)(1-D_f)(1-k)D^{-k}}{D_f^{1-k} - D_0^{1-k}} \qquad (5)$$

For a defined initial defect of D_0 and for a defect leading to damage D_f, relationship (5) will have the following form:

$$\frac{dE_{d_s}(D)}{dD} = (1-k)E_{D_0,f}(k)D^{-k} \qquad (6)$$

Let us note that parameter $E_{D_0,f}$ is an exponential function of power *k*, similarly as the whole derivative. While referring to the second rule of thermodynamics for irreversible processes we will assume the following in the contemplated model:

$$\frac{dE_{d_s}(D)}{dD} \ge 0 \qquad (7)$$

Thus for the assumed model to be able to fulfil condition (7), the exponent must meet the requirement of $k \le 1$. In addition, while referring to the rule of minimization of dissipated energy, the conditions of permissible wear process [8] show that the change of exponent k is possible as the defect develops.

To examine this problem let us assume that the exponent shows a straight line dependence on the extent of damage:

$$k(D) = a + bD \qquad (8)$$

For damage of small magnitude the linear approximation seems to be sufficient and enables description of defects whose emergence is characterized by small growth of defect energy (see Figure 1).

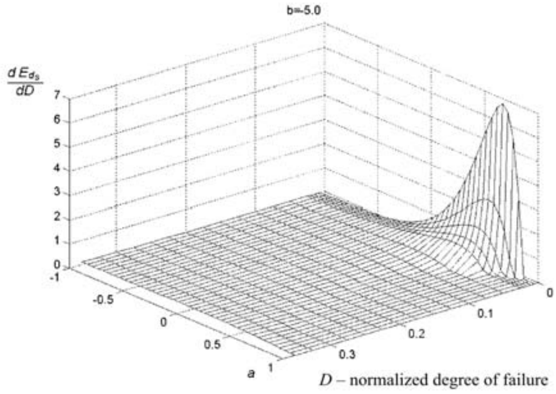

Fig. 1. Change of energy of defect development for small D

Thus while defining the set of diagnostic parameters we should pay attention to the need for selecting such a criterion so that it will be possible to identify defects whose emergence is characterized by small growth of defect-related energy.

While contemplating this issue let us assume that vibroacoustic signal is real and meets the cause-and-effect requirement, which means that it can be the base for creating an analytical signal.

In accordance with the theory of analytical functions, the real and the imaginary components are the functions of two variables and meet Cauchy-Riemann requirements.

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Please be reminded that the analysis of the run of the analytical signal will be conducted while relying on the observation of changes of the length of vector *A* and phase angle *φ*:

$$z(x,y) + jv(x,y) = A(\cos\varphi + j\sin\varphi) \qquad (9)$$

Thus,

$$z\big(x(\tau),y(\tau)\big) = A(\tau)\cos\varphi(\tau) \qquad v\big((x(\tau),y(\tau)\big) = A(\tau)\sin\varphi(\tau) \qquad (10)$$

means that the signal measured is the orthogonal projection of vector *A* on real axis.

Ultimately, while exploiting the Cauchy-Riemann conditions for variables *A(τ)* and *φ(τ)* we will obtain:

$$\frac{dz}{d\tau} = \frac{dA}{d\tau}\cos\varphi - A\sin\varphi\frac{d\varphi}{d\tau} \qquad (11)$$

As we expected the obtained relationship presents an equation that enables the analysis of the measured signal while observing *A* and *φ*. At the same time it should be noted that for low-energy processes, when we can neglect the changes of the vector's length and assume $A \cong const$, the whole information on the changes of the measured signal is contained in the phase angle, or more precisely in the run of momentary angular velocity.

While accounting for the obtained results of the analysis of the process of low-energy defect emergence and detection of diagnostic information associated with the changes of momentary values of amplitude and angular velocity, let us analyze the conditions that must be fulfilled by a diagnostic model intended to enable observation of the influence of such disturbance on the form of the system's dynamic response.

Occurrence of errors can lead to change of the conditions of contact in kinematic node, including: the qualitative change of the process of coming into / out of contact, disturbance of the linearity which as a result lead to non-linear changes of the system's parameters, in particular of rigidity and damping. This has essential impact upon the frequency structure of generated vibration. To make this problem more familiar, let us note that if we assume that the function $y:C \rightarrow C_0$ is holomorphic in the neighbourhood of a set point t_0, then Taylor's expansion defines *y(t)* for points close to t_0 with any arbitrarily selected precision. The first term of this expansion shows linear changes of *y(t)* which depend upon $(t - t_0)$. In points at which the first derivative $y'(t_0)$ disappears, it is only the analysis of the second derivative, responsible for second order increments, that gives us information about the way the function behaves around point t_0. This simple observation turns our attention to the models in which the *x(t)* signal is processed according to the following pattern:

$$y(t) = h_0(t) + h_1(t) * x(t) + h_2(t) * x(t) * x(t) + \ldots \qquad (12)$$

and the Fourier transform will respectively assume the following form:

$$Y(\omega) = Y_0(\omega) + H_1(\omega)X(\omega) + H_2(\omega)X^2(\omega) + \dots \quad (13)$$

This entails the necessity of analysis of non-linear signals, and particularly of examination and modelling of linear and bilinear components.

In comparison to input signals, the bilinear component will be characterised by additional frequencies having the form of sums and differences of frequencies found at the input point. Let us note that similar disturbance will be caused by the phenomenon of amplitude and phase modulation. Referring to the presented model of generation process, let us assume that the bilinear part of the signal is transmitted to the measuring point by a linear transmission channel. Then, after including the linear part of the signal and the non-linear one, which in accordance with the procedure proposed by Eykhoff [4] will be presented in the form of a process containing linear elements and multiplying terms, we shall obtain the following relationship:

$$z(t) = \int_0^t h_1(\tau_1)x(t-\tau_1)d\tau_1 + \int_0^t\int_0^t h_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 \quad (14)$$

where:

$$h_1(\tau_1) = h_{Q_1}(\tau_1) * h_{p_1}(\tau_1) \quad (15)$$

$$h_2(\tau_1,\tau_2) = h_{Q_{22}}(\tau_1-\tau_3)\cdot h_{Q_{21}}(\tau_2-\tau_3) * h_{p2}(\tau_3) \quad (16)$$

Let us note that the complex mechanism of the influence of non-linearity on the system's response was brought down to generation of a signal by a system with components that are relatively simple, which refers back to the model in the form of a Volterra series [13].

3. Detection of non-linear disturbance

Let us note that low-energy impulse disturbances cause broad-band response with small amplitude and that is why typical spectral analysis of averaged power spectrum in the domain of frequency, as well as the correlation function in the domain of time can contain information of second order process. For example, a sufficient characteristic of a Gauss process with an average value equal to zero is its covariance function. However, in the case of process with non-Gauss distribution of probability, respectively the correlation function or the power spectrum supplies only partial information about the process. Analysis of higher-order spectra, properly defining the non-linear effects, is required to obtain a more precise description. This observation pointed to the need for the analysis of the frequency structure of a signal in the plane defined by time and frequency [9].

Additionally, in many cases it becomes necessary to increase the resolution in order to detect the diagnostic information, which most often leads to the increase of the size of the data block and to extending of the required calculation time. Since the analysis of the effects of amplitude and phase modulation of a vibroacoustic signal requires examination of the distribution of the signal's power, both in time and with respect to individual frequency components, then apart from the fulfilment of the resolution-related requirements it also calls for the necessity to define the frequency of sampling. Let us note that conducting of such signal processing is justified in a situation when the general structure of a signal is known and when it is possible, without losing any essential diagnostic information, to select such parameters of a sample so as to obtain a locally stationary signal. Further investigation can be then conducted with the use of the known methods of stationary process analysis, e.g. Fourier transform:

$$S(\varpi) = \int_{-\infty}^{\infty} s(t)e^{-j\varpi t}dt \quad (17)$$

Assuming that the above mentioned assumptions regarding the stationary character of a signal can be implemented by selecting the parameters of a time window which will be moved with respect to a non-stationary signal, we will achieve the possibility to create a spectrogram picturing the frequency curve in the domain of time:

$$S(t,\varpi) = \int_{-\infty}^{\infty} s(\tau)w(\tau-t)e^{-j\varpi\tau}d\tau \quad (18)$$

In practice it is synonymous with dividing the signal into many brief samples by means of a rectangular time window and with the related discontinuity at the ends of the records thus created, as well as with an essential drop in terms of resolution. In work [1], devoted to the analysis of non-stationary character of the signals generated by the heart, we point to the necessity of fulfilment of a compromise criterion of selection of the time window which ensures that the requirements of the sample's stationary character and proper spectrum resolution will be maintained. Wang and McFadden [15], while presenting the possibility of using spectrograms in the tasks related to the identification of damage in toothed gears, stress the importance of the proper function of a window. What we have in mind here is particularly such a function of a window, which will prevent the appearance of additional rolling and the unwelcome Wigner-Ville complication [7]:

$$W_{f_z}(t,\varpi) = \int_{-\infty}^{\infty} f\left(t+\frac{\tau}{2}\right)f^*\left(t-\frac{\tau}{2}\right)e^{-j\varpi\tau}d\tau \quad (19)$$

This transform gives the possibility of differentiating between the phenomena of amplitude and phase modulation. Kumar and Carrol [5], who pointed to the possibility of using the Wigner-Ville distribution for analysing the signals, compare this method to an analysis performed with the use of a two-dimensional cross correlation function. Starting from an assumption that the change of the technical parameters leads to specific changes of the spectral picture, they proposed the evaluation of the status by comparing the obtained distribution with the characteristics of the reference picture. Such an approach enables us for example to define the carrier frequencies and the modulated bands, build a relevant analytical signal, and carry out further analysis on this basis, in particular the quantitative evaluation of the parameters of amplitude and phase modulation. To carry out the proposed procedure it is necessary to apply the Hilbert's transform [10]:

$$\hat{a}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{a(\tau)}{t-\tau}d\tau = a(t) * \left(\frac{1}{\pi t}\right) \quad (20)$$

which allows for the creation of an analytical signal:

$$\tilde{a}(t) = a(t) + j\hat{a}(t) \quad (21)$$

The discussed methods of generating the time and frequency power distributions, while taking into account relevant selection of the windows, turn out to be generally usable in the analysis of signals in which the diagnostically-essential features have similar time scale. On the other hand, their effectiveness drops significantly when examining signals, which contain features with various scales. In this case, as is shown for example in [1,15], much better results can be achieved by applying wavelet analysis:

$$W(t,\alpha) = \frac{1}{-\sqrt{a}} \int_{-\infty}^{\infty} x(\tau) g^* \left(\frac{\tau - t}{a} \right) d\tau, \qquad a(>0) \qquad (22)$$

where respectively:

x(t) is the signal analysed, a is the scale parameter which decreases as the frequency increases, while function *g(t)* describes a variable size window, which allows for selecting the proper sampling frequency while maintaining the expected resolution.

Let us note that in contrast with the Fourier transformation, where the base for decomposing the signal is constituted by the harmonic functions, in this case the choice of the shape of the function's base may account for both, the disturbances and the requirements of the digital processing [14]. Such a manner of analysis is particularly essential in a situation when for a series of modulated bands around the defined carrier frequencies it is necessary to define the type and size of modulation on the basis of the analysis of envelope and phase functions.

On the other hand, the attempt of modelling non-linear effects and generating a disturbed vibroacoustic signal requires accounting for additional effects, e.g. in the form of consecutive carrier frequency. Let us note that in these cases it is the multi-dimensional analysis in the domain of frequency that becomes particularly significant.

Some of these, and in particular the spectrogram, multi-dimension Fourier transform, Wigner-Ville distribution, and bispectral analysis were discussed in [9], in the context of their usability to examine the modulated vibroacoustic signals. Let us note at this place that as a result of signal modulation additional components will appear in the spectrum, respectively in the form of sums and differences of carrier frequencies and modulating frequencies in the case of modulation by a harmonic function. This points to the significance of the bispectrum, which is responsible for "third-order information". If the generally accepted interpretation of the spectrum of vibroacoustic signal's power does not raise any objections, the attempts to interpret the information found in the higher-order spectrum are not so clear. From the interesting to us point of view of using the time and frequency representation to analyse the phenomena of modulation of a signal generated by disturbance of the meshing and contact conditions, it was assumed that time and frequency distribution represents the momentary power spectrum – Wigner distribution:

$$W_{2,x}(t,f) = \int C_{2,x}(t,\tau) e^{-j2\pi f\tau} d\tau \qquad (23)$$

where: $C_{2,x}(t,\tau)$ - local autocorrelation function, second order cumulant.

Let us note that in this way, by using $C_{3,x}(t,\tau_1,\tau_2)$ - a third order cumulant it is possible to present a relationship defining the bispectral Wigner distribution:

$$W_{3,x}(t,f_1,f_2) = \iint C_{3,x}(t,\tau_1,\tau_2) e^{-j2\pi(f_1\tau_1 + f_2\tau_2)} d\tau_1 d\tau_2 \qquad (24)$$

With the assumption of relevant stationary character of a signal in accordance with Gerr's proposal [15], the relationship is in force for a bispectrum and for a bispectral Wigner's distribution:

$$E\left[W_{3,x}(t,f_1,f_2)\right] = \int W_{3,x}(t,f_1,f_2) dt = S_{3,x}(f_1,f_2) \qquad (25)$$

Thus a question appears, can more data be also obtained, both quality and quantity related data concerning the type and the extent of signal's modulation, by using the bispectral Wigner's distribution. Taking into account the fact that multiparameter modulation phenomenon, and the related additional complication of the spectrum's structure appear in a toothed gear, obtaining of a positive answer can have essential application significance.

4. Intermodulation and mutual modulation phenomena

Thus the occurrence of a defect and development of its low-energy phases are accompanied by a disturbance of the operation of kinematic node leading to change of power distribution between the spectrum components. The shares of respective components will be determined by multi-parameter modulation processes of four different modulated functions whose carrier frequencies correspond to the basic frequencies of harmonics appearing in the solution.

Let us note that the presented models showing the influence of defect origin and development are clearly associated with the development of the phenomenon of modulation of the signal's parameters. This is only partly confirmed by the results of analyses of the spectra generated by defective gears. Another effect that should be in addition taken into account is the occurrence of non-linear effects. For that reason the vibroacoustic signal generated by a defect should presented as a higher order components, which includes cases of non-linearity of the second, third or even fourth order:

$$y(t) = x(t) + \varepsilon\left(x(t)\right)^2 + \sigma\left(x(t)\right)^3 + \delta\left(x(t)\right)^4 \qquad (26)$$

It is sufficient in the case of systems or sets of machines with not so complex dynamic and kinematic structure. Let the results of simulation, as presented in [2], be an example of the fact that such a model of signal generation is unable to explain in a sufficient degree the spectrum's change in connection with a developing defect.

Figures 2-3 present the example of evolution of the spectrum simulated by the model of non-defective and defective two-stage toothed gear.

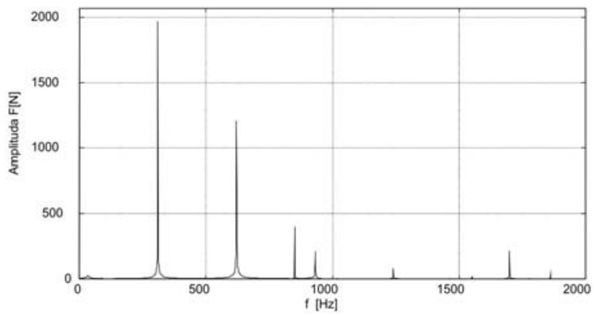

Fig. 2. Amplitude spectra of simulated response of bearings in an "ideal" toothed gear

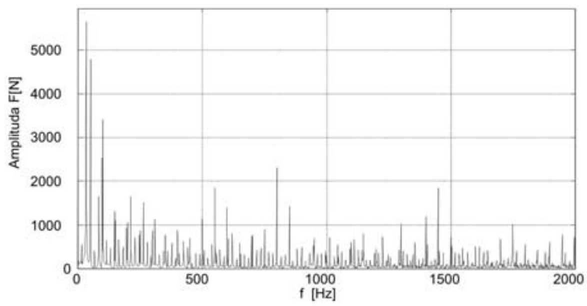

Fig. 3. Amplitude spectra of simulated response of bearings in a toothed gear with 2nd degree pitch error the pinion

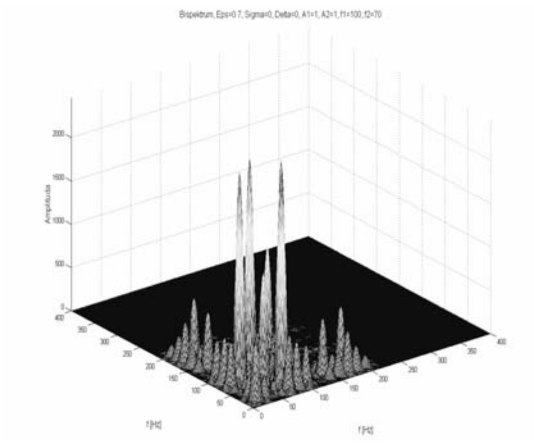

Fig. 4. Bispectrum of a signal with mutual modulation and squared non-linearity

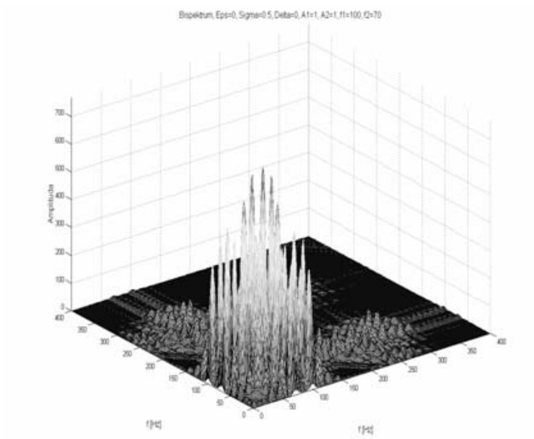

Fig. 5. Bispectrum of a signal with mutual modulation and non-linearity of the third order

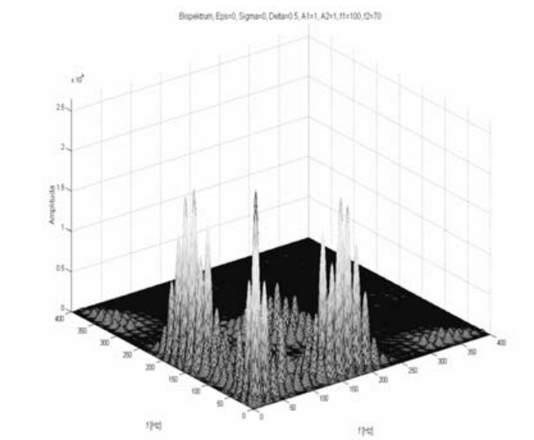

Fig. 6. Bispectrum of a signal with mutual modulation and non-linearity of the fourth order

While explaining the problem let us note that by taking into account the two-stage toothed gear, we as result are able to follow the vibroacoustic signal generated by two pairs of toothed wheels, that is by two sources. This means that the measured vibroacoustic signal is the sum of the minimum values from the two signals whose complexity, in this case the modulation of the parameters, depends on defect development. Bearing in mind the associated developing impact of non-linear effects one should expect coincidence of the influence of both phenomena. Thus one should include the phenomena of mutual modulation and inter-modulation in the description of changes of the frequency structure of the signal generated by the defective two-stage toothed gear [11].

Let us note that the phenomena of inter-modulation and mutual modulation enable, on the one hand, the explanation of the mechanism of emergence of additional components in the spectrum, and on the other they constitute interesting basis for diagnostic inference. Above all thanks to the analysis of the relations between the emerging defect and the developing modulation effect we can observe the growing role of mutual modulation in shaping the frequency structure of a vibroacoustic signal.

Assuming that numerous components are related to each other due to phase coupling, one should expect that significantly better results of diagnostic inference will be brought by applying multi-dimensional spectra, especially the bi-spectrum.

The relevant results of analyses are presented in Figures 4÷6. What is worth noting is the extension of the frequency band as the degree of non-linearity increases. Also the structure of bands which are characteristic for a square phase coupling can be a distinctive feature for a given type of non-linearity.

5. Applying of symbolic time series

One of problems conditioning the effective and reliable diagnostic-and-programming inference is the possibility of defining and building a relevant, well-defined functional model of such environment. It enables us to correctly determine the physical values and to select the relevant measurement and registration devices, which will enable proper measurement.

The awareness that we are not always able to construct a well-defined dynamic diagnostic model which is characterized by controlled uncertainty, directs the attention of people responsible for diagnosis towards direct utilization of the time series that are variable in time for the purpose of constructing such models.

The results we have obtained to date confirm that the right choice of relevant parameters of signal analysis, including the range of equipment dynamics, sampling frequencies and the ratio of useful signal to noise have significant impact on the credibility of results.

Thus there is increasing focus on the analytical methods which apply relevant weights to the structure of the experimental data set, particularly in the context of establishing or detecting the physical relations and links between the results of measurements and the occurring phenomena which result from or are associated with the processes taking place during operations. Such methods include the symbolic analysis of a time series, which has attracted a lot of attention in recent publications related to experimental data analysis.

The symbolic analysis of a time series is strictly connected with symbolic dynamics, which has developed together with the examination of complex dynamic systems. The closest to experimental research use of symbolic dynamics is presented in [6, 8].

While looking from our point of view we should stress that there exist no general rules of creation of measurement signal partitions that contain random noise.

The awareness of the limitations associated with symbolic dynamics leads to a situation that the selection of parameters in the symbolic analysis of a time series takes place on the basis of arbitrary decisions or experience in conducting experimental data analysis.

An essential element of symbolic analysis is the determination of sequences of symbolic values selected from a predefined set of symbols. The sequencing procedure by assumption contains a kind of a diagnostic model having the form of a template with a defined length which, when moved step by step in time, sets a different sequence each time. Each of such models of sequences is a new expression of the symbolic time series. Let us note that such a method of construction of symbolic sequences can be compared to immersion in a multi-dimensional space generated by a relevant time lag. In reality no such analogy exists and thus it is hard to expect that symbolic time series will have the same information value as a series obtained as a result of the immersion procedure.

Let us note that for a defined dynamic system and observed process the length of a symbolic sequence is correlated with the selection of sampling frequency on the one hand and the models of the observed phenomenon on the other. Too high sampling frequency can cause the effect of redundancy of information while too small frequency leads to loss of essential diagnostic information and impossibility of accomplishing the assumed diagnostic goal. Various methods are applied to evaluate the adopted sampling method, including the mutual information function:

$$I(\tau) = \sum p_{ij}(\tau) \frac{p_{ij}(\tau)}{p_i p_j} \qquad (27)$$

where τ – the lag resulting from the assumed symbol of digitization of measurement results.

Thus, in the case of analysis of results of measurements of complex systems with complex dynamics we can use symbolic transformation which enables transformation of original measurement results into a limited set of discrete symbols.

In the simplest case, when we apply a series composed of binary symbols and while assuming a 3-element sequence, we have the possibility of analyzing 2^3=8 various sequences and examining of dynamics while using a histogram of symbol sequences. For the same binary series and expression length D=2 we will get a four-element set of conditions {00, 01, 10, 11} for which the transitional matrix shall take the form of [12]:

	00	01	10	11
00	P_{00}	$1-P_{00}$	0	0
01	0	0	P_{01}	$1-P_{01}$
10	P_{10}	$1-P_{10}$	0	0
11	0	0	$1-P_{11}$	P_{11}

While using the measure in the form of a matrix norm**(?)** or applying the method of state equations for Markov models, we can detect the occurrence of a defect. Let us note that in such a case our k nowledge about the occurring phenomenon is based on the analysis of results of observation.

6. Conclusions

Progress in technical diagnosis, in combination with development of micro technology and sensorics, offers the possibility of developing new methods which enable formulation of more reliable forecasts of technical condition changes, thus reducing the uncertainty in the process of operational decisions. At the same time the cost of such systems enable their application in technical objects which present smaller threats for the environment and are characterized by relatively low prices, e.g. general purpose toothed gears.

Due to this an item which is of particular interest is the possibility of forecasting the fatigue-related destruction while relying on the analysis of vibroacoustic signal's structure. This is particularly related to examination of the process of generation and transmission of diagnostic information during the early stages of defect development.

Generally attention is drawn to the fact that defects of contact surfaces, corrosive and erosive wear, emergence of cracks and chipping are the reasons of occurrence of amplitude, phase and multi-parameter modulation of vibroacoustic signals. As a result, apart from accounting for the changes in power distribution for a defined harmonic or between harmonics, the model should distinguish the modulating and modulated functions and it should also describe the occurring modulation phenomena. Additional difficulty is that, as has been proven, along with the development of defects, the set of modulating and carrier functions that contain diagnostic information can change. If in parallel we account for the difficulties occurring during analysis of evolution of signals modulated by many parameters and caused by non-linear effects, then the unsatisfactory, till now, effectiveness of such models in diagnosis of defect development process becomes more comprehensible. On the other hand the low level of the useful signal vs. the noise and the need for applying the relevant selection of signal features make the selection of diagnostic signals with high information content the central issue.

While accounting for the natural feature of vibroacoustic diagnosis, which results from the possibility of registration of big number of vibration runs (leading to excessive information that is most often not fully utilized), the paper presents the issues of data compression and useful diagnostic information selection. In the case of multi-dimensional diagnostic signals the information of defect development is often contained not in the variability of absolute values of respective variables' measures but in the changes of relations between the variables which describe the course of a given phenomenon.

The set of main components, obtained thanks to the applied transformation and reduced in terms of dimensions, can enable extraction of a hidden structure of variables which serve as the basis for diagnostic models. Such a model combines the information on the course of operation and the accompanying wear and tear as well as degradation processes with the information on permitted boundary values for emergency/failure states. On the one hand the projection of the hidden structure enables the reduction of the dimensionality in measurement data and the vector of observed technical state, while on the other hand it enables defining the set of the most correlated components of the diagnostic vector and the technical condition vector, thus enabling not only the explanation of changes in the symptom vector but also ensuring the possibly most effective prediction of technical condition.

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