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FUZZY PROBABILISTIC MODELS IN STRUCTURAL RELIABILITY

Two types of uncertainties can be generally recognised in structural reliability: natural randomness of basic variables and vagueness of performance requirements. While the randomness of basic variables is handled by common methods of the probability theory, the vagueness of the performance requirements is described by the basic tools of the theory of fuzzy sets. Both the types of uncertainties are combined in the newly defined fuzzy probabilistic measures of structural reliability, the damage function and the fuzzy probability of failure. The proposed measures can be efficiently applied in a similar way as conventional probabilistic quantities for the verification and optimisation of structural reliability. Adequate data are however needed for further development of the outlined concepts.

Keywords: *structural reliability, fuzzy probabilistic measures, damage function*

1. Introduction

The performance requirements (serviceability constraints, structural resistance) of buildings and engineering works are often affected by various uncertainties that can hardly be described by traditional probabilistic models. As a rule, the transformation of human desires, particularly of those describing occupancy comfort and aesthetical aspects, to performance requirements often results in an indistinct or imprecise specification of the technical criteria for relevant performance indicators (for example permissible deflection, acceleration). Thus, in addition to the natural randomness of basic variables, the performance requirements may be affected by vagueness in the definition of technical criteria. Two types of the uncertainty of performance requirements are identified here: randomness, handled by commonly used methods of the theory of probability, and fuzziness, described by the basic tools of the theory of fuzzy sets (Brown 1983, Shiraishi 1983). Similarly as in the previous studies (Holický 1993, 1996 and 2001), the performance condition $S \leq R$, relating an action effect S and a relevant performance requirement R, is analysed assuming the randomness of S and both the randomness and the fuzziness of R.

2. Fuzzy probabilistic models of performance requirements

Fuzziness due to vagueness and imprecision in the definition of performance requirement R is described by the membership function $\nu_R(x)$ indicating the degree of the membership of a structure in a fuzzy set of damaged (unserviceable) structures (Holický 1993, 1996 and 2001); here x denotes a generic point of a relevant performance indicator (a deflection or a root mean square of acceleration) considered when assessing structural performance. A common experience indicates that a structure is loosing its ability to comply with specified requirements gradually within a certain transition interval $<r_1, r_2>$. The membership function $\nu_R(x)$ describes the degree of structural damage (lack of functionality). If the rate $d\nu_R(x)/dx$ of the "performance damage" in the interval $<r_1, r_2>$ is constant (a conceivable assumption), then the membership function $\nu_R(x)$ has a piecewise linear form as shown in Figure 1. It should be emphasized that $\nu_R(x)$ describes the non-random (deterministic) part of uncertainty in the requirement R related to economic and other consequences of inadequate performance. The randomness of R at any damage level $\nu = \nu_R(x)$ may be described by the probability density function $\varphi_R(x|\nu)$ (see Figure 1), for which the normal distribution having a constant standard deviation σ_ν is considered here.

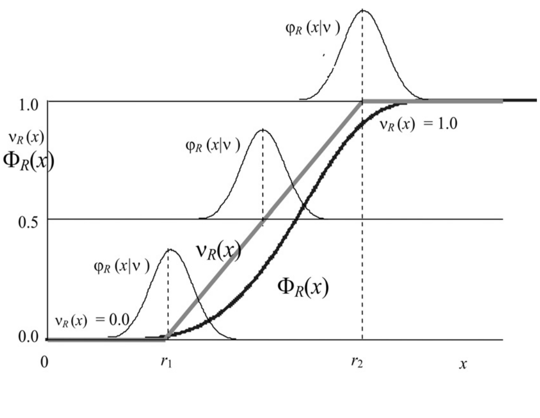

Fig. 1. The fuzzy probabilistic model of the performance requirement R

The transition region $< r_1 , r_2 >$, where the structure is gradually losing its ability to perform adequately and its damage increases, may be rather broad depending on the nature of the performance requirement. For common serviceability requirements (deflections) the upper limit r_2 may be a multiple of the lower limit r_1 (for example $r_2 = 2\ r_1$). An extreme example is the case of continuous vibration in buildings specified in the International Standards (ISO 1989 and 1991) and discussed by Bachmann (1987) (see also a previous study by Holický (1996)). In general the acceleration constraints for continuous vibration are considered within a range from 0.02 to 0.06 ms^{-2}. There is a low probability of an adverse comment for accelerations below the lower limit $r_1 = 0.02$ ms^{-2}. On the other hand adverse comments are almost certainly expected for accelerations above the upper limit $r_2 = 0.10$ ms^{-2}, thus, in that case $r_2 = 5\ r_1$.

3. Fuzzy probabilistic measures of structural performance

The damage function $\Phi_R(x)$ is defined as the weighted average of damage probabilities reduced by the corresponding damage level (Holický 1993, 1996 and 2001)

$$\Phi_R\left(x\right)=\frac{1}{N}\int_0^1 \nu\ (\int_{-\infty}^{x} \varphi_R(x'\,|\,\nu)dx')d\nu \qquad (1)$$

where N denotes a factor normalising the damage function $\Phi_R(x)$ to the conventional interval <0, 1> (see Figure 1) and is a generic point of x. The damage function $\Phi_R(x)$ defined by equation (1) may be considered as a generalised distribution function of the performance requirements R that can be used for the specification of the design (or characteristic) value of the requirements R corresponding to a given level of the total expected damage. The density of the damage $\varphi_R(x)$ follows from (1) as

$$\varphi_R\left(x\right)=\frac{1}{N}\int_0^1 \nu\ \ \varphi_R(x\,|\,\nu)\ \ d\nu \qquad (2)$$

Figure 2 shows variation of the statistical parameters of the performance requirement R with $\sigma_\nu/(r_2 - r_1)$. It appears that Beta distribution with the origin at zero can be used as an approximation of $\varphi_R(x)$. If the standard deviation $\sigma_\nu = 0.2\ (r_2 - r_1)$, then $\mu_R = r_1 + 0.67(r_2 - r_1)$, $\sigma_R = 0.31(r_2 - r_1)$ and $\sigma_R = -\ 0.25$, Beta distribution has the bounds $a = 0$, $b = r_1 + 1.65(r_2 - r_1)$ and the shape parameters $c = 10.07$ and $d = 5.88$.

The fuzzy probability of performance failure π can be defined provided that the probability density function of the action effect S, denoted $\varphi_S(x)$ is known as

$$\pi\ =\ \int_{-\infty}^{\infty} \varphi_S(x)\Phi_R(x)dx \qquad (3)$$

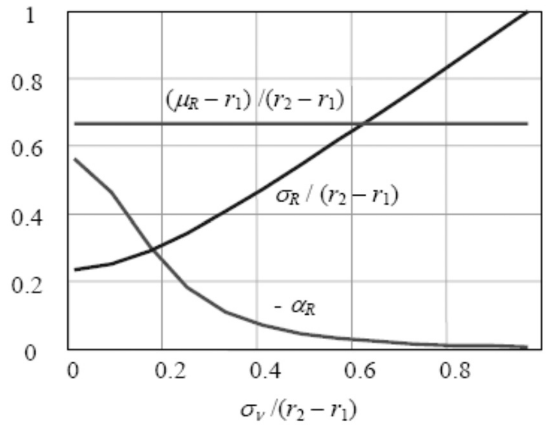

Fig. 2. Variation of the statistical parameters of the performance requirement R with $\sigma_\nu/(r_2 - r_1)$

An asymmetric three parameter lognormal distribution of S is accepted in earlier studies (Holický 1993, 1996 and 2001). The damage function $\Phi_R(x)$ defined by equation (1) and the fuzzy probability of performance failure π defined by equation (3) enable the formulation of various design criteria in terms of relevant randomness and fuzziness parameters. However, adequate data for the specification of the fuzziness parameters r_1, r_2, the membership function $\nu_R(x)$ and its standard deviation σ_ν (describing the requirement R), the probability density $\varphi_S(x)$ of the load effect S and its characteristics are needed.

4. Optimisation

The optimum value of the fuzzy probability of performance failure can be estimated using the technique of design optimisation (Holický 1996 and 2001). It is assumed that the objective function is given by the total cost $C(\xi)$ expressed approximately as the sum

$$C(\xi) = C_0(\xi) + \pi(\xi)\ C_D \qquad (4)$$

where $C_0(\xi)$ is given as the sum of the construction and maintenance cost, $\pi(\xi)\ C_D$ is the expected malfunction cost; here C_D denotes the cost of full damage (full malfunction or serviceability failure) and ξ denotes the decision parameter (for example the mass per unit length or the cross section area). It has been shown (Holický 1996 and 2001) that this equation can be used if the malfunction cost due to the damage level ν is given as the multiple $\nu_R(x)C_D$ (in the example of continuous vibration it represents the cost due to disturbance and the lower efficiency of occupancies in the offices). Further, it is assumed that both the initial cost $C_0(\xi)$ and the fuzzy probability of performance failure $\pi(\xi)$ are dependent on a decision parameter ξ (for example on the mass per unit length of a floor component) while the cost of full damage C_D is independent of ξ. If $C_0(\xi)$ is proportional to the decision parameter ξ, and the load effect S is proportional to a power ξ^{-k} $(k\geq 1)$, then the optimum ratio $C_D/C_0(\xi)$ may be expressed as

$$C_D / C_0(\xi) = \left(k \frac{\partial \pi(\xi)}{\partial \mu_S(\xi)} \mu_S(\xi) + (k+1) \frac{\partial \pi(\xi)}{\partial \sigma_S(\xi)} \sigma_S(\xi) \right)^{-1} \quad (5)$$

where the quantities $C_0(\xi)$, $\mu_S(\xi)$, $\sigma_S(\xi)$ are dependent on the decision parameter ξ. Partial derivatives of the fuzzy probability of failure π in equation (5) are to be determined using equation (3) and numerical methods of integration and derivation. Previous optimisation studies of various structural aspects indicate that commonly used performance requirements including the deformation and acceleration constraints may be uneconomical (Holický 1996 and 2001).

5. Concluding remarks

(1) Performance requirements on structural behaviour are generally affected by two types of uncertainty: randomness and vagueness due to indistinct or imprecise definitions and perception.
(2) The newly developed fuzzy probabilistic concepts provide valuable measures enabling the reliability analysis and optimisation of structural performance.
(4) Previous optimisation studies indicate that commonly used performance criteria for serviceability constraints concerning deflection and continuous vibration may be uneconomical.
(5) Further development and practical applications of the fuzzy probabilistic concepts require appropriate experimental data enabling an adequate specification of initial theoretical models.

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