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SIMULATION APPROACH FOR MODELING OF DYNAMIC RELIABILITY USING TIME DEPENDENT ACYCLIC GRAPH

The dynamic reliability approach takes into account changes (evolution) of the system structure (hardware). For instance, the dynamic reliability allows modeling a human operator (or an electronic control system) naturally. In these cases, the structure of the system is usually changed in order to keep the functionality and/or safety of the system. The main purpose of the paper is to illustrate, by means of a model example, the ability of acyclic oriented graph, terminal nodes of which are programmable components, to model simple dynamic system and to assess its performance via Monte-Carlo simulations. To demonstrate the availability of our framework a test case study with the deterministic evolution is presented. The here presented numerical results are in agreement with the exact analytical solution.

***Keywords**: reliability, complex dynamic systems, oriented acyclic graphs, Monte Carlo simulation*

1. Principles of Dynamic Reliability Approach

One of the main problems in modeling of reliability of complex dynamic systems is to take into account time dependencies of the system structure resulting from changes of its physical parameters. The evolution of the system can be modeled by modifications of values of so called process variables (Pasquet et al., 1998, Labeau, 2000, Chabot et al., 2002). Unfortunately, the traditional modeling techniques see Fig. 1, which are usually based on Boolean modeling, such as Fault Trees and Event Trees, are not suitable for modeling of general dynamic systems because of statistical dependency between values of physical parameters and state of components (Chabot et al., 2002). Recently, Neural Networks and Petri Nets approaches were used as tools for reliability analysis of dynamic systems (Pasquet, et. al., 1998, Chabot et al., 2002).

In special cases when times of the structural changes are deterministically scheduled according to a considered time interval, it is possible to solve the problem of reliability assessment. The time partition may be given for example, as a result of evolution of a process variable (Labeau, 2000). The aims of this paper are to present, by means of a P.E. Labeau test-case benchmark demonstrated at ESREL 2002, (i) the efficiency of oriented acyclic graphs (Bris et al., 2002, 2003) to model dynamic systems and (ii) assess their performance using the direct Monte Carlo (MC) simulation technique. Moreover, exactly the same test-case was successfully solved by the Petri Net approach (Chabot et al., 2002).

The structure of this paper is organised as follows. In Section 2, a Dynamic Reliability test-case is described. Section 3 clarifies ability of acyclic oriented graphs as a tool for modeling of dynamic systems. Finally, numerical results of MC calculations and future works are presented and discussed in Section 4.

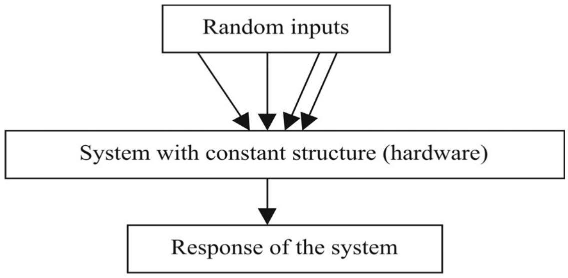

Fig. 1. Traditional Probabilistic Safety Assesment (PSA). Stochastic character of system inputs is described by random variables. The output of PSA is a response of the system, for instance probability of failure. In contrast of Dynamic Reliability approach, methodology of PSA assumes that the structure of the system is constant, non-changing, stable

2. Dynamic Reliability Test-Case Benchmark Description

This example was first proposed by P. E. Labeau. The exact problem description, an analytical solution and an alternative solution can be found at (Labeau, 2000). Let us shortly cite the problem description. For more details, please see (Labeau, 2000) and (Chabot et al., 2002).

"Consider a system (a tank) described by one process variable x (pressure), with its steady state value x_0. At time t_0, a transient is initiated in state 1, and the evolution of x follows an exponential law.

When the level $x = l$ is reached, a protection device is solicited (component C_1); it can either fail (with a probability p_0), or work correctly (with a probability $1 - p_0 - p_1$, state 2) or be imperfectly triggered (with a probability p_1, state 3). In the last two cases, x starts to decrease, and a safe shutdown is reached as soon as $x < x_0$. But if $x > L$, the system fails (tank rupture).

We add to this description the possibility of an additional component failure (component C_2; λ) that accelerate the transient, i. e. a transition from state i to $i + 3$, $i = 1, 2, 3$.

This more severe transient can still be mitigated in case of a perfect working of the protection device, but it is only slowed down in case of partial triggering. The evolution of x is now as follows:

$$\frac{dx}{dt} = \begin{cases} a_i x, & i = 1,4,6 \\ -a_i x, & i = 2,3,5 \end{cases} \quad a_i > 0, \ \forall i,$$

with $a_2 > a_3$, $a_4 = a_1 + b$, $a_5 = a_2 - b$, $a_6 = -a_3 + b$, and $b > 0$. The transition rate is assumed to be constant on $[x0, L]$ and to be 0 outside this range. This makes both final situations (failure and safe shutdown) absorbing".

Let T be the lifetime of the tank before its rupture, i.e., in other words, its failure time. Then the estimation of the probability $PL(t) = P(T \le t)$ is the objective of the calculations carried out via the time dependent acyclic graph model. Following (Labeau, 2000) and (Chabot et al., 2002), we will assume the following numerical values of the parameters of the system, see Tab 1.

Table 1. Parameters of the system

x_0	1	p_0	0.02	a_1	0.2	a_4	0.35
l	3	p_1	0.04	a_2	0.25	a_5	0.1
L	4	λ	4.10^{-2}	a_3	0.1	a_6	0.05

Dynamic reliability approaches suppose the deterministic evolution of the process variables, see Fig.2. In other words, we will assume that all changes of hardware configurations are determined (caused) by stochastic events and spontaneous changes of the hardware configuration are not considered (Pasquet et al., 1998).

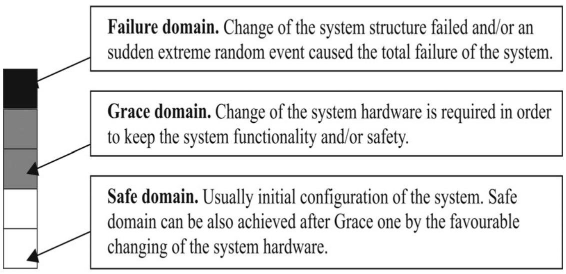

Fig. 2. Three possible states of the process variable in Dynamic Reliability approach

3. The Time Dependent Acyclic Graph Model

It can be shown that the benchmark has the deterministic evolution of the process variable. Moreover, the time partition may be computed analytically. Following (Chabot et al., 2002), we will use the following partitioning of the time-interval in the model, see Tab. 2:

Table 2. The time partition of the system mission time

T1	T2	T3	T4	T5	T6
0	3.961	6.931	8.892	11.247	44.205

The continuous evolution of the process variable *x(t)* and the discrete behaviour of components can be modeled by using the time dependent acyclic graph model, see Figure 3. The model uses the positive logic, so the top event is "Success". The value in a triangle represents the number of inputs, which must indicate "Success" in order to send the "Success" to the higher level. Let us shortly describe how the model works. Analyzing Tab.1 and Tab.2, the probability of the non-fail of the component K_1 is $[p_0] = 1 - p_0 = 1 - 0.02 = 0.998$. The component K_2 is modeled by the shifted exponential distribution with the shift 3.961. This component is switched off (permanent failure) at the time $t = 6.931$. Farther, the component K_1 is switched off after the time $t = 8.892$, etc.

Consequently (see Figure 3), during the time interval denoted as **a**, only corresponding part of the acyclic graph represents the system behaviour. The block inside the dashed box in the same Figure 3, represents the situation "C_2 fails in $[0, \tau)$ and C_1 is imperfectly triggered", with $\tau = 7/3\{t - [20 \ln 4 - (120/7) \ln 3]\}$. In this case, C_2 is modeled by the component K_3 with the shifted exponential distribution, where

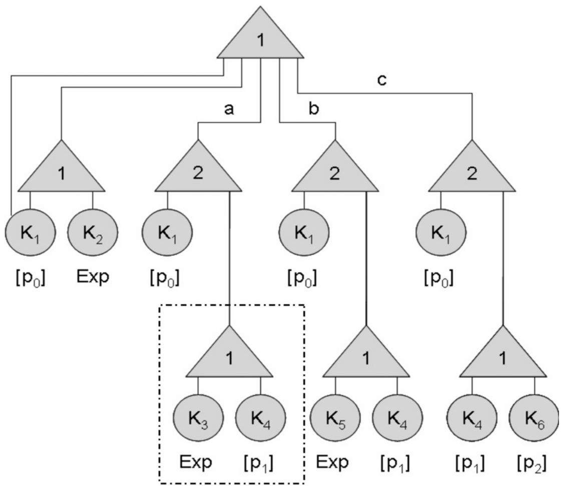

Fig. 3. The time dependent acyclic graph representation of the dynamic reliability test-case. Symbols a, b, c denotes the following time intervals: a = (8.892; 11.247), b = (11.247; 44.205), c = (44.205; ∞)

$\lambda = (7*4.10^{-2})/3$ and the shift is 20 ln 4-(120/7) ln 3. The K_5 is also modeled by the shifted exponential distribution with $\lambda = (4.10^{-2})/3$ and the shift: 20 ln 4 – 30 ln 3. Finally, the probabilities $[p_0]$ and $[p_1]$ are given as the complements of p_0 and p_1, see Tab. 1, and $[p_2] = 1 - \exp(-4.\ 10^{-2} *15 \ln 3) = 0.48272$, see (Chabot et al., 2002).

4. Results and Future Works

Using our updated simulation software (Bris R., 2003) and above described acyclic graph we have successfully verified, with high level of accuracy, numerical results of the benchmark, which correspond to the analytical results (Labeau P. E., 2000). The computed failure probability *PL(t) = P(T≤t)* is presented at Fig. 4. We concluded on the basis of our results that the time dependent acyclic graph can be successfully applied for the description and modeling of the dynamics in the benchmark.

At future time, we would like to solve dynamic reliability problems with maintained components (Bris et al., 2002) for more general class of problems from practice than the benchmark. To increase performance of the algorithm, we will test variance reduction techniques based on Importance Sampling (Praks et al., 2003).

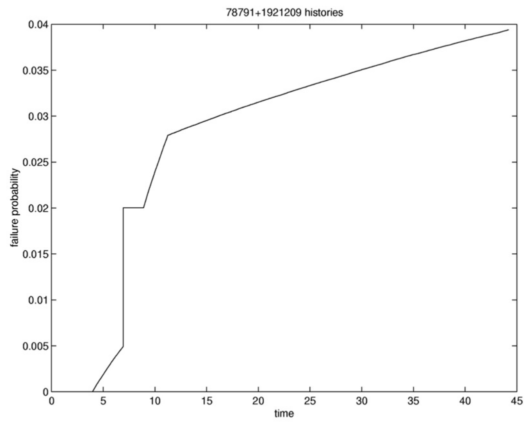

Fig. 4. Evolution with time of PL(t) for $\lambda = 4.10^{-2}$ computed via direct Monte Carlo simulation

5. Acknowledgements

This work is supported by The Ministry of Education, Youth and Sports of the Czech Republic. Project „Research of the reliability of power systems in connection with non-traditional ecological sources of energy and valuation of unsupplied energy“, CEZ MSM6198910007.

6. References

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