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# MODELING OF ALGORITHMIC PROCESS RELIABILITY WITH FUZZY SOURCE DATA

*This paper proposes the method, which allows predicting such reliability figures of a discrete algorithmic process as the fuzzy time and the fuzzy probability of correct execution. Fuzzy numbers represents the uncertain source modeling data. Fuzzy rule bases used for taking into account dependence of source data on many influencing factors. Fuzzy logic inference, fuzzy extension principle together the crisp reliability models of algorithmic processes are used for modeling.*

*Keywords: Fuzzy Reliability, Algorithmic Process, Fuzzy Number, Fuzzy Extension Principle*

## 1. Introduction

Many discrete-behavior systems can be analyzed in a unified framework if combined into a class of so-called algorithmic processes. Typical algorithmic processes include information processing in computer systems, performance of research or design projects, technological production processes etc. Each of these processes involves a sequence of operations or jobs unfolding in time whose execution leads to the goal achievement. When designing a specific algorithmic process (AP), we need to estimate of the following reliability figures:

- $p_{AP}$ – the probability of correct AP execution; this may be interpreted as the reliability of output information, defect-free quality of the output products, reliability of system functioning;
- $t_{AP}$ – the time or other resources required to execute the AP.

Models to estimate $p_{AP}$ and $t_{AP}$ are widely used in reliability theory of man-machine systems [1, 2, 3]. In these studies, the modeling is based on the theory of semi-Markov processes [4] whose states correspond to the operators and logical conditions of the given algorithm. Successful application of AP reliability theory envisages construction of databases with reliability characteristics of the basic elementary operations. However, new operations do not have ex-post statistical estimates of outcomes under real-life conditions. Complex-system designers are therefore often forced to make decisions on the basis of following expert judgments: "*if the human operator is tired, then the number of errors is approximately doubled*" or "*if the equipment is properly maintained and is operated under appropriate conditions, then the reliability is high*".

The probabilistic algorithmic reliability theory [1, 2, 3] is incapable of utilizing input data expressed in the form of natural-language expert judgments. It is therefore relevant to try and develop a so-called "fuzzy reliability theory of algorithmic processes" [5, 6, 7], which together the probabilistic approach also uses fuzzy set theory [8], [9] that can manipulate linguistic expert information.

In this article we propose an approach that extends the probabilistic AP's reliability models to the case of fuzzy input data and allows for the dependence of data on influential factors through fuzzy inference. In terms of fuzzy reliability [10], extended AP's reliability models one can account as a branch of probist reliability theory with fuzzy probabilities.

## 2. Language for description the algorithmic processes

For formal description of AP we use the language of Glushkov's algorithmic algebras [11]. In this language, the algorithm operators are denoted by Latin capital letters ($A$, $B$, $C$, …) and logical conditions are denoted Greek lower-case letters ($\alpha$, $\beta$, $\gamma$, …). By the regularization theorem [11], every algorithm is representable a superposition of the following operator structures:

- $B=A_1A_2$ – linear structure consists of the process of consecutive operators $A_1$ and $A_2$ execution in the order of their registration;
- $C = \underset{\alpha}{(A_1 \vee A_2)}$ – $\alpha$-disjunction representing operator $A_1$ execution when condition $\alpha$ is true ($\alpha$=1), and execution of operator $A_2$ when condition $\alpha$ is false ($\alpha$=0);
- $D = \underset{\alpha}{\{A\}}$ – $\alpha$-iteration representing cyclic execution of operator $A$ till condition $\alpha$ has become true.

## 3. Probabilistic models of algorithm reliability

Let us assume that in execution of any operator A and logical condition $\omega$ the following events are possible:

$A^1(A^0)$ – correct (incorrect) execution of operator $A$;
$\omega^1(\omega^0)$ – condition $\omega$ is a priori true (false);
$\omega^{11}(\omega^{10})$ – an a priori true condition $\omega$ is recognized as true (false) during a check;
$\omega^{00}(\omega^{01})$ – an a priori false condition $\omega$ is recognized as false (true) during a check.

The above-listed events are assumed pairwise mutually exclusive. The probability (Prob) of these events is denoted by:

$$p_A^1 = Prob\ A^1 \quad p_A^0 = Prob\ A^0 \quad p_\omega = Prob\ \omega^1 \quad p_{\bar\omega} = Prob\ \omega^0$$

$$k_\omega^{11} = Prob\ \omega^{11} \quad k_\omega^{10} = Prob\ \omega^{10} \quad k_\omega^{00} = Prob\ \omega^{00} \quad k_\omega^{01} = Prob\ \omega^{01}$$

Note that $k_\omega^{10}$ and $k_\omega^{01}$ are the probabilities of type I and type II errors when checking condition $\omega$. The time for execution the operator $A$ and check the logical condition ω are denoted by $t_A$ and $t_\omega$.

Error-free execution of operator structures is defined by following logical functions:

$$B^1 = A_1^1 \wedge A_2^1;$$

$$C^1 = \left(\omega^1 \wedge \omega^{11} \wedge A_1^1\right) \vee \left(\omega^0 \wedge \omega^{00} \wedge A_2^1\right);$$

$$D^1 = a \vee (b \wedge a) \vee (b \wedge b \wedge a) \vee (b \wedge b \wedge b \wedge a) \ldots$$

where

$$a = A^1 \wedge \omega^{11}; \quad b = \left(A^1 \wedge \omega^{10}\right) \vee \left(A^0 \wedge \omega^{00}\right)$$

Given the logical functions of error-free execution of operator structures, we obtain the following rules for estimating the algorithm execution reliability:

$$B = A_1 A_2 \quad \Rightarrow \quad p_B^1 = p_{A_1}^1 \cdot p_{A_2}^1 \quad , \quad t_B = t_{A_1} + t_{A_2} \tag{1}$$

$$C = \underset{\omega}{(A_1 \vee A_2)} \Rightarrow \begin{cases} p_C^1 = p_\omega\, k_\omega^{11} p_{A_1}^1 + p_{\bar\omega}\, k_\omega^{00} p_{A_2}^1 \\ t_C = t_\omega + t_{A_1}\left(p_\omega\, k_\omega^{11} + p_{\bar\omega}\, k_\omega^{01}\right) + t_{A_2}\left(p_\omega\, k_\omega^{10} + p_{\bar\omega} k_\omega^{00}\right) \end{cases} \tag{2}$$

$$D = \underset{\omega}{\{A\}} \quad \Rightarrow \quad p_D^1 = \frac{p_A^1\, k_\omega^{11}}{1-\left(p_A^1\, k_\omega^{10} + p_A^0\, k_\omega^{00}\right)}, \quad t_D = \frac{t_A + t_\omega}{1-\left(p_A^1\, k_\omega^{10} + p_A^0\, k_\omega^{00}\right)} \tag{3}$$

## 4. Representation of uncertain source data by fuzzy numbers

Let q be an uncertain parameter that corresponds to the probability of error-free execution or the cost of executing the operator A or logical condition ω. The uncertain parameter q is treated as a linguistic variable [7] whose levels are formalized by fuzzy sets with convex membership functions defined on the universal set $U = [\underline{q},\ \overline{q}]$, where $\underline{q}$ and $\overline{q}$ are the smallest and greatest allowed values of the parameter $q$. In this case, the uncertain parameter $q$ is fuzzy number $\tilde{q}$. We represent the fuzzy number in following 3 forms: $l$-, $l(X)$-, and $\alpha$- forms.

**Definition 1.** The l-form of the uncertain parameter q is the triple:

$$\tilde{q} = <\underline{q},\ \overline{q},\ l>$$

where $l$ is the linguistic assessment (e.g. "Low", "Average", "High") of the parameter $q$ in the range $[\underline{q},\ \overline{q}]$ selected from the term-set $L=\{l_1, l_2, \ldots, l_m\}$ such that $l_j = \int_{q \in [\underline{q},\overline{q}]} \mu_{l_j}(q)/q$,

where $\mu_{l_j}(q)$ is the membership function of the value $q$ in the term $l_j \in L$, $j=1,m$.

**Definition 2.** The $\alpha$ -form of the uncertain parameter $q$ is the union of the pairs

$$\tilde{q} = \bigcup_{\alpha \in [0,1]} \left(\underline{q}_\alpha, \overline{q}_\alpha\right) \tag{4}$$

where $\underline{q}_\alpha(\overline{q}_\alpha)$ is the smallest (greatest) allowed value of q at the $\alpha$ -level of the membership function, i.e.:

$$\mu(q_\alpha) = \mu(\overline{q}_\alpha) = \alpha, \quad \mu(\underline{q}) = \mu(\overline{q}) = 0$$

**Definition 3.** The $l(X)$-form of the uncertain parameter q is the triple

$$\tilde{q} = <\underline{q},\ \overline{q},\ l(x)>$$

where $l(X)$ is the expert knowledge base in the form of systems of fuzzy logical propositions:

if $\left(x_1 = a_1^{j1}\right)$ and $\left(x_2 = a_2^{j1}\right)$ and...
...and $\left(x_n = a_n^{j1}\right)$ or …

if $\left(x_1 = a_1^{jk_j}\right)$ and $\left(x_2 = a_2^{jk_j}\right)$ and...
...and $\left(x_n = a_n^{jk_j}\right)$, then $l = l_j$,

where

$$a_i^{jp} = \int\limits_{x_i \in [\underline{x}_i, \overline{x}_i]} \mu^{jp}(x_i)/x_i, \quad i=\overline{1,n}, \quad p=\overline{1,k_j}$$

where $k_j$ is the number of fuzzy rules for $l = l_j$, and $\mu^{jp}(x_i)$ is the membership function of the variable $x_i$ to the fuzzy term $a_i^{jp}$ estimating the factor $x_i$ in rule with number jp, $i=\overline{1,n}, \quad j=\overline{1,m}, \quad p=\overline{1,k_j}$.

The $l(X)$-form ties the level 1 of the parameter $q \in [\underline{q}, \ \overline{q}]$ with the vector of influential factors $X=(x_1, x_2, ..., x_n)$. The $l(x)$ – form is transformed into $l$–form by fuzzy inference [7]. Transition from $l$–form to $\alpha$ –form is carried out via the membership function of fuzzy number.

## 5. Extending the reliability models to the fuzzy case

**Definition 4.** *Extension principle* [7]. If the function $y=f(q_1, q_2, ..., q_n)$ of n independent variables is given and its arguments $q_i$ are fuzzy numbers $\tilde{q}_i$ in $\alpha$ -form (4) ($i=\overline{1,n}$), then the value of function $\tilde{y} = f(\tilde{q}_1, \ \tilde{q}_2, ..., \ \tilde{q}_n)$ is fuzzy number $\tilde{y}$ represented in $\alpha$ -form:

$$\tilde{y} = \bigcup_{\alpha\in[0,1]} \left(\underline{y}_\alpha, \overline{y}_\alpha\right)$$

where

$$\underline{y}_\alpha = \inf_{\substack{q_{i_\alpha} \in [\underline{q}_{i_\alpha}, \overline{q}_{i_\alpha}] \\ i=\overline{1,n}}} \left( f(q_{1_\alpha}, q_{2_\alpha}, ..., \ q_{n_\alpha}) \right)$$

$$\overline{y}_\alpha = \sup_{\substack{q_{i_\alpha} \in [\underline{q}_{i_\alpha}, \overline{q}_{i_\alpha}] \\ i=\overline{1,n}}} \left( f(q_{1_\alpha}, q_{2_\alpha}, ..., \ q_{n_\alpha}) \right)$$

The extension principle easily produces fuzzy analogues of reliability models of algorithm execution (1) – (3). They are listed below (for each $\alpha$-set):

- linear structure $B=A_1A_2$:

$$\underline{p}_B^1 = \underline{p}_{A_1}^1 \cdot \underline{p}_{A_2}^1, \quad \overline{p}_B^1 = \overline{p}_{A_1}^1 \cdot \overline{p}_{A_2}^1$$

$$\underline{t}_B = \underline{t}_{A_1} + \underline{t}_{A_2}, \quad \overline{t}_B = \overline{t}_{A_1} + \overline{t}_{A_2}$$

- $\alpha$ -disjunction $C = (A_1 \vee A_2)_\alpha$:

$$\underline{p}_C^1 = min\left( p_\omega \ \underline{k}_\omega^{11} \underline{p}_{A_1}^1 + (1\text{-}p_\omega) \underline{k}_\omega^{00} \ \underline{p}_{A_2}^1 \right)$$

$$\overline{p}_C^1 = max\left( p_\omega \ \overline{k}_\omega^{11} \overline{p}_{A_1}^1 + (1-p_\omega) \ \overline{k}_\omega^{00} \overline{p}_{A_2}^1 \right)$$

$$\underline{t}_C = min(\underline{t}_\omega + \underline{t}_{A_1}(p_\omega \ k_\omega^{11} + (1-p_\omega)(1\text{-}k_\omega^{00})) + \\ + \underline{t}_{A_2}(p_\omega \ (1\text{-}k_\omega^{11}) + (1-p_\omega)k_\omega^{00}))$$

$$\overline{t}_C = max(\overline{t}_\omega + \overline{t}_{A_1}(p_\omega \ k_\omega^{11} + (1-p_\omega)(1\text{-}k_\omega^{00})) + \\ + \overline{t}_{A_2}(p_\omega \ (1\text{-}k_\omega^{11}) + (1-p_\omega)k_\omega^{00}))$$

where: $p_\omega \in \{\underline{p}_\omega, \ \overline{p}_\omega\}$, $k_\omega^{11} \in \{\underline{k}_\omega^{11}, \ \overline{k}_\omega^{11}\}$,

$k_\omega^{00} \in \{\underline{k}_\omega^{00}, \ \overline{k}_\omega^{00}\}$.

- $\alpha$ -iteration $D = \{A\}_\alpha$:

$$\underline{p}_D^1 = \frac{\underline{p}_A^1 \ \underline{k}_\omega^{11}}{1-\underline{p}_A^1 \ (1\text{-}\underline{k}_\omega^{11}) - \ \underline{k}_\omega^{00}(1-\underline{p}_A^1)}$$

$$\overline{p}_D^1 = \frac{\overline{p}_A^1 \ \overline{k}_\omega^{11}}{1-\overline{p}_A^1 \ (1\text{-}\overline{k}_\omega^{11}) - \ \overline{k}_\omega^{00}(1-\overline{p}_A^1)}$$

$$\underline{t}_D = min\left( \frac{\underline{t}_A + \underline{t}_\omega}{1-p_A^1 \ (1\text{-}\overline{k}_\omega^{11}) - (1-p_A^1) \ \underline{k}_\omega^{00}} \right)$$

$$\overline{t}_D = max\left( \frac{\overline{t}_A + \overline{t}_\omega}{1-p_A^1 \ (1\text{-}\underline{k}_\omega^{11}) - (1-p_A^1) \ \overline{k}_\omega^{00}} \right)$$

where $p_A^1 \in \{\underline{p}_A^1, \ \overline{p}_A^1\}$.

An example of application the fuzzy reliability models for assessment probabilistic-time characteristics of a ticket-booking information system is described in [7].

## 6. Optimization of algorithm reliability under fuzziness

The problem of optimization we can formulate by the next way. It is known:

- initial variant of AP: $Y=f(A_1,A_2, ..., An, \omega_1, \omega_2, ..., \omega_m)$;
- variants of realization of operators $A_i \in \{A_i^1, A_i^2, ...\}$ and logical conditions $\omega_j \in \{\omega_j^1, \omega_j^2, ...\}$, $i=\overline{1,n}$, $j=\overline{1,m}$;
- fuzzy probabilistic-time characteristics of each variant of operators and conditions realizations.

It is necessary to find such variant of AP structure (vector $X$) that provides the best level of AP time ($T$) and probability of correct execution ($P$):

$\tilde{T}(X) \rightarrow$ min subject to $\tilde{P}(X) \geq \tilde{P}*$ or

$\tilde{P}(X) \rightarrow$ max subject to $\tilde{T}(X) \leq \tilde{T}*$

where $\tilde{P}*$ and $\tilde{T}*$ are the admissible time threshold and the admissible threshold for error-free execution of the AP.

Reasonable techniques for the optimization are genetic algorithms or method of branches and boundaries. Compare fuzzy numbers $\tilde{P}(X)$ and $\tilde{T}(X)$ may be done via defuzzification.

## 6. Conclusions

The main obstacle to the application of probabilistic reliability models is the absence of input data that reflect real-life conditions describing the operation of the system. The method proposed in this paper for estimating the reliability of algorithms is one of the formal approaches to resolving the difficulty with source data by means of linguistic expert information and fuzzy extension principle. Contrary to semi-Markov models used in reliability theory, the proposed technique is free from time-consumed procedures for convolution of the distribution functions of the system sojourn time in a given state.

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