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THE USE OF IMPORTANCE MEASURES FOR THE OPTIMIZATION OF MULTI-STATE SYSTEMS

In this paper we propose an approach to the multiobjective optimization of a multi-state system (MSS) design, based on incorporating information from importance measures (IMs). More specifically, IMs come into play at the objective functions level in order to drive the search towards a MSS which, besides being optimal from the points of view of economics and safety, is also 'balanced' in the sense that all components have similar IMs values, without bottlenecks or unnecessarily high-performing components.

Keywords: *importance measures, multi-state systems, multiobjective optimization*

1. Introduction

Importance measures (IMs) quantify components contribution to the system performance (reliability, availability, risk, throughput) and allow tracing bottlenecks and weaknesses in the system design (Hřyland and Rausand 1994). The use of IMs is emerging also for Multi-State Systems (MSS), for which the performance can achieve multiple levels, e.g. 100%, 80%, 50% of the nominal capacity (Lisnianski and Levitin 2003).

This paper proposes an approach to system design in which the information provided by IMs is incorporated in the formulation of a multiobjective optimization problem to drive the design towards a solution which, besides being optimal from the points of view of economics and/or safety, is also 'balanced' in the sense that all components have similar importance values, without bottlenecks or unnecessarily high-performing components.

2. Importance measures for Multi-State Systems

Consider a MSS made up of n components. The performance level of component *j*, X_j, j=1, 2, …, *n*, can assume one of m_j+1 values, x_{j0}, x_{j1}, ..., x_{jmj} ($0=x_{j0}\le x_{j1}\ldots\le x_{jmj}$) and the system performance W can assume one of $m+1$ values, w_0, w_1, …, w_m ($w_0 \le w_1 \le\ldots\le w_m$).

In genral, IMs for MSS address the importance, with respect to the MSS output performance measure (e.g. reliability, availability, risk, throughput), that component j achieves a given level of performance α. For example, the α-level Fussell-Vesely IM, FV_j^α, of component j is (Zio and Podofillini 2003):

$$FV_j^\alpha = \left(E\left[W\right] - E\left[W_j^{\le\alpha}\right]\right) / E\left[W\right] \qquad (1)$$

FV_j^α represents the ratio of the decrement in the expected system performance due to the component *j* operating below α ($X_j \le \alpha$) to the nominal value of $E[W]$. Such measure quantifies the criticality of a reduction in performance of component *j* below level α.

3. Balanced optimization problem

Formally, in system optimization problems one has to optimise a vector of *Nf* objective functions (e.g. the system unreliability, unavailability, risk, profit):

$$f(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_{Nf}(\mathbf{x})) \qquad (2)$$

subjected to a vector of *Ng* constraints:

$$g(\mathbf{x}) = (g_1(\mathbf{x}), g_2(\mathbf{x}), \ldots, g_{Ng}(\mathbf{x})) \le 0 \qquad (3)$$

where $\mathbf{x}$ is the vector of the decision variables encoding a particular system design and/or maintenance strategy.

A desirable property of the optimal system is that of being 'balanced' with respect to the contributions of the components to its performance, having no bottlenecks or unnecessarily over-performing components.

This paper introduces an importance balancing objective in the multiobjective optimization problem formalized by eqs. (2) and (3).

More specifically, balance function σ_I is added to the usual optimization targets. With reference to the generic IM I_j^α, *j*=1, 2, …, *n*, we compute:

$$\sigma_I^\alpha = \sqrt{\overline{(I^\alpha)^2} - \overline{I^\alpha}^2}\;;\; \overline{(I^\alpha)^2} = \frac{1}{n}\sum_{j=1}^{n}(I_j^\alpha)^2;\; \overline{I^\alpha} = \frac{1}{n}\sum_{j=1}^{n} I_j^\alpha; \qquad (4)$$

If, for example, for a given α every component has the same importance, then

$$I_1^\alpha = I_2^\alpha = \ldots = I_n^\alpha \text{ and } \sigma_I^\alpha = 0.$$

Then:

$$\sigma_I = \sum_\alpha \sigma_I^\alpha \qquad (5)$$

is taken as a measure of the system balance. If $\sigma_I^\alpha = 0$ for every α, then $\sigma_I = 0$ and the system is fully balanced, free of bottlenecks or over-reliable components.

4. Application to multi-state system design

Consider a system made up of $n = 3$ multi-state components in series logic. Each component has $m_j = 4$ nonzero performance states with values of performance $x_{j0} = 0$, $x_{j1} = 25$, $x_{j2} = 50$, $x_{j3} = 75$, $x_{j4} = 100$, $j=1, 2, 3$. Let p_{jk} be the probability of component j of being in state k. Each of the three components has to be properly selected among the 11 available choices in Table 1, respectively. The alternatives differ in their state probability distributions and costs.

The design objectives to optimize are the expected system performance $E[W]$ and the system balance σ_{FV} with respect to the generalized Fussell-Vesely IM, FV_j^α of eq. (1).

In optimisation problem 1, we do not consider constraints on the total allocation cost, whereas the case of a maximum allocation cost C_{max} is considered in optimisation problem 2. The search space (11 ·11 ·11 alternative designs) has been spanned in both cases. The non-dominated solutions are listed in decreasing order of $E[W]$ in Table 2 and Table 3 for optimisation problems 1 and 2, respectively.

For problem 1, we report in Figure 1 the values of $FV_j^{\leq\alpha}$ (left) $\alpha = 0, 25, 50, 75$, $j = 1, 2, 3$, and p_{jk} (right), $k = 0, 1, \ldots, 4$, $j = 1, 2, 3$, for solution 1 of Table 2, the best with respect to the expected performance $E[W]$, and in Figure 1. the values for solution 16, the best with respect to σ_{FV}.

Table 1. Data for the eligible alternatives

	Component 1					Component 2					Component 3					
#	p_{10}	p_{12}	p_{12}	p_{13}	p_{14}	p_{20}	p_{22}	p_{22}	p_{23}	p_{24}	p_{30}	p_{32}	p_{32}	p_{33}	p_{34}	Cost
1	0.500	0.000	0.000	0.000	0.500	0.450	0.000	0.100	0.000	0.450	0.400	0.100	0.000	0.100	0.400	1.0
2	0.001	0.001	0.997	0.001	0.000	0.050	0.000	0.900	0.050	0.000	0.100	0.100	0.700	0.100	0.000	2.0
3	0.001	0.500	0.400	0.001	0.098	0.001	0.400	0.500	0.001	0.098	0.100	0.400	0.400	0.100	0.000	2.2
4	0.000	0.250	0.500	0.250	0.000	0.000	0.200	0.600	0.200	0.000	0.000	0.200	0.600	0.200	0.000	3.0
5	0.200	0.200	0.200	0.200	0.200	0.200	0.300	0.200	0.100	0.200	0.350	0.200	0.100	0.200	0.150	3.5
6	0.001	0.001	0.500	0.200	0.298	0.001	0.200	0.500	0.001	0.298	0.050	0.050	0.400	0.200	0.300	3.7
7	0.000	0.001	0.001	0.998	0.000	0.000	0.010	0.040	0.900	0.050	0.000	0.100	0.000	0.800	0.100	4.0
8	0.001	0.001	0.001	0.600	0.397	0.000	0.050	0.050	0.600	0.300	0.050	0.100	0.050	0.400	0.400	4.2
9	0.001	0.001	0.200	0.300	0.498	0.001	0.001	0.100	0.400	0.498	0.000	0.100	0.200	0.350	0.350	4.5
10	0.050	0.050	0.100	0.400	0.400	0.050	0.050	0.100	0.300	0.500	0.050	0.050	0.000	0.300	0.600	4.7
11	0.001	0.000	0.000	0.000	0.999	0.050	0.000	0.000	0.050	0.900	0.000	0.100	0.000	0.100	0.800	5.0

Table 2. Non-dominated designs (problem 1).

Design #	E[W]	σ_I	Cost	j = 1	j = 2	j = 3
1	84.420	0.616	15.0	11	11	11
2	78.730	0.535	14.7	11	11	10
3	71.460	0.424	13.7	8	9	11
4	69.220	0.315	13.4	8	9	10
5	68.580	0.137	12.0	7	7	7
6	66.560	0.106	12.2	7	8	7
7	63.840	0.103	12.5	7	7	9
8	62.070	0.086	12.7	7	8	9
9	56.930	0.079	12.7	9	9	6
10	52.820	0.075	11.9	6	9	6
11	46.120	0.071	10.2	2	9	6
12	45.860	0.052	11.5	4	7	9
13	44.170	0.029	11.0	9	9	2
14	42.380	0.028	8.5	2	9	2
15	41.040	0.024	10.0	4	7	4
16	37.250	0.009	9.0	4	4	4

Table 3. Non-dominated designs (problem 2).

Design #	E[W]	σ_I	Cost	j = 1	j = 2	j = 3
1	49.520	0.090	11.0	7	7	4
2	46.120	0.071	10.2	2	9	6
3	44.170	0.029	11.0	9	9	2
4	42.380	0.028	8.5	2	9	2
5	41.040	0.024	10.0	4	7	4
6	37.250	0.009	9.0	4	4	4

The best solution with respect to $E[W]$ (solution 1 in Table 2) corresponds to the design vector $\underline{x}^1 = (11; 11; 11)$, where alternative 11 is the most performing for all three components, i.e. the one with state probability distribution most shifted towards high-performance states. Figure 1 (left) shows the values of $FV_j^{\leq\alpha}$, $j = 1$,

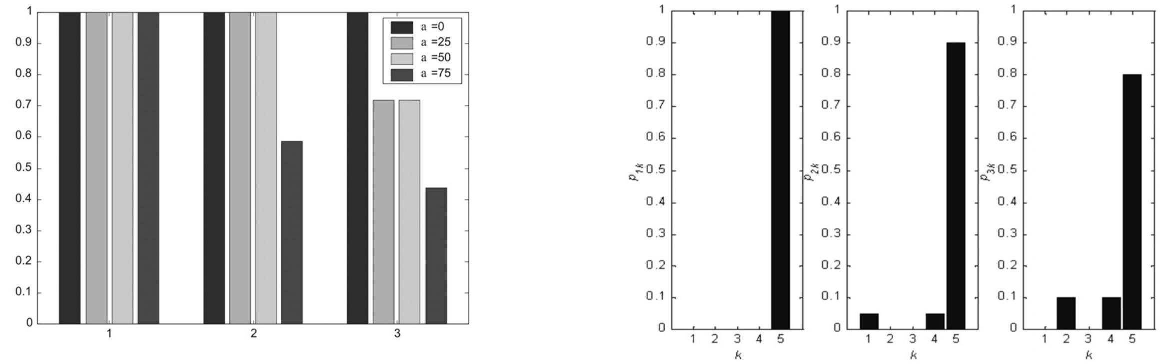

Fig. 1. $FV_j^{<\alpha}$ measures for solution 1 in Table 2 (selected components 11; 11; 11)

2, 3, $\alpha = 0, 25, 50, 75$, corresponding to solution 1. The importance measures assume rather different values for the three components, thus reflecting an unbalanced system configuration. This is due to the fact that the three selected solutions 11 are characterized by rather different performance distributions (Figure 1, right).

Solution 16 in Table 2, $\underline{x}^{16} = (4; 4; 4)$, shows features dual to solution 1, with high system balance and low overall system performance. Indeed, Figure 2 (left) shows that the $FV_j^{\leq\alpha}$, $\alpha = 0, 25, 50, 75$ are almost identical for the three components $j = 1, 2, 3$: this is due to the fact that, as Figure 2 (right) reveals, the three solutions have very similar performance distributions.

In order to highlight the improvements provided by the proposed multiobjective optimisation approach we report in Figure 3 the $FV_j^{\leq\alpha}$ (left) $\alpha = 0, 25, 50, 75$, $j = 1, 2, 3$, and p_{jk} (right), $k = 0, 1, \ldots, 4$, for solution 15, $\underline{x}^{15} = (4; 7; 4)$ to be compared with those of solution $\underline{x}^{16} = (4; 4; 4)$. The latter represents the most balanced system, ($\sigma_{FV} = 0.009$), with $E[W] = 37.250$ and differs

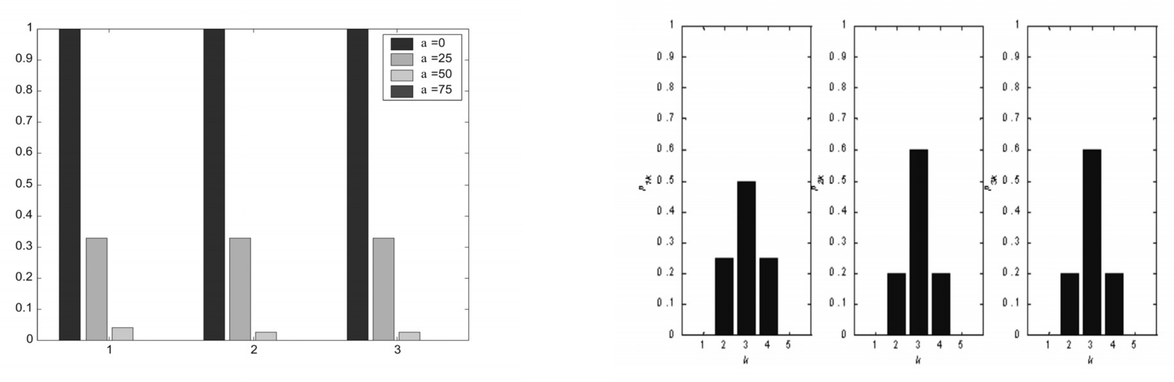

Fig. 2. $FV_j^{<\alpha}$ measures for solution 16 in Table 2 (selected components 4; 4; 4)

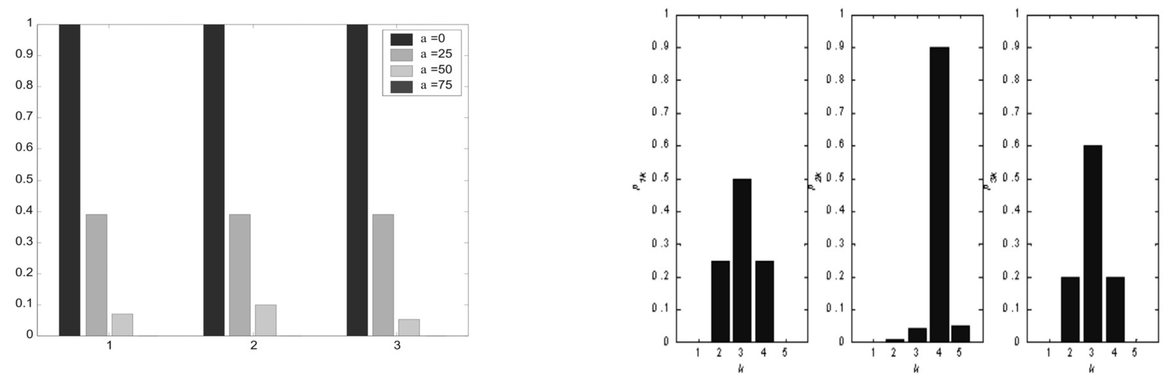

Fig. 3. $FV_j^{<\alpha}$ measures for solution 15 in Table 2 (selected components 4; 7; 4)

from solution $\underline{x}^{15}$ for the choice of allocating alternative 4 instead of alternative 7 at component 2. Although the expected performance of alternative 7 is 1.5 times as high as that of alternative 4 ($E[X_7]$ = 74.75 and $E[X_5]$ = 50.05 from the data of Table 1), the resulting increment in $E[W]$ is only of a factor of 1.1 ($E[W(4; 7; 4)]$ = 41.06, $E[W(4; 4; 4)]$ = 37.25, Table 2). The reason for this stands in the 'bottleneck' effect of the first and the third components when alternative 7 is allocated as the second component: the first and third components most likely operate at performance 50 so that an amount of extra performance of alternative 7 as component 2 remains actually unexploited.

The example shows that incorporating IMs at the objective function level allows identifying design solutions in which the components have balanced performances, in the sense that they are chosen coherently with their role within the system and with the features of the other components of the system. By so doing, over-performances and bottleneck effects of components can be reduced.

5. References

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