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FAILURE ANALYSIS OF SERIES AND PARALLEL MULTI-STATE SYSTEM

The reliability of the Multi-State System is investigated by Dynamic Reliability Indices in this paper. These indices estimate influence upon the Multi-State System reliability by the state of a system component. Structure function and mathematical tools of Multiple-Valued Logic calculate them. Dynamic Reliability Indices for failure of parallel and series systems are examined in detail.

***Keywords**: Reliability, Multi-State System, Dynamic Reliability Indices, Multiple-Valued Logic*

1. INTRODUCTION

Interest in reliability analysis has increased substantially in resent years, because in many technical systems, reliability has been considered as an important design measure, e.g. in manufacturing system, in telecommunication system, in system for pattern recognition and power system (Ball et al. 1995; Lisnianski and Levitin 2003). As a rule the reliability analysis problem in design of a technical system is: given the characteristics of system components, compute a measure of system reliability. Generally, the system reliability model and its indices are required for the desicion of this problem.

Discrete probability models are typically employed in reliability analysis. In the most commonly studied model to which are investigated, system component can take on one of two states: failure or functioning. Similarly, the system model itself is in one of two states too. This model is named Binary System. Many problem of the Binary System have been settled. But this approach fails to describe many situations where the system can have more than two distinct states (Ball et al. 1995; Lisnianski and Levitin 2003).

Alternative decision for reliability analysis of technical system has been proposed as a *Multi-State System* (MSS). In this system, both the system and its components may experience more than two states, for example, completely failed, partially functioning and perfect functioning. The MSS is frequently required for applied problem. But reliability analysis for MSS with multi-state components is a complex subject in reliability (Lisnianski and Levitin 2003). Many theoretical problems remain to be solved in this area. One of them is crucial to identify the weakness of the system and how failure of each individual component affects properly to improve the system reliability. There are two tools for solving of this problem: Markov processes to analyze the system state transition process and the structure function to investigate the system topology. Thus the structure function does not allow to estimate the dynamic behavior of the MSS (Ball et al. 1995; Barlow and Wu 1978; Boedigheimer and Kapur 1994; Lisnianski and Levitin 2003).

We propose the approach for evaluation of dynamic properties of the MSS reliability by the *Dynamic Reliability Indices* (DRI). Basic and theoretical conceptions of this approach were determined in (Zaitseva 2003; Zaitseva et al. 2005). DRI are calculated with respect to structure function by the Logical Differential Calculus of *Multiple-Valued Logic* (MVL). These indices characterize the change of the MSS reliability that is caused by the change of a component state (Zaitseva et al.2005).

In this paper we investigate the special case of the MSS. It is a parallel system and a series system. These types of system are typically employed in reliability analysis (Coit and Smith 1996; Nahas and Nourelfath 2005). The DRI of parallel and series systems are dermined in this paper. New expressions for the probability of the parallel or series MSS are presented in this paper for two occasions: for the failure and for restoration of the MSS.

2. MSS MATHEMATICAL MODEL

A MSS has m states of reliability from 0 (it is the complete failure) to m-1 (it is the perfect functioning). Each component has m states too and it is denoted as x_i ($i = 1,\ldots, n$). The dependence of the system reliability (system state) on its components state is defined by the structure function identically:

$$(\boldsymbol{x}): \{0, 1, \ldots, m\text{-}1\}^n \rightarrow \{0, 1, \ldots, m\text{-}1\} \qquad (1)$$

In this paper we use the following assumptions for structure function (Boedigheimer and Kapur 1994; Lisnianski and Levitin 2003; Zaitseva 2003): *(a)* it is the MVL function; *(b)* the structure function is monotone i.e. $\phi(\boldsymbol{x})$ is non-decreasing in each argument and $\phi(s) = s$, $s \in \{0, \ldots, m\text{-}1\}$; *(c)* all components are s-independent and are relevant to the system.

The assumption (*a*) permits to use mathematical tools of MVL for structure function analysis. We use the Direct Partial Logic Derivatives (as the part of Logical Differential Calculus). The Direct Partial Logic Derivatives reflect changing the value of investigation function when the value of its variable. So we have possibility to investigate the changes of system reliability over the change of one of component states.

3. THE DIRECT PARTIAL LOGIC DERIVATIVE IN RELIABILITY ANALYSIS OF MSS

The Direct Partial Logic Derivative $\partial\phi(j\rightarrow h)/\partial x_i(a\rightarrow b)$ of function $\phi(x)$ (1) with respect to variable x_i reflects the fact of changing of function from j to k when the value of variable x_i is changing from a to b (Zaitseva 2003):

$$\partial\phi(j\rightarrow k)/\partial x_i(a\rightarrow b)=\begin{cases} m-1, & \text{if } \phi(a_i,\mathbf{x})=j \ \& \ \phi(b_i,\mathbf{x})=h \\ 0, & \text{in the other case}\end{cases} \quad (2)$$

where $\phi(a_i, x) = \phi(x_1,...,x_{i-1}, a, x_{i+1},...,x_n)$ and $\phi(b_i, x) = \phi(x_1,...,x_{i-1}, b, x_{i+1},...,x_n)$.

So, the Direct Partial Logic Derivative of the structure function allows to examine the influence the state change of i-th component into the system reliability.

The MSS failure in Direct Partial Logic Derivative terminology is represented as chenging of the function value $\phi(x)$ from j into zero ($\phi(x)$: $j\rightarrow 0$). The reliability decrease of the i-th system component is presented by the modification of the variable x_i from a to b (x_i: $a\rightarrow b$). The Direct Partial Logic Derivative in this case is (Zaitseva 2003):

$$\partial\phi(j\rightarrow 0)/\partial x_i(a\rightarrow b) \text{ for } a\in\{1, \ldots, m-1\} \text{ and } b\in\{0, \ldots, m-2\},\ b < a$$

Because the structure function is monotone (assumption (*b*)) this derivative is

$$\partial\phi(1\rightarrow 0)/\partial x_i(a\rightarrow a-1),\ a\in\{1, \ldots, m-1\} \quad (3)$$

4. THE DYNAMIC RELIABILITY INDICES

There are three groups of DRI (Zaitseva 2003; zaitseva et al. 2005). The first group is *Dynamic Deterministic Reliability Indices* (DDRI). These indices are sets of the boundary system states when the change of component states cause to the MSS failure or repairing of its. Note, these states are used for different measures of MSS reliability frequently (Boedigheimer and Kapur 1994; Meng 2005). They conform to the minimal paths and minimal cuts of system that are well know in reliability ananlysis (Ball et al.1995; Meng 2005).

The Direct Partial Logic Derivative (3) allows to formalize the calculation of DIRI, but the dimension of thise sets are a very high for real application. So, the *Component Dynamic Reliability Indices* (CDRI) and *Dynamic Integrated Reliability Indices* (DIRI) are used for applied problem.

CDRI characterize probabilities of MSS failure as the changes of the i-th component states:

$$P_f(i)=\sum_{a=1}^{m-1} p(i)^{1\rightarrow 0}_{a\rightarrow a-1}\cdot p_a(i) \quad (4)$$

$p_a(i)$ is the component probability of state a; $p(i)^{1\rightarrow 0}_{a\rightarrow a-1}$ is the structural probability of i-th component failure where the system fail:

$$p(i)^{1\rightarrow 0}_{a\rightarrow a-1}=\frac{\rho(i)^{1\rightarrow 0}_{a\rightarrow a-1}}{\rho_1} \quad (5)$$

$\rho(i)^{1\rightarrow 0}_{a\rightarrow a-1}$ is number of system states when the breakdown of the i-th component forces the system failure; ρ_1 is numbers of one values of structure function (if $\phi(\mathbf{x})=1$).

Note, number $\rho(i)^{1\rightarrow 0}_{a\rightarrow a-1}$ is obtained as number of values of Direct Partial Logic Derivative $\partial\phi(1\rightarrow 0)/\partial x_i(a\rightarrow a-1)$ whit respect to i-th variable which are not equal zero.

DIRI determine probability of a system failure with a modification of one of the component state. We take account of the assumption *(c)* for structure function of MSS and define DIRI as:

$$P_f=\sum_{i=1}^{n} P_f(i)\prod_{\substack{q=1\\ q\neq i}}^{n}(1-P_f(q)) \quad (6)$$

where $P_f(i)$ is determined in (4).

We consider the CDRI and DIRI for parallel and series systems in detail below.

5. DRI OF PARALLEL SYSTEM

We use the OR MVL functions for mathematical description of the parallel MSS:

$$\phi_p(\mathbf{x}) = \mathrm{OR}(x_1, x_2,..., x_n) = \max(x_1, x_2,..., x_n) \quad (7)$$

CDRI calculation are started with equations (4) and (5). The function (7) is symmetric function and its measures on defferent variables are equal (Xue and Yang 2003). So, the number $\rho(i)^{1\rightarrow 0}_{a\rightarrow a-1}$ in (4) is coincided for different variables of structure function (7):

$$\rho(i)^{1\rightarrow 0}_{a\rightarrow a-1}=\rho(t)^{1\rightarrow 0}_{a\rightarrow a-1} \quad \text{for } i\neq t$$

We calculated numbers $\rho(i)^{1\rightarrow 0}_{a\rightarrow a-1}$ for m = 2, 3, 4, 5 and n = 2, …, 10 by the Direct Partial Logic Derivates $\partial\phi(1\rightarrow 0)/\partial x_i(a\rightarrow a-1)$. Number ρ_1 for structure function of parallel MSS is calculated by the structure function (7). These experimental results are presented in Table 1.

The analysis of datas in Table 1 allows to make next results. Numbers $\rho(i)^{1\rightarrow 0}_{a\rightarrow a-1}$ exist for a=1 only and aren't for other values of this parameter:

$$\rho(i)^{1\rightarrow 0}_{a\rightarrow a-1}=\rho(i)^{1\rightarrow 0}_{1\rightarrow 0}=\rho_{fp}=1 \quad (8)$$

and the number of structure function values ρ_1 is defined as:

$$\rho_1 = 2^n - 1 \qquad (9)$$

Structural probabilities of the parallel MSS are defined according to (5) subject to (8) and (9):

$$p(i)^{1\to 0}_{1\to 0} = \frac{\rho(i)^{1\to 0}_{1\to 0}}{\rho_1} = \frac{1}{2^n - 1} \qquad (10)$$

CDRI for parallel system declared by (4) and (10):

$$P_{fp}(i) = \frac{p_1(i)}{2^n - 1} \qquad (11)$$

DIRI for parallel system are defined by CDRI as:

$$P_{fp} = \frac{1}{2^n - 1}\sum_{i=1}^{n} p_1(i) \prod_{\substack{q=1 \\ q\neq i}}^{n} \left(1 - \frac{p_1(q)}{2^n - 1}\right) \qquad (12)$$

So, the CDRI and DIRI for parallel MSS have next feature:

- these measures do not dependent on the value m of structure function (number of discrete level of reliability);
- the probability of the parallel MSS failure (11) and (12) decreases if the number of system component increases.

6. DRI OF SERIES SYSTEM

The AND MVL functions is used for mathematical description of the series MSS:

$$\phi_s(\boldsymbol{x}) = \mathrm{AND}(x_1, x_2, \ldots, x_n) = \min(x_1, x_2, \ldots, x_n) \qquad (13)$$

The function (13) is symmetric too. So, we can analyze only one in variables. Firstly, numbers $\rho(i)^{1\to 0}_{a\to a-1}$ and ρ_1 are determined by the Direct Partial Logic Derivates $\partial\phi(1\to 0)/\partial x_i(a\to a\text{-}1)$ and by the structure function (13) for m = 2, 3, 4, 5 and n = 2, …, 10 (Table 2).

According to data in Table 2, the breakdown of the series MSS is possible for parameter a = 1 only and numbers $\rho(i)^{1\to 0}_{a\to a-1}$ in this case are:

$$\rho(i)^{1\to 0}_{a\to a-1} = \rho(i)^{1\to 0}_{1\to 0} = \rho_{fs} = (m-1)^{n-1} \qquad (14)$$

Number of structure function values ρ_1 are by next equations:

$$\rho_1 = (m-1)^n - (m-2)^n \qquad (15)$$

Structural probabilities (5) of the series system subject to (14) and (15) is:

$$p(i)^{1\to 0}_{1\to 0} = \frac{(m-1)^{n-1}}{(m-1)^n - (m-2)^n} \qquad (16)$$

and CDRI of this MSS is defined as:

$$P_{fs}(i) = \frac{(m-1)^{n-1}}{(m-1)^n - (m-2)^n} \cdot p_1(i) \qquad (17)$$

The probability of the MSS failure if one of system component fails (DIRI) (6) for the series MSS is:

$$P_{fs} = \sum_{i=1}^{n} \frac{(m-1)^{n-1}}{(m-1)^n - (m-2)^n} \cdot p_1(i) \times \prod_{\substack{q=1 \\ q\neq i}}^{n} \left(1 - \frac{(m-1)^{n-1}}{(m-1)^n - (m-2)^n} \cdot p_1(q)\right) \qquad (18)$$

Table 1. Numbers $\rho(i)^{1\to 0}_{a\to a-1}$ and ρ_1 for for parallel system

n	$\rho(i)^{1\to 0}_{a\to a-1}$										ρ_1			
	m=2	m=3		m=4			m=5				m=2	m=3	m=4	m=5
	a=1	a=1	a=2	a=1	a=2	a=3	a=1	a=2	a=3	a=4				
2	1	1	0	1	0	0	1	0	0	0	3	3	3	3
3	1	1	0	1	0	0	1	0	0	0	7	7	7	7
4	1	1	0	1	0	0	1	0	0	0	15	15	15	15
5	1	1	0	1	0	0	1	0	0	0	31	31	31	31
6	1	1	0	1	0	0	1	0	0	0	63	63	63	63
7	1	1	0	1	0	0	1	0	0	0	127	127	127	127
8	1	1	0	1	0	0	1	0	0	0	255	255	255	255
9	1	1	0	1	0	0	1	0	0	0	511	511	511	511
10	1	1	0	1	0	0	1	0	0	0	1023	1023	1023	1023

Table 2. Numbers $\rho(i)^{1\to 0}_{a\to a-1}$ and ρ_1 for series system

n	$\rho(i)^{1\to 0}_{a\to a-1}$										ρ_1			
	m=2	m=3		m=4			m=5				m=2	m=3	m=4	m=5
	a=1	a=1	a=2	a=1	a=2	a=3	a=1	a=2	a=3	a=4				
2	1	2	0	3	0	0	4	0	0	0	1	3	5	7
3	1	4	0	9	0	0	16	0	0	0	1	7	19	37
4	1	8	0	27	0	0	64	0	0	0	1	15	65	175
5	1	16	0	81	0	0	256	0	0	0	1	31	211	781
6	1	32	0	243	0	0	1024	0	0	0	1	63	665	3367
7	1	64	0	729	0	0	4096	0	0	0	1	127	2059	14197
8	1	128	0	2187	0	0	16384	0	0	0	1	255	6305	58975
9	1	256	0	6561	0	0	65536	0	0	0	1	511	19171	242461
10	1	512	0	19683	0	0	262144	0	0	0	1	1023	58025	989527

DIRI for the series MSS in equation (18) depends on the value of structure function m (the number of reliability discrete levels) and number of its variables *n* (number of system component). So, these indices can be defined for MSS with a large dimensionality, because Direct Partial Logic Derivatives have not used for they calculation as distinct from algorithms in papers (Zaitseva 2003; Zaitseva et al. 2005).

7. CONCLUSION

DRI are different from measures which well known in reliability analysis of MSS (Coit and Smith 1996; Meng 2005; Xue and Yang 2003). The probability that a certain threshold of system performance has been attained is calculated for MSS in these paper as a rule: $P_s = Pr[\phi(\mathbf{x})\geq s]$, $s = \{0, \ldots, m\text{-}1\}$.

DRI are probabilities of changes of the system states depending on modifications of components states. But the algorithm for its calculation in (Zaitseva 2003; Zaitseva et al. 2005) has restriction, because for estimation of DRI needs to compute Direct Partial Logic Derivatives that have high complexity calculation. One of the ways for decision of this problem serves as investigation of special type of MSS, for example, series and parallel system.

The CDRI and DIRI for special system (parallel and series MSS) are examined in this paper firstly. The expressions (11), (12) and (17), (18) define the dependence of the MSS failure on breakdown of a system component by the component probability, parameters m (number of reliability levels) and n (number of system components) only. Direct Partial Logic Derivates are not calculated in these cases and a complexity of CDRI and DIRI calculation reduces.

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