

ON THE SYSTEM: TRUCK – WORKSHOP IN THE QUEUE THEORY TERMS

There are a lot of technical systems in engineering world that apply tire haulage as significant component of their structure. It is quite obvious that in order to maintain a haulage fleet in proper conditions the system must possess a preservative subsystem to meet planned maintenance and repair requirements. A suitable organization of this subsystem determines the number of transport units accomplishing the given transportation task.

In the paper the problem of proper mathematical description of the system: truck – workshop is being considered employing models taken from the queue theory. The main characteristic that is obtained is the probability distribution of a number of failed trucks as the function of several parameters such as the fleet size and possession of a reserve, reliability indices of machines and parameters characterizing intensity of realized repairs and number of repair stands.

Keywords: Marjanovitch model, Sivazlian and Wang model, heavy traffic situation.

1. Introduction

The system that comprises trucks and workshop is widely employed in the engineering world. Civil engineering, mining, earthmoving, ordinary transporting firms are – among the other things – examples of scope of application of that system.

In some areas of applications, the system is the only one, self-dependant. In some areas the system is only a certain component of larger system e.g. in surface mining. But there is no doubt that its characteristics determine the whole systems' character.

It is quite astonishing that the main point of interest for the system that comprises tire transportation units was and still is the system: loading units – transporting machines. And nothing but that. The preserving subsystem that is obviously needed was usually considered taking into account planned maintenance and possible preventive actions for carrying devices. Mathematical identification of the generated stream of machines for repair was considered reluctantly, usually assuming the Poisson character of it, rarely presuming more general nature of this process.

In Polish literature the above statements can be easily proved making a review of more important papers that have occurred in last fifty years in this regard. Almost all papers neglected the problem of mathematical determination of number of repair stands for the maintenance shop.

Before the publication [10] made by Takács in 1962 where the mathematical description of model $M/G/1/m$ was done, application of Palm's system was used exclusively, i.e. the incoming stream of clients was identified as Poisson one and the service was described by exponential distribution. Some years later Kopocińska in article [5] considered employment of Takács model for the system: loading shovel – transporting trucks for proper organization of the system applied in the gravel mine. The system comprised of one loading machine and a certain number of trucks. All machines were totally reliable therefore a workshop was not needed. Mathematicians considering properties of the real system were aware of the fact that the employed model was significantly simplified versus reality. Therefore Huk and Łukasiewicz published article [3] making some small steps towards generalizations, mainly by application of the simulation technique. A year later Kapliński in paper [4] considered optimization of technological system employing model $M/M/1+m$ for analysis of selected problem in the civil engineering. A few years later Strykowski in paper [9] described a queue model for the system: shovels – trucks – crusher assuming Poisson character of incoming process and exponential time of service. Again, all

machines were totally reliable and the repair shop obviously did not exist. It should be added that even in textbook [1] published just recently – where machines were considered unreliable – the problem of the workshop has not arisen. However, one of the models presented in this work was suitable for modeling the system with repair stands. Finally, last year occurred the dissertation [2] in which the problem: trucks - workshop is well thought-out for proper organization of machinery system in the surface mining field.

The goal of this paper is to present some important outcomes of the cited dissertation that should be interested for engineers from different areas as well as to show some extensions in those considerations, results obtained just recently.

2. The system considered and mathematical model applied

The general model that can be applied to analyze the problem of operation of trucks – workshop system is Sivazlian and Wang one, remembering that the exploitation situation should fulfill *the heavy traffic situation* [8]. The operating scheme of the system can be illustrated as it is shown in Fig. 1.

This system can be described as follows. There are m trucks given to accomplish the transportation task and r trucks are in reserve (cold one). Working machines can fail with intensity δ and repair is being realized with intensity of γ . The number of repair stands is k . The standard deviations of random variables – work time between failures and repair time – are known and are identified as σ_p and σ_n respectively.

Let us neglect here the problem of proper selection of the reserve size. As it was proved in dissertation [2] three system parameters must be selected simultaneously: $\langle m, r, k \rangle$. But here we assume that these three parameters have been determined in a proper way for the system. If so, basing on the procedure given in cited dissertation, we are able to construct the probability distribution of number of trucks in failure P_j .

At first we have to specify relationship between the number of repair stands and the reserve size because this relation determines the set of patterns that should be taken into consideration [8].

Usually, in practice the following inequalities hold: $r < k < m$.

If, additionally, we check that the condition determining heavy traffic situation, is fulfilled:

$$k \leq \frac{m}{0.75} \frac{1 - A_w}{A_w} \quad (1)$$

where A_w is the truck long run availability, we are able to select the suitable set of patterns.

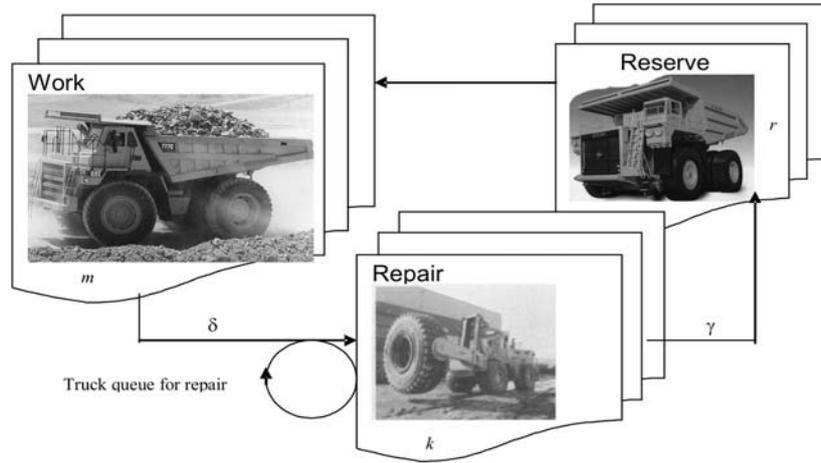


Fig. 1. Operating scheme of truck – workshop system

Here is the procedure that allows finding this distribution in such a way that is convenient for a pc application.

- Determination of power exponents:

$$\beta_1 = 2m\xi \frac{C_M + C_R}{(C_R)^2} \quad \beta_5 = 2(m+r)\xi \frac{C_M + C_R}{(C_R - \xi C_M)^2} \quad (2)$$

$$\beta_3 = \frac{2k}{\xi C_M} \left(1 + \frac{C_R}{C_M} \right)$$

where:

$$C_M = (\delta\sigma_p)^2, \quad C_R = (\gamma\sigma_n)^2, \quad \xi = \delta/\gamma; \quad (3)$$

- Construction of functions:

$$g_4(x) = \frac{1}{m\xi C_M + xC_R} \left(\frac{m\xi C_M + xC_R}{m\xi C_M} \right)^{\beta_1} \exp\left(\frac{-2x}{C_R}\right)$$

$$g_5(x) = \frac{1}{(m+r)\xi C_M + xC_R} \left(\frac{(m+r)\xi C_M + xC_R}{m\xi C_M + rC_R} \right)^{\beta_5} \exp\left(\frac{-2(\xi+1)(x-r)}{\xi C_M - C_R}\right) \quad (4)$$

$$g_6(x) = \frac{1}{(m+r)\xi C_M + kC_R} \left(\frac{(m+r-x)\xi C_M + kC_R}{(m+r-k)\xi C_M + kC_R} \right)^{\beta_3} \exp\left(\frac{2(x-k)}{C_M}\right)$$

- Two conditions to assure function continuity:

$$\alpha_1 = \frac{g_6(k)}{g_5(k)} \quad \alpha_2 = \frac{g_5(r)}{g_4(r)} \quad (5)$$

- Three constants to close probability to unity:

$$K_6 = \left(\alpha_1 \alpha_2 \int_0^r g_4(x) dx + \alpha_1 \int_r^k g_5(x) dx + \int_k^{m+r} g_6(x) dx \right)^{-1} \quad (6)$$

$$K_5 = \alpha_1 K_6 \quad K_4 = \alpha_1 \alpha_2 K_6$$

- The probability distribution of number of trucks in failure P_j is given by:

$$P_0 = \int_0^{0.5} K_4 g_4(x) dx$$

$$P_j = \int_{j-0.5}^{j+0.5} K_4 g_4(x) dx \quad \text{for } j = 1, 2, \dots, r-1;$$

$$P_r = \int_{r-0.5}^r K_4 g_4(x) dx + \int_r^{r+0.5} K_5 g_5(x) dx$$

$$P_j = \int_{j-0.5}^{j+0.5} K_5 g_5(x) dx \quad \text{for } j = r+1, \dots, k-1;$$

$$P_k = \int_{k-0.5}^k K_5 g_5(x) dx + \int_k^{k+0.5} K_6 g_6(x) dx$$

$$P_j = \int_{j-0.5}^{j+0.5} K_6 g_6(x) dx \quad \text{for } j = k+1, \dots, m+r-1;$$

$$P_{m+r} = \int_{m+r-0.5}^{m+r} K_6 g_6(x) dx \quad (7)$$

The above probability distribution is a function of seven variables: $m, r, k, \delta, \gamma, \sigma_p, \sigma_n$.

Depending on the data system, i.e. values of these seven parameters, we are now able to analyze and – in the next step – we are able to find directions for improvement of the system considered. This can be achieved by tracing changes in the basic system operation parameters as the result of changes in values of the variables. Two additional parameters could be pieces of important information for the system.

- The mean time that truck spends in a queue waiting for repair:

$$T_{ow} = \gamma^{-1} \sum_{j=k+1}^{m+r} (j-k) K_6 \int_{j-0.5}^{j+0.5} g_6(x) dx \quad (8)$$

- The mean truck time spent in failure, i.e. the sum of two means: that one determined by pattern (8) plus the mean time of truck repair T_n :

$$T_{ns} = T_n (1 + \phi) \quad (9)$$

$$\phi = K_6 \int_{k+0.5}^{m+r} g_6(x) dx \sum_{j=k+1}^{m+r} (j-k) K_6 \int_{j-0.5}^{j+0.5} g_6(x) dx$$

3. The problem of service saturation

It is obvious that parameter T_{ov} is a measure of system imperfection. The greater value of the parameter, the greater losses in production/truck service. The best solution that can be achieved in this field is when the number of repair stands equals the total number of trucks in the system. In such a case, we can say that the system is of full service saturation – no losses due to machines in failure without service.

From the theoretical point of view such a case was considered by Marjanovitch under the assumption that the machine work time between failures can be described by the exponential distribution (comp.[6]). Marjanovitch model is identified according to the Kendall system of notation as: $M/G/m+r/r$. Fortunately, in a lot of cases this exponential assumption is acceptable for operating trucks, Therefore the Marjanovitch model has been applied widely before Sivazlian and Wang model was developed.

It was proved (comp. [6]) that the probability distribution of number of machines in repair in the Marjanovitch model is given by:

$$P_k^{(n)} = \begin{cases} P_0^{(n)} (\kappa^k / k!) m^k & \text{for } 1, 2, \dots, r \\ P_0^{(n)} (\kappa^k / k!) m^r m(m-1) \dots (m-k+r+1) & \text{for } k = r+1, \dots, m+r \end{cases} \quad (10)$$

where: $\kappa = (1 - A_w) / A_w$ and obviously $\sum_{k=0}^{m+r} P_k^{(n)} = 1$.

Thus, if we consider Sivazlian and Wang model and we are going to increase the number k of repair stands we are moving towards Marjanovitch solution.

Let us notice that very often the probability of occurrence of event that all machines are down is very small, especially for larger system and/or for trucks of high reliability. So, it is of high interest to find such a number of repair stands for which the mean time in which a truck spends in failure waiting for repair is negligible. We presume speculatively that this number could be smaller than the total number of machines in the system. And it is true in reality. As it was shown by calculation [2] – considering

truck system for open pit mining – it is enough when the number of repair stands equals approximately 1/3 of the total number of trucks in the system. For lower reliability of machines this fraction is smaller. Fig. 2 shows general relationship between the mean number of trucks in failure E_u versus the number of repair stands k .

Generally, the following regularities have been identified:

- for trucks of high reliability the function $E_u(k)$ runs quickly towards the asymptote defined by Marjanovitch model; conclusion: the required number of repair stands for the shop is significantly smaller than the total number of trucks in the system,
- for trucks of low reliability the function $E_u(k)$ runs slowly towards the asymptote; the required number of repair stands for the shop is smaller than the total number of trucks in the system, but usually high enough to generate significant cost,
- considerations associated with this problem are of special value for large systems.

Interesting properties posses also the standard deviation function $S(X)$ of random variable – number of trucks in failure. Fig. 3 illustrates three plots of this function obtained analyzing the system operating in one of southern African open pits.

4. The probability distribution of number of trucks in work state

Our hitherto consideration was generally directed to find such a number of repair stands to reduce losses connected with the truck state in which this unit waits idly for repair. This problem was interested due to the fact that we would like to extend the work time of transporting means.

Having specified the probability distribution of machines in failure we are able to determine the probability distribution in work state. Denoting by $P_{wm}^{(p)}$ probability that machine is in work state, we can define:

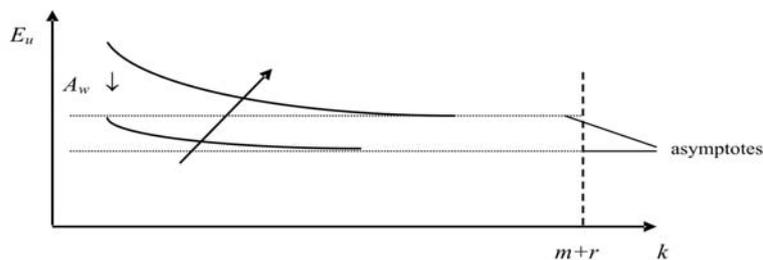


Fig. 2. The mean number of trucks in failure E_u as a function of number of repair stands k for different values of truck availability A_w .

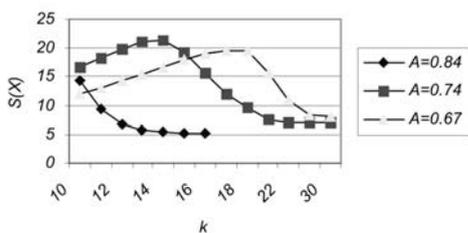


Fig. 3. The function of standard deviation of number of trucks in failure versus number k of repair stands for different levels of truck availability A_w .

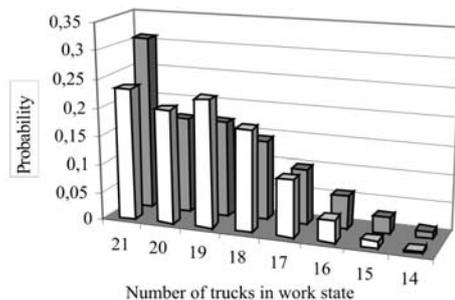


Fig. 4. The probability distribution of number of trucks in work state for two different systems with different reserve

$$\begin{cases} P_{wm}^{(p)} = \sum_{j=0}^r P_j \\ P_{w(m-j)}^{(p)} = P_{r+j} \quad \text{for } j = 1 + 2, \dots, m \end{cases} \quad (11)$$

The typical feature of this probability distribution is the accumulation of the mass of probability in point m because of the reserve. If in a system there are no spare units such regularity does not exist. The greater number of machines in reserve, the greater accumulation of the probability mass. Fig. 4 shows typical distributions for two different machinery systems having different size of reserve.

The probabilistic function (11) is the essential tool to compute the system output. In some modeling procedures, this is only a certain item being the information input to obtain the analyzed system important characteristics how it was shown in dissertation [2].

5. Final remarks

There are two important problems connected with the above considerations, namely:

- selection of three basic system parameters $\langle m, r, k \rangle$,
- repair stands can fail.

The first problem is of great significance. Selection of these parameters has influence on the amount of money spent to purchase transportation units (in some cases one truck can cost more than 3 billion US\$ and we need more than 100 such units

6. Preferences

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[1, 2]) and to arrange a maintenance bay (further 3 to 4 billion of US\$). Sometimes one hour of operation of a truck can reach several hundreds of US\$. It is easy to calculate what amount of money is involved in such a business. Therefore the selection of these three parameters can result in great money savings or great money losses. Generally, these parameters should be chosen to accomplish a transportation task formulated in a proper way. This mission should be attained in the cheapest way, assuring proper level of safety. Sometimes it is impossible to find the best economic solution, but fortunately, we are usually able to formulate optimizing criterion in a different way, not moving far from a financially viable key. Such a case was considered in monograph [2].

The second problem requires randomization of parameter k . At first glance, the problem looks simple. Usually it is assumed that repair stands operate independently on each other, all units are of the same nature and the workshop system consists of k such elements. Nevertheless, even in such case construction of the probability distribution of number of trucks in failure can make some troubles. It is connected with areas of validity of sets of patterns determined by values of basic system parameters. Situation becomes much more complicated when the assumption that all stands are of the same nature must be rejected. Immediately the problem of sequences of repair stands for a given type of repair comes together with the probability of their occurrence. For the time being it is much better to consider a particular system in this regard than to find general solutions.

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